Some Review of Signals and Systems

Lecture #1

1.1 – 1.3

What Is this Course All About ?

- To Gain an Appreciation of the Various Types of Signals and Systems
- To Analyze The Various Types of Systems
- To Learn the Skills and Tools needed to Perform These Analyses.

What are Signals?

- A Signal is a term used to denote the information carrying property being transmitted to or from an entity such as a device, instrument, or physiological source
- Examples:
  - Radio and Television Signals
  - Telecommunications and Computer Signals
  - Biomedical Engineering Signals

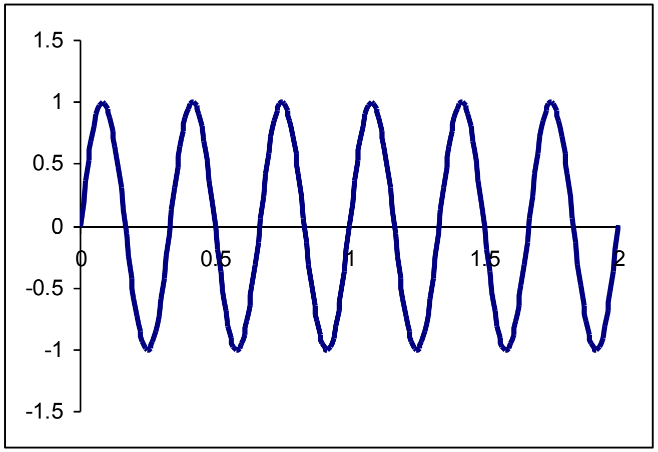
What is a System?

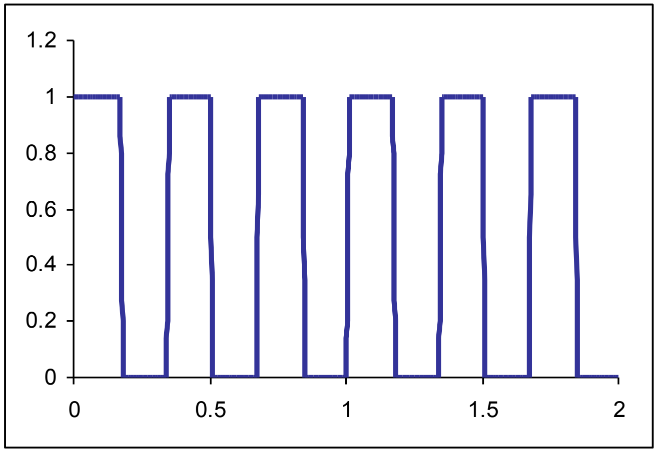
- A System is a term used to denote an entity that processes a Signal
- A System has inputs and outputs
- Examples
  - Amplifiers, Radios, Televisions
  - Telephone, Modem, Computer
  - Oscilloscopes, EKG, EEG, EMG

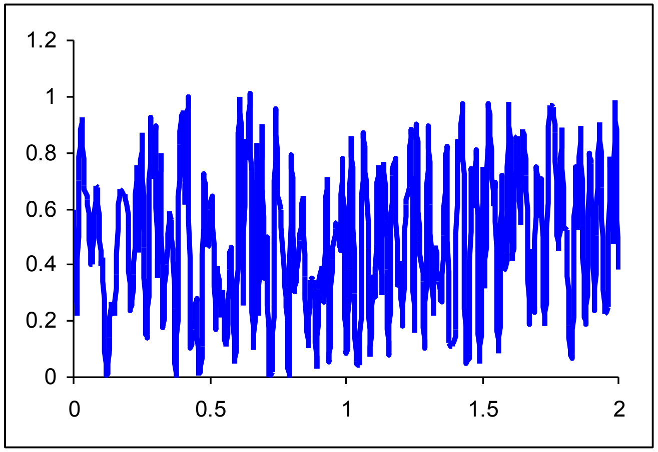
How do we describe Signals?

- Signals are associated with an independent variable(s): e.g., time, single or multivariate spatial coordinate
  - Most instrumentation signals have time as their independent variable
  - A digital photograph or image has spatial coordinates as its independent variables
- Signal Independent Variables can be either Continuous or Discrete

Continuous-Time Signals

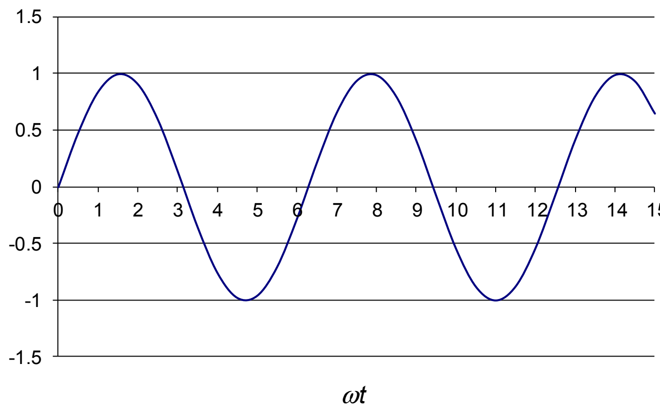


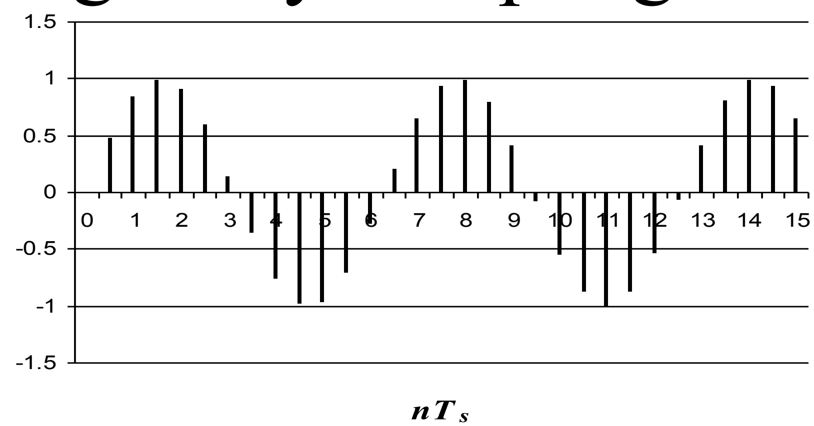




Discrete-Time Signals

A Discrete-Time Signal can be obtained from a Continuous-Time signal by Sampling.





