Spectrum of Biomedical Signals

Lecture 8a

How can we use Fourier Series to calculate a biomedical signal.

- Fourier Series requires that the signal is periodic
- What biomedical signal is periodic?
- Can we take a non-periodic signal and make it periodic?

Assumptions

- Let's take a ECG as an example
- If we know what the range of frequencies of our non-periodic signal
  - 50 beats per minutes
  - 200 beats per minutes maximum for a 20 year old
- Estimate the spectrum using these facts.

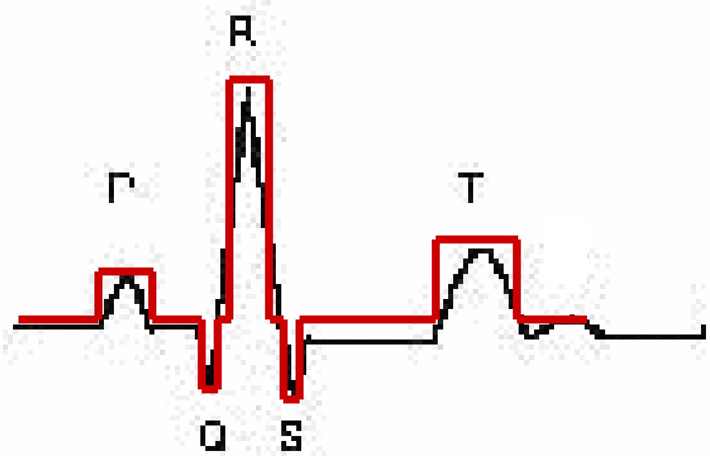
Steps to Estimating ECG Spectrum

1. Model the ECG as a periodic signal with fundamental frequency f_o
2. Calculate the Fourier coefficients, a_k, of the ECG.
3. Plot the spectrum of the ECG signal, (i.e., a_k) as a function of k and, therefore, frequency, $f = kf_o$
4. Note how the spectrum changes as f_o varies between its maximum and its minimum

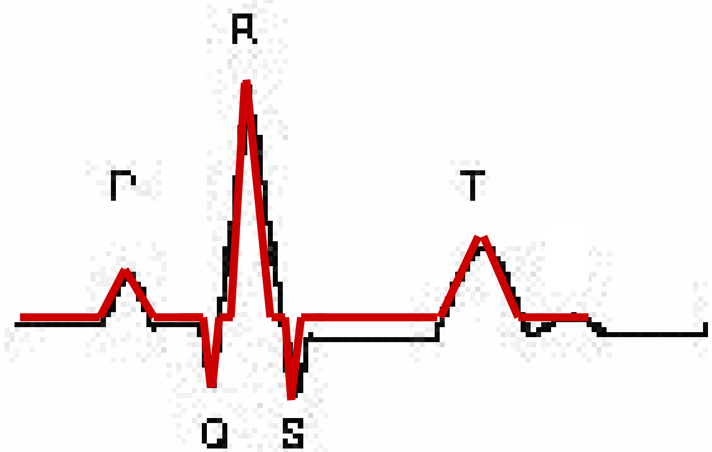
How do we do step 1

- Step 1: Model the ECG as a periodic signal with fundamental frequency f_o
- Model the ECG as a set of simple signals that we already know
  - Square Wave Model
  - Triangular Wave Model
  - Quadratic Model
  - Mixture Model

ECG Modeling Examples



Square Wave Modeling



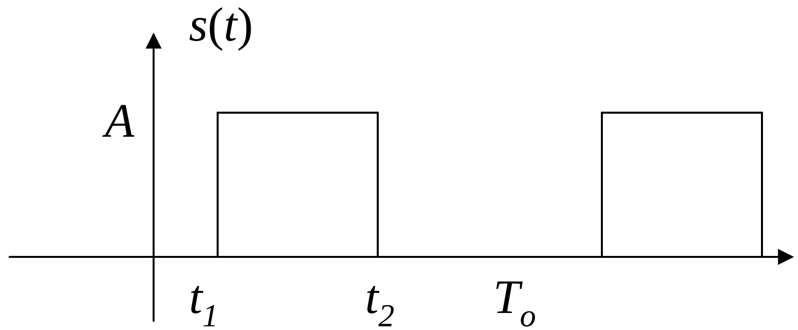
Triangle Wave Modeling

- Which is a better model?
- Which is easier to calculate?

How do we do step 2

- Calculate the Fourier coefficients, a_k, of the ECG.
- Calculate a_k for the general model
- Duplicate this calculation for each component of the ECG
- Use superposition for all of the model components

Fourier Series of a ECG Using a Square Wave Model



$$s(t) = \begin{cases} A & \text{for } t_1 \le t \le t_2 \\ 0 & \text{elsewhere} \end{cases}$$

