

Z Transforms

Lesson 20

6DT

Homework

- Problems: 2.16-17,2-19,2-25

Homework Answers #1

- Problems: 2.16

$$y[n] - \frac{1}{3}y[n-1] = x[n] = u[n]$$

$$Y(z) - \frac{1}{3}z^{-1}Y(z) = X(z) = \frac{z}{z-1}$$

$$(1 - \frac{1}{3}z^{-1})Y(z) = \frac{z}{z-1}$$

$$(z - \frac{1}{3})Y(z) = \frac{z^2}{z-1}$$

$$Y(z) = \frac{z^2}{(z-1)(z-\frac{1}{3})}$$

$$\frac{Y(z)}{z} = \frac{z}{(z-1)(z-\frac{1}{3})} = \frac{K_1}{(z-1)} + \frac{K_2}{(z-\frac{1}{3})}$$

$$K_1 = \frac{z}{(z-\frac{1}{3})} \Big|_{z=1} = \frac{3}{2}$$

$$K_2 = \frac{z}{(z-1)} \Big|_{z=\frac{1}{3}} = \frac{1/3}{-2/3} = -\frac{1}{2}$$

$$Y(z) = \frac{3/2 z}{(z-1)} - \frac{1/2 z}{(z-\frac{1}{3})}$$

$$y[n] = 3/2 u[n] - 1/2 \left(\frac{1}{3}\right)^n u[n]$$

Homework Answers #2

- Problems: 2.17

$$X(z) = \frac{3 - \frac{5}{6}z^{-1}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z^{-1})}$$

$$= \frac{z(3z - \frac{5}{6})}{(z - \frac{1}{4})(z - \frac{1}{3})}$$

$$\frac{X(z)}{z} = \frac{(3z - \frac{5}{6})}{(z - \frac{1}{4})(z - \frac{1}{3})} = \frac{K_1}{(z - \frac{1}{4})} + \frac{K_2}{(z - \frac{1}{3})}$$

$$K_1 = \left. \frac{(3z - \frac{5}{6})}{(z - \frac{1}{3})} \right|_{z=\frac{1}{4}} = \frac{(3\frac{1}{4} - \frac{5}{6})}{(\frac{1}{4} - \frac{1}{3})} = \frac{(\frac{18}{24} - \frac{20}{24})}{(\frac{3}{12} - \frac{4}{12})} = \frac{-\frac{2}{24}}{-\frac{1}{12}} = 1$$

$$K_2 = \left. \frac{(3z - \frac{5}{6})}{(z - \frac{1}{4})} \right|_{z=\frac{1}{3}} = \frac{(3\frac{1}{3} - \frac{5}{6})}{(\frac{1}{3} - \frac{1}{4})} = \frac{(1 - \frac{5}{6})}{(\frac{4}{12} - \frac{3}{12})} = \frac{\frac{1}{6}}{\frac{1}{12}} = 2$$

$$\frac{X(z)}{z} = \frac{1}{(z - \frac{1}{4})} + \frac{2}{(z - \frac{1}{3})}$$

$$X(z) = \frac{1z}{(z - \frac{1}{4})} + \frac{2z}{(z - \frac{1}{3})}$$

$$x[n] = \left(\frac{1}{4}\right)^n u[n] + 2\left(\frac{1}{3}\right)^n u[n]$$

Homework Answers #3

- Problems: 2-19

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[n]$$

$$Z\{x[n]\} = X(z) = \frac{z}{z - \frac{1}{2}} + \frac{z}{z - \frac{1}{3}}$$

$$X(z) = \frac{z(2z - \frac{5}{6})}{(z - \frac{1}{2})(z - \frac{1}{3})}$$

Homework Answers #4

- Problems: 2-25

$$H(z) = \frac{1}{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 - \frac{1}{6}z^{-1}\right)}$$

$$Y(z) = \frac{1}{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 - \frac{1}{6}z^{-1}\right)} X(z)$$