Continuous-Time Signal

$$x(t) = \sin(\omega t)$$

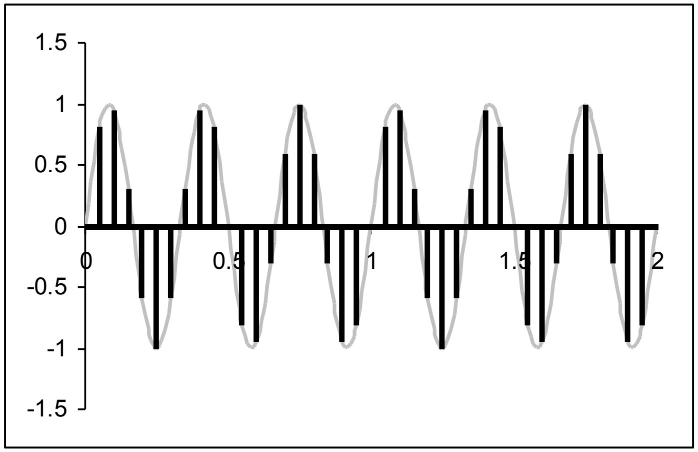
Discrete-Time Signal

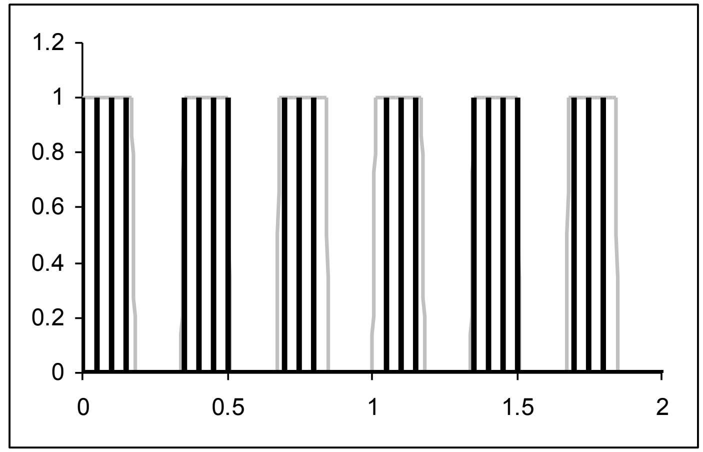
$$x(t \Rightarrow nT_s) \Rightarrow x(nT_s) = x[n] = \sin\ (\omega n T_s)$$

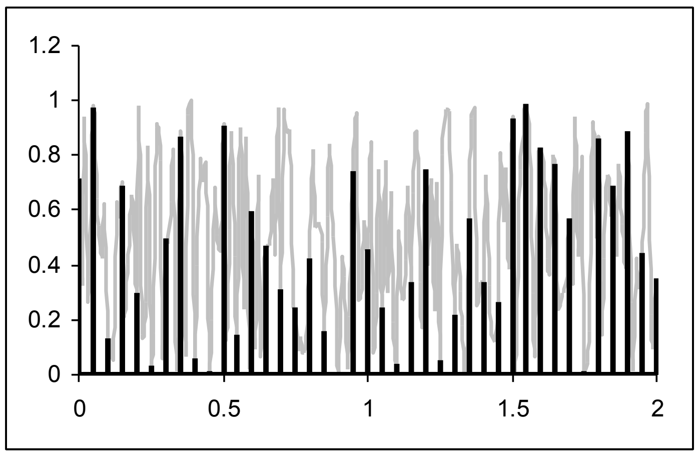
where n is an integer: $N_1 < n < N_2$

and T_s is the sampling period

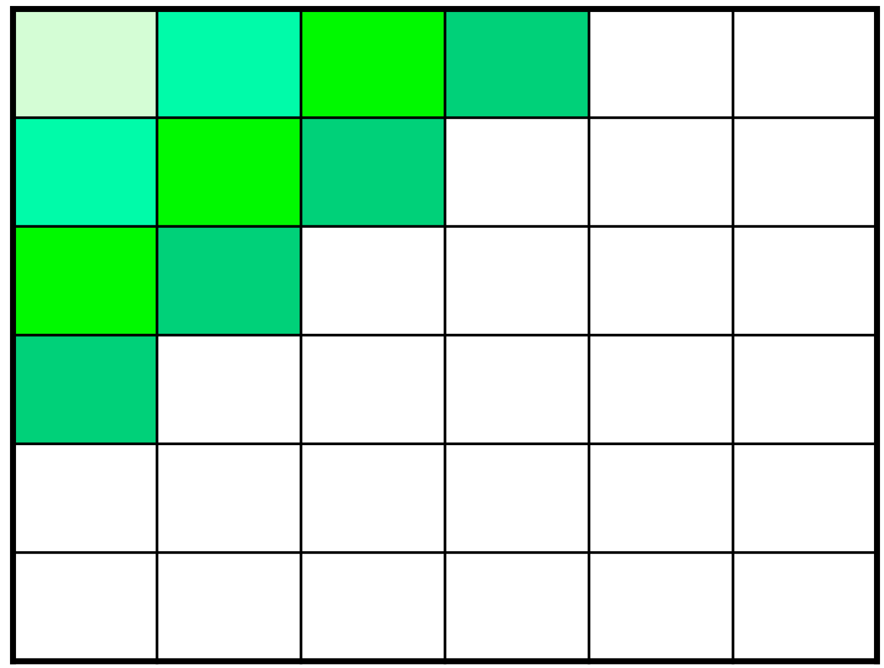
Discrete-Time Signals







Discrete Spatial Signal



This image consists of 200 x 158 pixels where each pixel can take on a value representing the color displayed in the form of [r,g,b].