Fourier Series of a ECG Using a Square Wave Model

$$a_k = \frac{1}{T_o}\int_0^{T_o} s(t)e^{-j(\frac{2\pi}{T_o})kt}dt$$

By superposition, we have for each wave

$$a_k = a_k^P + a_k^Q + a_k^R + a_k^S + a_k^T$$

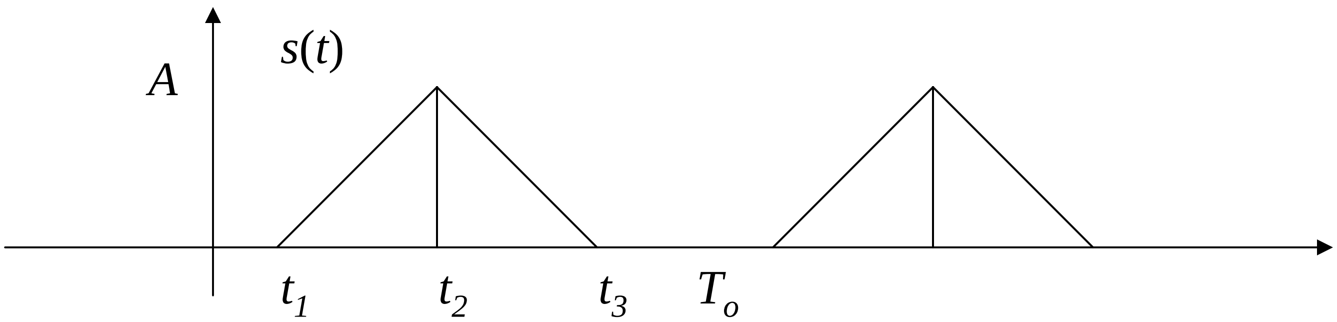
$$\Re e[a_k] = \Re e[a_k^P] + \Re e[a_k^Q] + \Re e[a_k^R] + \Re e[a_k^S] + \Re e[a_k^T]$$

$$\Im m[a_k] = \Im m[a_k^P] + \Im m[a_k^Q] + \Im m[a_k^R] + \Im m[a_k^S] + \Im m[a_k^T]$$

$$a_k = \sqrt{\Re e[a_k]^2 + \Im m[a_k]^2}\, e^{j\tan^{-1}(\frac{\Im m[a_k]}{\Re e[a_k]})}$$

where for the *Mth* wave

Fourier Series of a ECG Using a Triangular Wave Model



$$s(t)\begin{cases} = A\left(\dfrac{t-t_1}{t_2-t_1}\right) & \text{for}\ t_1 \le t \le t_2 \\ = A\left(\dfrac{t_3-t}{t_3-t_2}\right) & \text{for}\ t_2 \le t \le t_3 \\ = 0 & \text{elsewhere} \end{cases}$$

Fourier Series of a ECG Using a Triangular Wave Model

$$a_k = \frac{1}{T_o}\int_0^{T_o} s(t)e^{-j(\frac{2\pi}{T_o})kt}dt = \frac{1}{T_o}\int_{t_1}^{t_2} A(\frac{t\text{-}t_1}{t_2\text{-}t_1})e^{-j(\frac{2\pi}{T_o})kt}dt + \frac{1}{T_o}\int_{t_2}^{t_3} A(\frac{t_3\text{-}t}{t_3\text{-}t_2})e^{-j(\frac{2\pi}{T_o})kt}dt$$

$$= \frac{A}{T_o}[\int_{t_1}^{t_2}(\frac{t\text{-}t_1}{t_2\text{-}t_1})e^{-j(\frac{2\pi}{T_o})kt}dt + \int_{t_2}^{t_3}(\frac{t_3\text{-}t}{t_3\text{-}t_2})e^{-j(\frac{2\pi}{T_o})kt}dt]$$

$$= \frac{A}{T_o}[\int_{t_1}^{t_2}(\frac{t\text{-}t_1}{t_2\text{-}t_1})e^{-j(\frac{2\pi}{T_o})kt}dt - \int_{t_2}^{t_3}(\frac{t\text{-}t_3}{t_3\text{-}t_2})e^{-j(\frac{2\pi}{T_o})kt}dt]$$

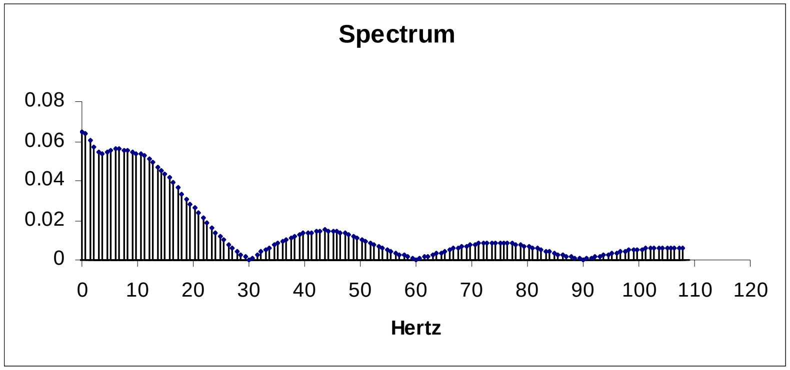
After 4 pages of calculations, I get:

$$a_k = \frac{A}{2\pi k}[e^{-j[(\frac{2\pi}{T_o})k\frac{(t_2+t_1)}{2}+\frac{\pi}{2}]}\text{sinc}[(\frac{2\pi}{T_o})k(\frac{t_2-t_1}{2})] + e^{-j[(\frac{2\pi}{T_o})k(\frac{t_3+t_2}{2})+\frac{\pi}{2}]}\text{sinc}[(\frac{2\pi}{T_o})k(\frac{t_3-t_2}{2})]]$$

Fourier Series of a ECG Using a Square Wave Model

Rate	80	1.333333

	Amplitude	Start	Width	End
Pwave	0.025	0	0.1	0.1
Qwave	-0.1	0.1	0.033333	0.133333
Rwave	2.5	0.133333	0.033333	0.166667
Swave	-0.1	0.166667	0.033333	0.2
Twave	0.05	0.2	0.1	0.35



Homework

- Calculate the “periodic” ECG model Fourier coefficients for a square wave model
- Use superposition to get the Fourier coefficients for the complete ECG.