$$\left(1 - \frac{1}{3}z^{-1}\right)\left(1 - \frac{1}{6}z^{-1}\right)Y(z) = X(z)$$

$$\left(1 - \frac{1}{2}z^{-1} + \frac{1}{18}z^{-2}\right)Y(z) = X(z)$$

$$y[n] - \frac{1}{2}y[n-1] + \frac{1}{18}y[n-2] = x[n]$$

$x[n] = u[n]$ **Homework Answers #4**

$$X(z) = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

• Problems: 2-25

$$Y(z) = \frac{1}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{6}z^{-1})(1-z^{-1})} = \frac{z^3}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)}$$

$$\frac{Y(z)}{z} = \frac{z^2}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)} = \frac{K_1}{(z-\frac{1}{3})} + \frac{K_2}{(z-\frac{1}{6})} + \frac{K_3}{(z-1)}$$

$$K_1 = \frac{z^2}{(z-\frac{1}{6})(z-1)} \Big|_{z=1/3} = \frac{1/9}{(\frac{1}{3}-\frac{1}{6})(\frac{1}{3}-1)} = \frac{1/9}{(\frac{1}{6})(-\frac{2}{3})} = -\frac{1/9}{(-\frac{2}{18})} = -1$$

$$K_2 = \frac{z^2}{(z-\frac{1}{3})(z-1)} \Big|_{z=1/6} = \frac{1/36}{(\frac{1}{6}-\frac{1}{3})(\frac{1}{6}-1)} = \frac{1/36}{(-\frac{1}{6})(-\frac{5}{6})} = \frac{1}{5}$$

$$K_3 = \frac{z^2}{(z-\frac{1}{3})(z-\frac{1}{6})} \Big|_{z=1} = \frac{1}{(1-\frac{1}{3})(1-\frac{1}{6})} = \frac{1}{(\frac{2}{3})(\frac{5}{6})} = 1.8$$

Homework Answers #4

- Problems: 2-25

$$\frac{Y(z)}{z} = \frac{-1}{\left(z - \frac{1}{3}\right)} + \frac{1/5}{\left(z - \frac{1}{6}\right)} + \frac{9/5}{(z-1)}$$

$$Y(z) = \frac{-1z}{\left(z - \frac{1}{3}\right)} + \frac{1/5z}{\left(z - \frac{1}{6}\right)} + \frac{9/5z}{(z-1)}$$

$$y[n] = -1\left(\frac{1}{3}\right)^n u[n] + \frac{1}{5}\left(\frac{1}{6}\right)^n u[n] + \frac{9}{5}u[n]$$

Homework Answers #4

- Problems: 2-25

Check

$$\frac{Y(z)}{z} = \frac{-1}{(z-\frac{1}{3})} + \frac{1/5}{(z-\frac{1}{6})} + \frac{9/5}{(z-1)}$$

$$\frac{Y(z)}{z} = \frac{-1(z-\frac{1}{6})(z-1)}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)} + \frac{1/5(z-\frac{1}{3})(z-1)}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)} + \frac{9/5(z-\frac{1}{3})(z-\frac{1}{6})}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)}$$

$$= \frac{-1(z^2 - \frac{7}{6}z + \frac{1}{6})}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)} + \frac{1/5(z^2 - \frac{4}{3}z + \frac{1}{3})}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)} + \frac{9/5(z^2 - \frac{3}{6}z + \frac{1}{18})}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)}$$

$$= \frac{[-1+1/5+9/5]z^2 + [-1(-\frac{7}{6})+1/5(-\frac{4}{3})+9/5(-\frac{3}{6})]z + [-1(+\frac{1}{6})+1/5(+\frac{1}{3})+9/5(+\frac{1}{18})]}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)}$$

$$= \frac{[1]z^2 + [(\frac{35}{30}) + (-\frac{8}{30}) + (-\frac{27}{30})]z + [(-\frac{15}{90}) + (+\frac{6}{90}) + (+\frac{9}{90})]}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)} = \frac{z^2}{(z-\frac{1}{3})(z-\frac{1}{6})(z-1)}$$