Signals have Properties

- Take on Real or Complex values
- Periodic or non-periodic
- Symmetries
- Bounded or Unbounded

Complex Signals

- Continuous Signals have to be solutions of differential equations they can be in the form:

$$x(t) = (A_1 + B_1 t + \cdots)e^{s_1 t} + (A_2 + B_2 t + \cdots)e^{s_2 t} + \cdots$$

- Discrete Signals have to be solutions of difference equations they can be in the form:

$$x[n] = (A_1 + B_1 n + \cdots)z_1^n + (A_2 + B_2 n + \cdots)z_2^n + \cdots$$

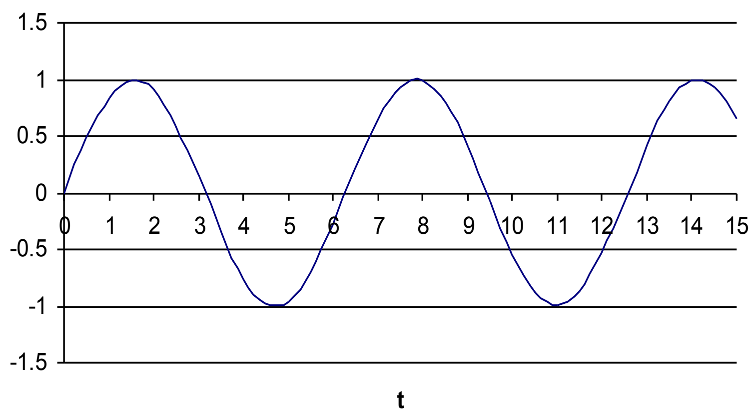
where A_i, B_i, etc., s_i and z_i can be complex numbers with real and imaginary parts.

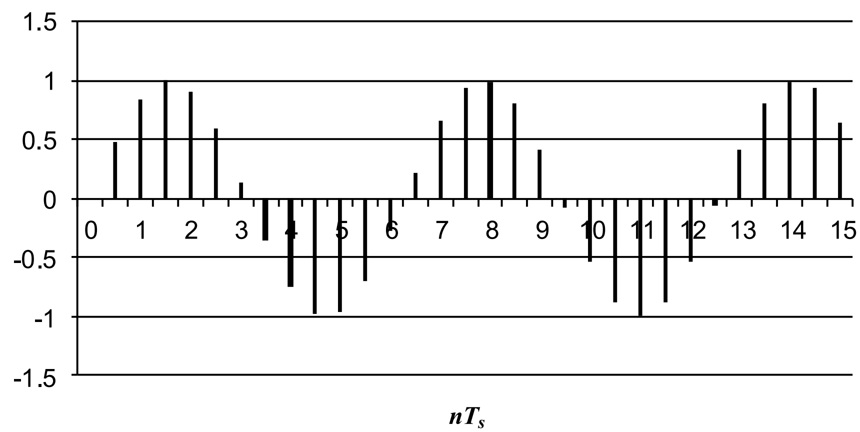
Periodic or non-periodic

- Periodic signals are those which satisfy

$$x(t+T) = x(t) \quad \text{for all t}$$

and T is called the Period.





Sinusoidal Continuous Signals

- Sinusoidal Signals are **<u>periodic</u>** functions which are based on the sine or cosine function from trigonometry.
- The general form of a Sinusoidal Signal

$$x(t)=A\cos(\omega_o t+\phi)$$

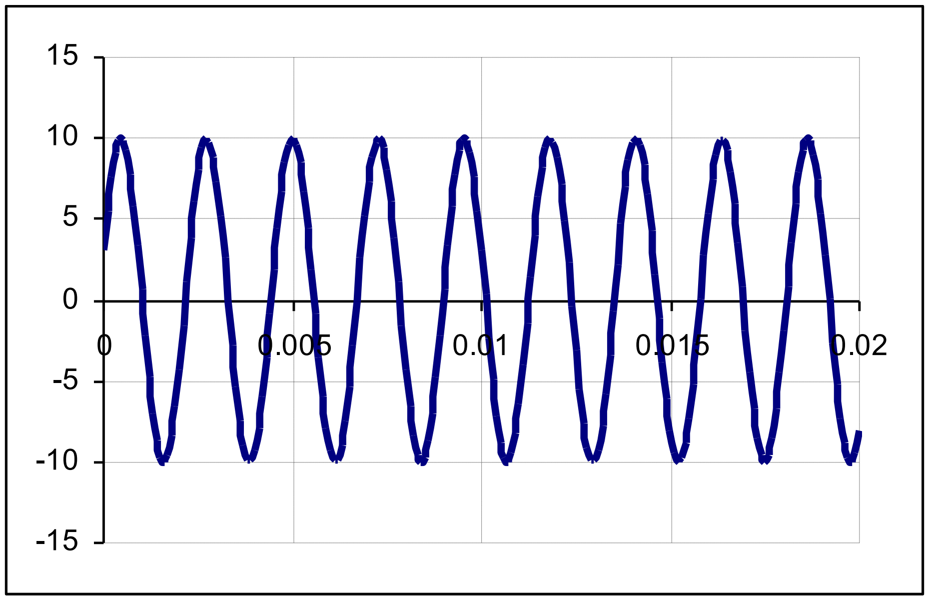
Or

$$x(t)=A\cos(2\pi f_o t+\phi)$$

  - where cos (·) represent the cosine function
    - We can also use sin(·), the sine function
  - $\omega_o t+\phi$ or $2\pi f_o t+\phi$ is angle (in radians) of the cosine function
    - Since the angle depends on time, it makes $x(t)$ a signal
  - ω_o is the radian frequency of the sinusoidal signal
    - f_o is called the cyclical frequency of the sinusoidal signal
  - ϕ is the phase shift or phase angle
  - A is the amplitude of the signal

Example

$x(t)$=10 cos(2π(440)t -0.4π)

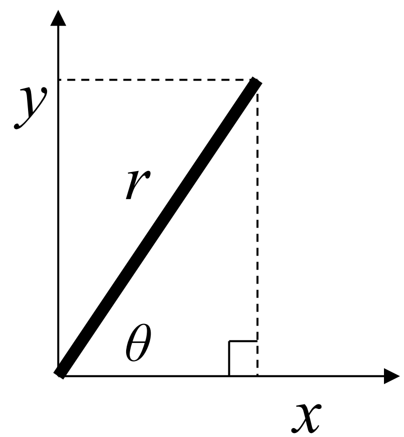


One cycle takes 1/440 = .00227 seconds

This is called the period, T, of the sinusoid and is equal to the inverse of the frequency, f

Sine and Cosine Functions

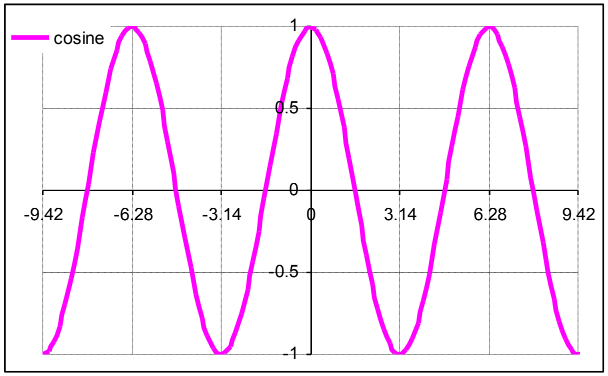
- Definition of sine and cosine

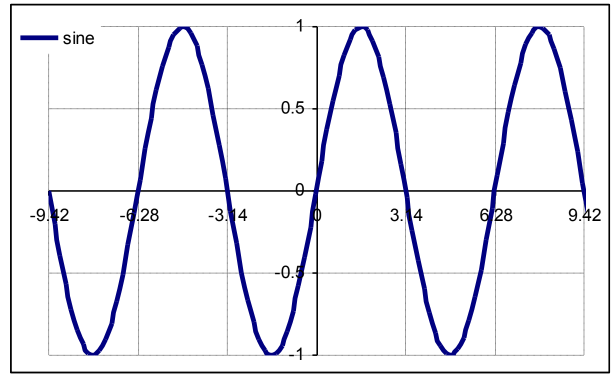


$$\sin\theta = \frac{y}{r}$$
$$\Rightarrow y = r\sin\theta$$
$$\cos\theta = \frac{x}{r}$$
$$\Rightarrow x = r\cos\theta$$

- Depending upon the quadrant of θ the sine and cosine function changes
  - As the θ increases from 0 to $\pi/2$, the cosine decreases from 1 to 0 and the sine increases from 0 to 1
  - As the θ increases beyond $\pi/2$ to π, the cosine decreases from 0 to -1 and the sine decreases from 1 to 0
  - As the θ increases beyond π to $3\pi/2$, the cosine increases from -1 to 0 and the sine decreases from 0 to -1
  - As the θ increases beyond $3\pi/2$ to 2π, the cosine increases from 0 to 1 and the sine increases from -1 to 0

Properties of Sinusoids

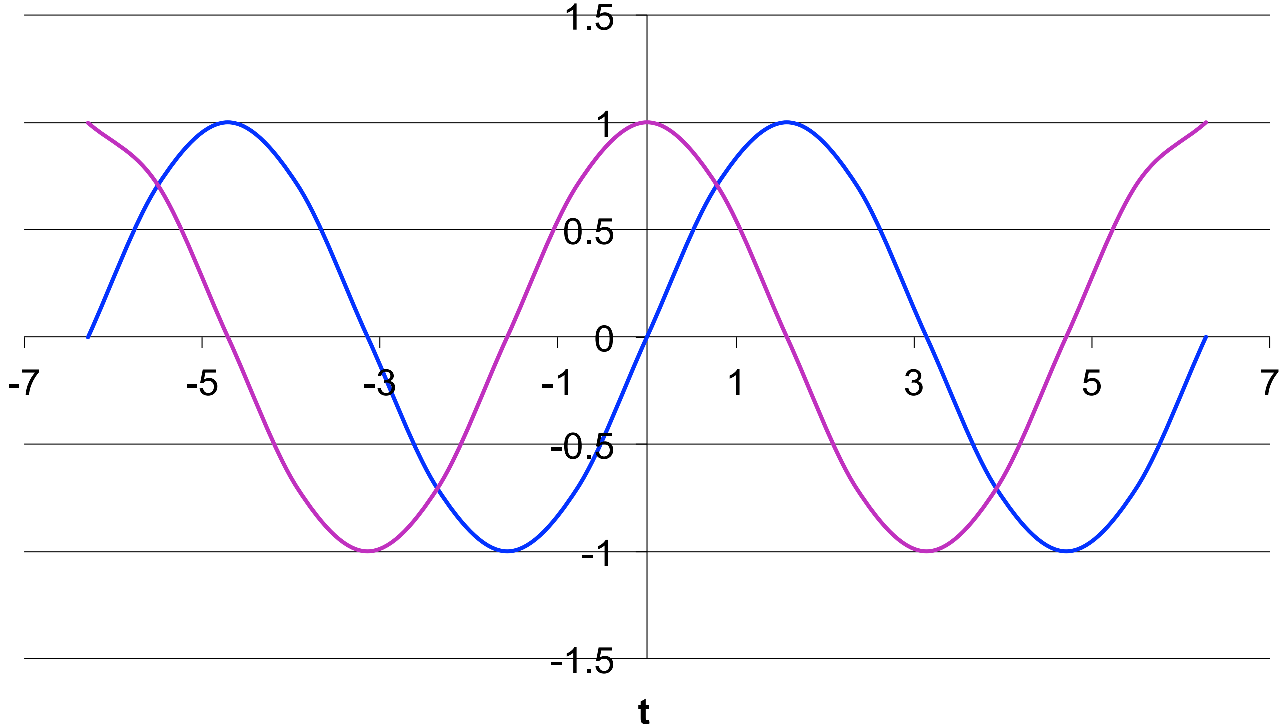




Property	Equation
Equivalence	$\sin\theta = \cos(\theta - \pi/2)$ or $\cos\theta = \sin(\theta + \pi/2)$
Periodicity	$\cos(\theta + 2\pi k) = \cos\theta$ or $\sin(\theta + 2\pi k) = \sin\theta$ where k is an integer
Evenness of cosine	$\cos\theta = \cos(-\theta)$
Oddness of sine	$\sin\theta = -\sin(-\theta)$
Zeros of sine	$\sin \pi k = 0$, when k is an integer
Zeros of cosine	$\cos[\pi(k+1)/2] = 0$, when k is an even integer; odd multiples of $\pi/2$
Ones of the cosine	$\cos 2\pi k = 1$, when k is an integer; even multiples of π
Ones of the sine	$\sin[\pi(k+1/2)] = 1$, when k is an even integer; alternate odd multiples of $\pi/2$
Negative ones of the cosine	$\cos[2\pi(k+1)/2] = -1$, when k is an integer; odd multiples of π
Negative ones of the sine	$\sin[\pi(k+1/2)] = -1$, when k is an odd integer; alternate odd multiples of $3\pi/2$

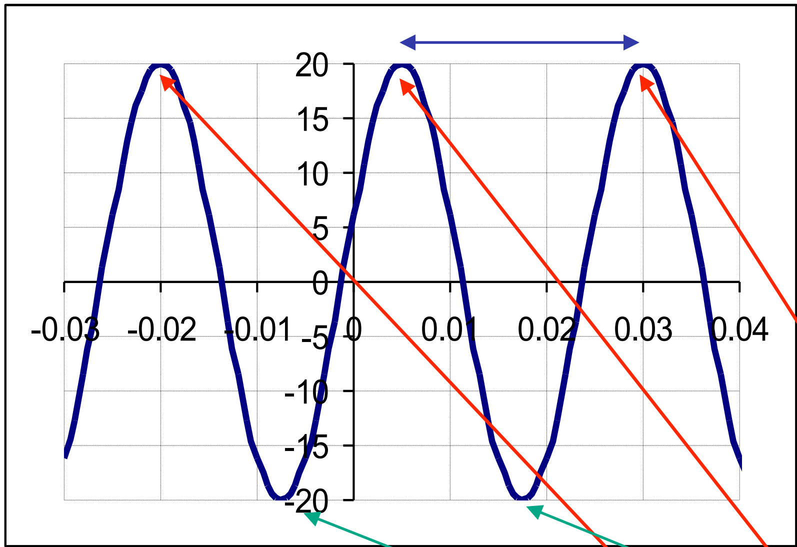
Signal Symmetries

- Even Signals are defined as $x_e(t) = x_e(-t)$
- Odd Signals are defined as $x_o(t) = -x_o(-t)$



Sinusoidal Signals

$x(t)=20\cos(2\pi(40)t - 0.4\pi)$



$A = 20$, $\omega_o = 2\pi(40)$, $f_o = 40$, $\theta = -0.4\pi$

Maxima at $2\pi(40)t-0.4\pi = 2\pi k$ *or when* $t = \ldots, -0.02, 0.005, 0.03, \ldots$

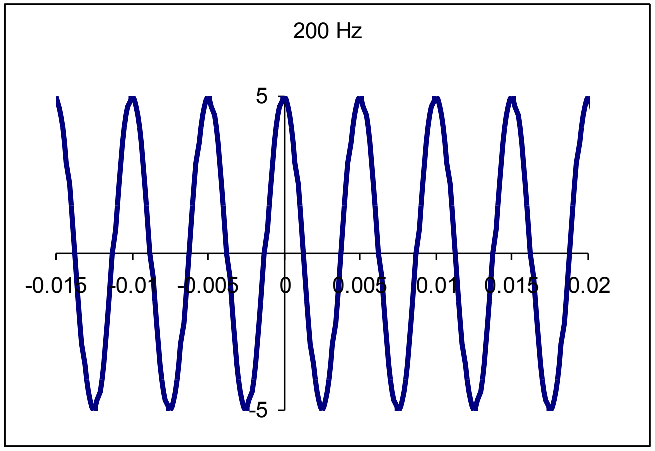
Minima at $2\pi(40)t-0.4\pi = 2\pi(k+1)0.5$ *or when* $t = \ldots -0.0075, 0.0175$

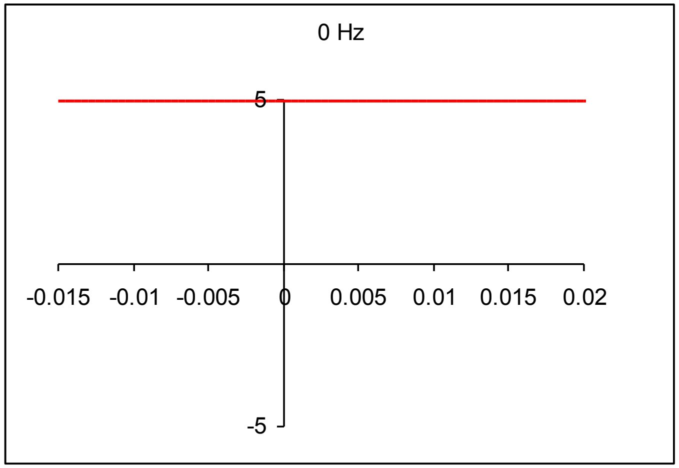
Time Period ($1/f_o$) between = *0.005*- (*-0.02*) =*0.025 sec*

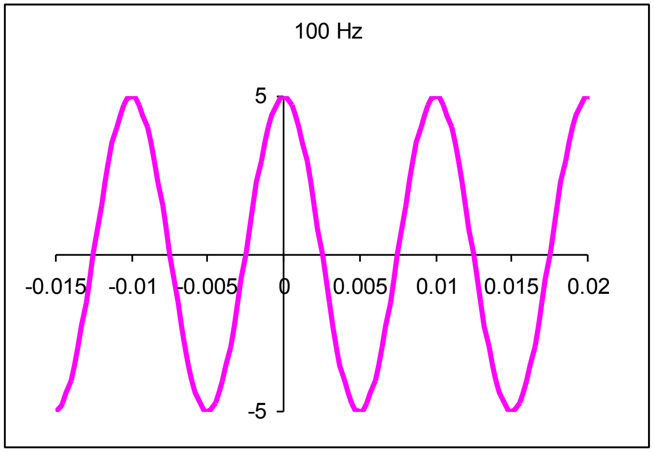
Frequencies

$$A\cos(2\pi f_o t + \theta)$$

for 200 Hz, 100 Hz, 0 Hz,







Relation of Period to Frequency

- Period of a sinusoid, T_o, is the length of one cycle and

$$T_o = 1/f_o$$

- The following relationship must be true for all Signals which are periodic (not just sinusoids)

$$x(t + T_o) = x(t)$$

- So

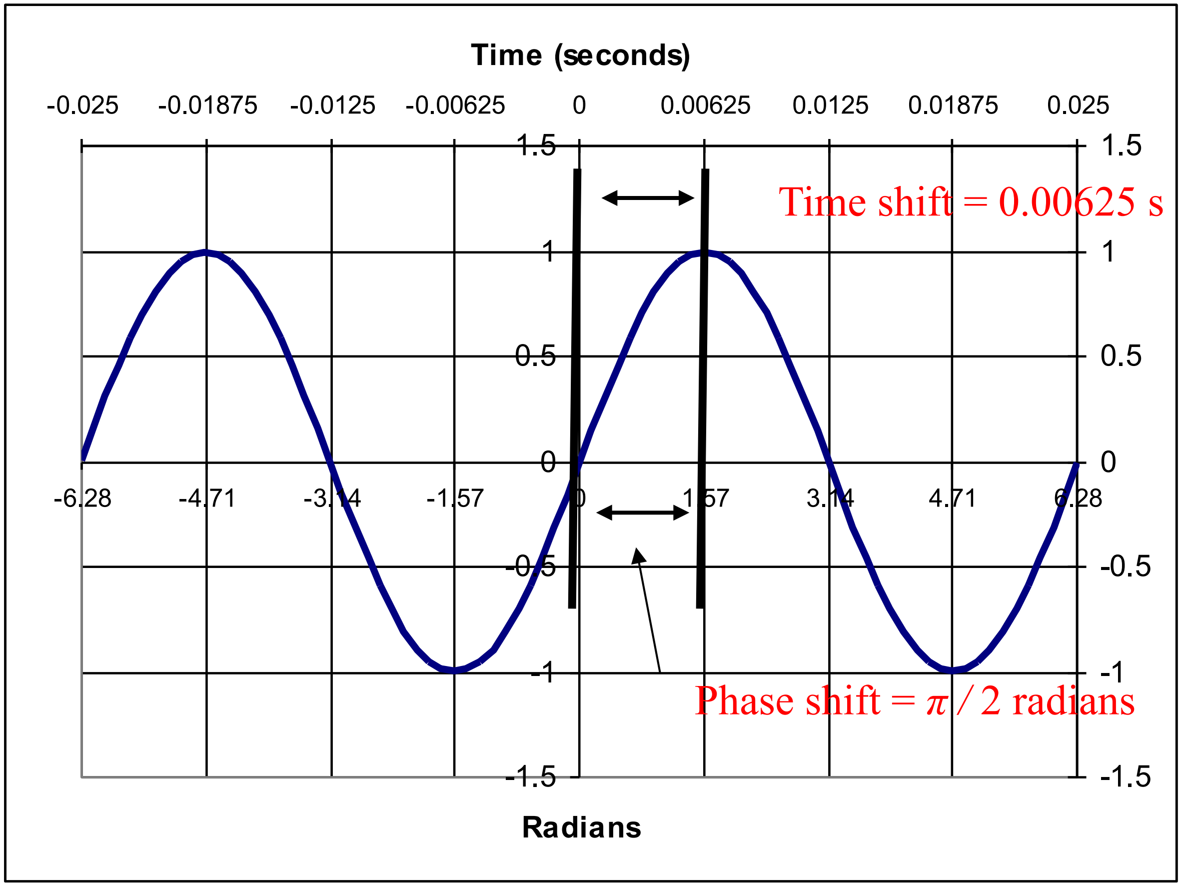
$$A\cos(\omega_o(t + T_o) + \theta) = A\cos(\omega_o t + \omega_o T_o + \theta)$$

$$A\cos(\omega_o t + \omega_o T_o + \theta) = A\cos(2\pi f_o t + 2\pi f_o T_o + \theta)$$

$$A\cos(2\pi f_o t + 2\pi f_o T_o + \theta) = A\cos(2\pi f_o t + 2\pi + \theta)$$

$$A\cos(2\pi f_o t + 2\pi + \theta) = A\cos(2\pi f_o t + \theta) = A\cos(\omega_o t + \theta)$$

Phase shift and Time Shift



$x(t) = \cos(2\pi 40t - \frac{\pi}{2})$

$f = 40Hz;$

$T = \frac{1}{40} = 0.025 \text{ sec}$

phase shift:

$\theta = -\frac{\pi}{2}$

time shift:

$t_s = -\frac{-\frac{\pi}{2}}{2\pi 40} = \frac{1}{160} = 0.00625 \text{ sec}$

$x(t) = \cos(2\pi 40(t - 0.00625))$

Phase Shift and Time Shift

- The phase shift parameter θ (with frequency) determines the time locations of the maxima and minima of the sinusoid.
- When $\theta = 0$, then for positive peak at $t = 0$.
- When $\theta \neq 0$, then the phase shift determines how much the maximum is shifted from $t = 0$.
- However, delaying a signal by t_1 seconds, also shifts its waveform.

$$x(t-t_1) = A\cos(\omega_o(t-t_1)) = A\cos(\omega_o t-\omega_o t_1)$$
$$\omega_o t-\omega_o t_1 = \omega_o t + \theta$$
$$-\omega_o t_1 = \theta$$
$$t_1 = -\theta / \omega_o = -\theta / 2\pi f_o$$

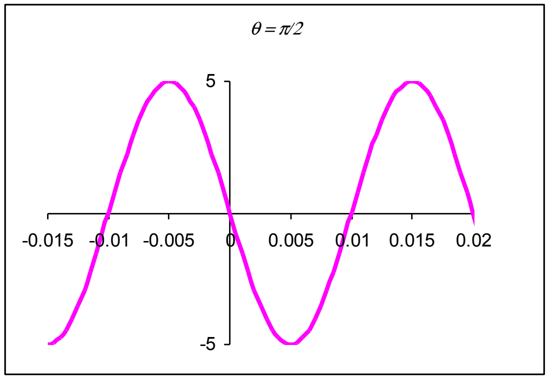
$$\theta = -2\pi f_o t_1 = -2\pi(t_1 / T_o)$$

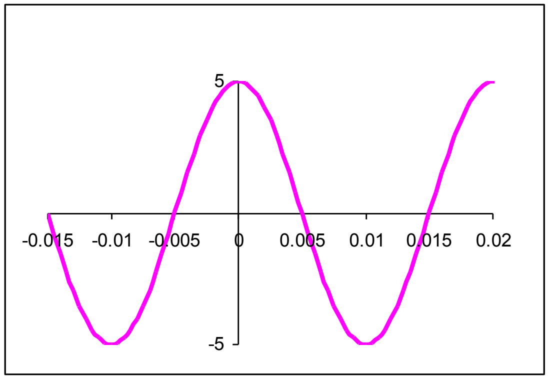
- Note that a positive (negative) value of t_1 equates to a delay (advance)
- And a a positive (negative) value of θ equates to an advance (delay)

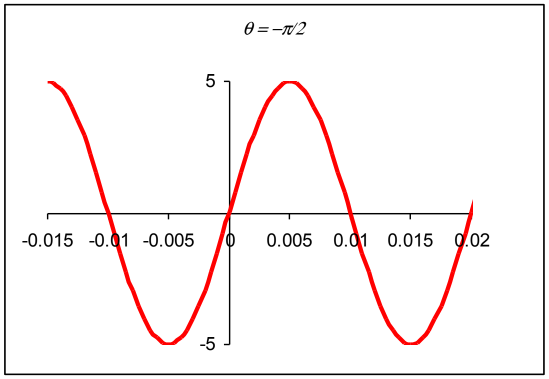
Phase and Time Shift

- Note that a positive (negative) value of t_1 equates to a delay (advance)
- And a a positive (negative) value of θ equates to an advance (delay)

$$x(t) = 5\cos(2\pi\, 50t + \theta)$$
$$\theta = \pi / 2; -\pi / 2$$
$$t_1 = -\pi / 2 / (2\pi\ 50) = -.005 \text{ sec};\ +.005 \text{ sec}$$





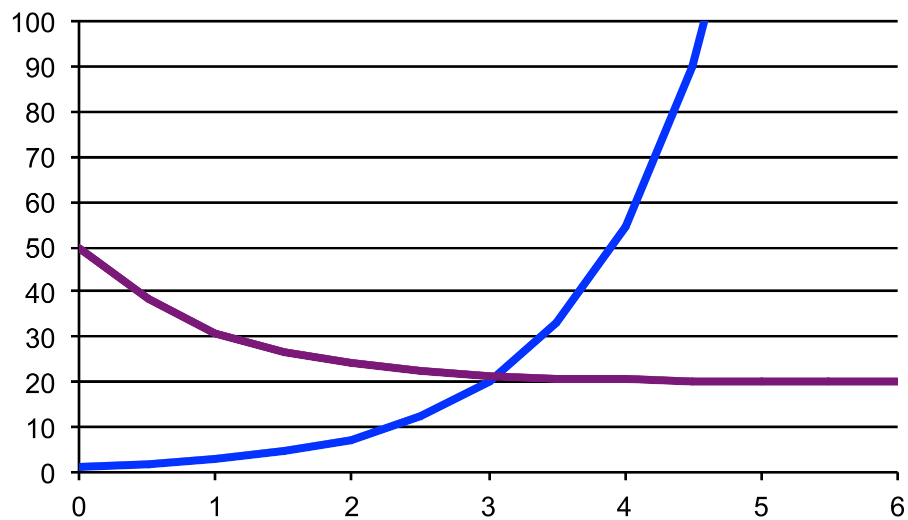


Identities and Derivatives

Number	Equation
1	$\sin^2\theta + \cos^2\theta = 1$
2	$\cos 2\theta = \cos^2\theta - \sin^2\theta$
3	$\sin 2\theta = 2 \sin \theta \cos \theta$
4	$\sin (a \pm b) = \sin a \cos b \pm \cos a \sin b$
5	$\cos (a \pm b) = \cos a \cos b \quad \sin a \sin b$
6	$\cos a \cos b = [\cos (a + b) + \cos (a - b)]/2$
7	$\sin a \sin b = [\cos (a - b) - \cos (a + b)]/2$
8	$\cos^2\theta = [1 + \cos 2\theta]/2$
9	$\sin^2\theta = [1 - \cos 2\theta]/2$
10	$d \sin \theta / d\theta = \cos \theta$
11	$d \cos \theta / d\theta = -\sin \theta$

Bounded or Unbounded

- For Bounded Signals $\int |f(t)| dt$ approaches a constant value as $t \rightarrow \pm\infty$
- Unbounded Signals approach infinity as $t \rightarrow \pm\infty$



Euler's Formula

Basic Formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$

(Note that: $e^{\theta} \neq \cos\theta + j\sin\theta$)

Also:

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

And

$$e^{j(\omega t+\phi)} = \cos(\omega t+\phi) + j\sin(\omega t+\phi)$$

And one more where $s = \alpha + j\omega$

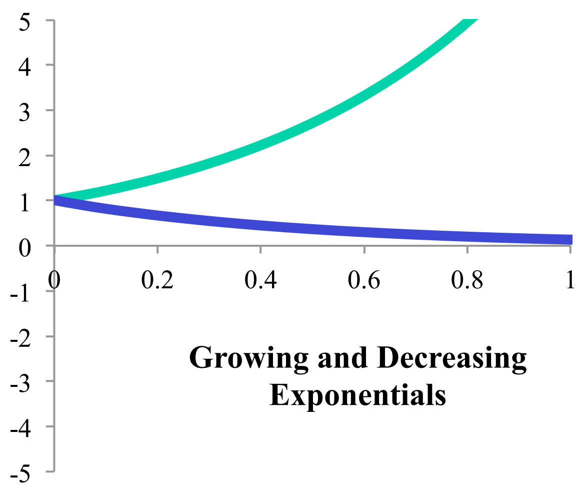
$$e^{st} = e^{(\alpha+j\omega)t} = e^{\alpha t}\cos(\omega t+\phi) + je^{\alpha t}\sin(\omega t+\phi)$$

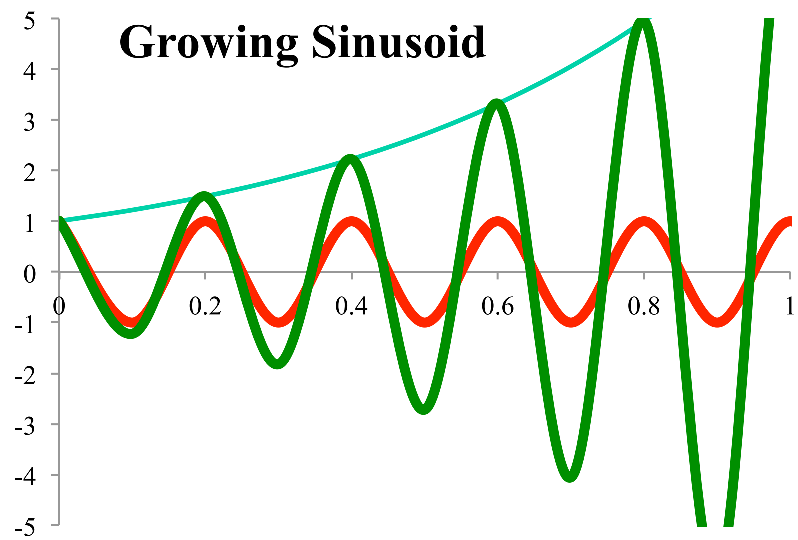
More on Complex Signals

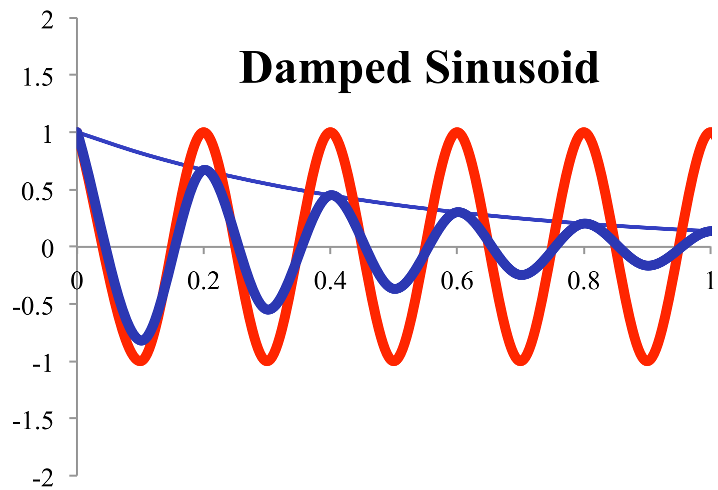
- Let's assume that $x(t)=Ae^{st}$ for all t, A is a real constant and s is complex and is given as $s = \alpha + j\omega$
- If $s = \alpha$ is real then $x(t)$ is a real exponential function $x(t)=Ae^{\alpha t}$
- If $s = j\omega$ is imaginary and using Euler's formula then $x(t)$ is a sinusoidal function $x(t)=Ae^{j\omega t} = A(\cos\omega t+j\sin\omega t)$ [Book uses, F: $\omega=2\pi F$]
- If s is complex then $x(t)$ is called a damped sinusoidal function for $s < 0$ and is of the form

$$x(t)=Ae^{st} = Ae^{\alpha t}(\cos\omega t + j\sin\omega t)$$

More on Complex Signals







Homework

1. Continuous and Discrete Signals Use Matlab to plot the signals; submit your code
   1. $f(t) = 1- e^{-t}$ is a continuous signal. Draw its waveform.
   2. Draw the discrete version of $f(t)$ for $T=0.25$.
2. Periodic Signals
   1. Show that *tan t* is periodic. What is its period?
   2. Is e^{-t} periodic? Why not?
   3. Is $e^{-t} \sin(t)$ periodic? Describe?
3. Bounded Signals
   1. Prove that $f(t) = e^{-t}$ is bounded for $t > 0$.
   2. What about $f(t) = e^{-t}$ for all t.
4. Biosignals
   1. For a typical EEG, EKG, and EMG signal, is the signal periodic? If so, what is it's period.
   2. For a typical EEG, EKG, and EMG signal, is the signal bounded? If so, describe why.

Homework cont'd

5. Symmetry
   1. Is *cos t* even or odd? *sin t*? *tan t*?
   2. What about *cos t x sin t* ? *tan t x cos t* ?
   3. What is the symmetry of the product of:
      1. Two even functions
      2. Two odd Functions
      3. Even and Odd function
6. CT.1.2.1,CT.1.2,3
7. DT.1.2.1,DT.1.2.3