BME 333 Biomedical Signals and Systems

Biomedical Signals and Systems Quiz #1

- There are 5 questions, you need to complete 4
- Questions 3, 4, and 5 are **mandatory**
- Choose 1 more

Biomedical Signals and Systems Quiz #1

1. a) Is $f(t) = (t+2)^{-1}$ bounded for $t > 0$; prove your point.

   b) Is $f(t)=(t+2)^{-100}$ bounded for $t > 0$; prove your point.

   c) Describe in **mathematical terms** the properties a system must exhibit to be considered LTI.

   d) Is $y(t) = (t+2)x(t+1)$ a linear system? Why?

2. a) Describe the components of the solution of an ordinary difference equation and what the terms transient and steady state mean.

   b) Describe the possible forms of the transient response of a 2nd order differential equation and the show the relationship which defines them. Describe the relationship of the parameters shown in the equation for $y(t)$.

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = x(t)$$

Biomedical Signals and Systems Quiz #1

3. For the following provide a written answer:
   a) Describe the impulse response for both continuous and discrete systems.
   b) Describe how the impulse response characterizes a system.
   c) How does the unit impulse **function** for continuous systems compare with the unit impulse **function** for discrete systems? Describe the properties of the continuous unit impulse function.

4. Derive the solution and its components for the following ODEs using the results of problem (2):

$$a)\ \dot{y} + 3y = 12; y(0) = 4$$

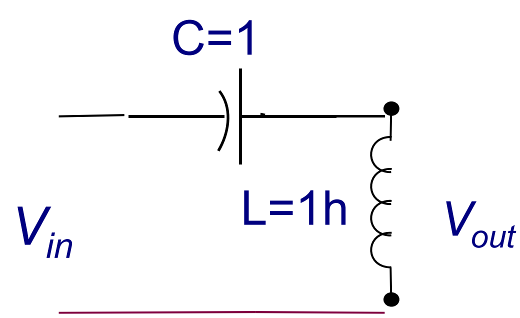
$$b)\ \dot{y} + 3y = 12e^{-3t}; y(0) = 4$$

$$c)\ \ddot{y} + 4\dot{y} + 3y = 9; y(0) = 0; \dot{y}(0) = 10$$

$$d)\ \ddot{y} + 2\dot{y} + y = e^{-10t}; y(0) = 10; \dot{y}(0) = 10$$

Biomedical Signals and Systems Quiz #1

5. Starting with the differential equation(s) and p operators, calculate the sinusoidal steady state response of the Transfer Function $V_{out}(j\omega)\ /V_{in}(j\omega)$ for this circuit and sketch the Bode plot. Show and calculate the important points on the plot



Biomedical Signals and Systems Quiz #1 Answers

1. a) Is $f(t) = (t+2)^{-1}$ bounded for $t > 0$; prove your point.

To be bounded $\int |f(t)| dt =$ constant

$$\int_0^\infty (t+2)^{-1} dt = \int_0^\infty \frac{1}{t+2} dt = \ln(t+2)\,|_0^\infty = \ln(\infty) - \ln(2) = \infty$$

Not bounded

b) Is $f(t)=(t+2)^{-100}$ bounded for $t > 0$; prove your point.

To be bounded $\int |f(t)| dt =$ constant

$$\int_0^\infty (t+2)^{-100} dt = \int_0^\infty \frac{1}{(t+2)^{100}} dt = -\frac{1}{99\left(t+2\right)^{99}}\,|_0^\infty = \frac{1}{99(2)^{99}} - \frac{1}{\infty} \to 6\times 10^{-60}$$

Bounded

Biomedical Signals and Systems
Quiz #1 Answers

c) Describe the 3 properties a system must exhibit to be consider linear and time invariant.

– All LTI systems must be

- Linear and support superposition

  $$x_k(t) \longrightarrow y_k(t)$$
  $$\Sigma_k \, a_k x_k(t) \longrightarrow \Sigma_k \, a_k y_k(t)$$

- Casual

  A system is causal if the output at any time depends **<u>only</u>** on the input values up to that time

  $y(t_o)$ does not depend on $x(t_i)$ that occur at times after t_o, $t_i > t_o$.

- Time Invariance

  $$x_k(t) \longrightarrow y_k(t)$$

  Delay x(t) by t_0 yields same response only later

  $$x_k(t\text{-}t_0) \longrightarrow y_k(t\text{-}t_0)$$

d) Is $y(t) = (t+2)x(t+1)$ a linear system? Prove your point.

$$y(t) = (t+2)x(t+1)$$

Superposition

$$x_1(t) \to (t+2)x_1(t+1) = y_1(t)$$
$$x_2(t) \to (t+2)x_2(t+1) = y_2(t)$$
$$x_3(t) = a_1x_1(t) + a_2x_2(t) \to (t+2)x_3(t+1) = (t+2)[a_1x_1(t+1) + a_2x_2(t+1)] =$$
$$(t+2)a_1x_1(t+1) + (t+2)a_2x_2(t+1) \stackrel{?}{=} a_1y_1(t) + a_2y_2(t) = a_1(t+2)x_1(t+1) + a_2(t+2)x_2(t+1)$$

Yes; superposition is supported

Time invariant

$$x(t) \to (t+2)x(t+1) = y(t)$$
$$x(t-t_0) \to (t+2)x(t-t_0+1) \stackrel{?}{=} y(t-t_0) = (t-t_0+2)x(t-t_0+1)$$

No; Time invariance is not supported

Causality

$$y(t) = (t+2)x(t+1)$$
$$e.g., y(1) = (1+2)x(1+1) = 3x(2);$$

No; $y(t)$ depends on the future, e.g, the output at time 1 seconds depends on the input at time 2 seconds which is the future.

Biomedical Signals and Systems Quiz #1 Answers

2. a) Describe the components of the solution of an ordinary difference equation.

- The Solution of an ODE consists of two components:
  - Response due to the Source which will take on the form of the Source, This is the steady state response
  - Source free response which will be the solution of the homogeneous equation and will take the form of the z^n eigenfunctions (e^{at} for continuous functions)
  - And with eigenvalues z (a for continuous functions) which are solution to the characteristic equation.. This is the transient response

Biomedical Signals and Systems
Quiz #1 Answers

2. b) Describe the possible forms of the transient response of a 2[nd] order ODE and the show the relationship which defines them.

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = x(t)$$

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = x(t)$$

$$(p^2 + 2\xi\omega_n p + \omega_n^2)y = A_3 x(t)$$

$$(p^2 + 2\xi\omega_n p + \omega_n^2) = 0$$

$$p_{1,2} = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4\omega_n^2}}{2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

ξ is the damping ratio

ω_n is the natural frequency

$\xi > 1;$ roots real, unequal, negative $\Rightarrow$ overdamped

$\xi = 1;$ roots real, equal, negative $\Rightarrow$ critically damped

$\xi < 1;$ roots complex, unequal, conjugates $\Rightarrow$ underdamped

$\xi = 0;$ roots imaginary, unequal, conjugates $\Rightarrow$ undamped

Final Answers

3. For the following provide a written answer:
   - a) Describe the impulse response for both continuous and discrete systems.
   - b) Describe how the impulse response characterizes a system.
   - c) How does the unit impulse **function** for continuous systems compare with the unit impulse **function** for discrete systems? Describe the properties of the continuous unit impulse function.

a) The impulse response is the output of LTI whose input is the unit impulse function.

b) The impulse response provides insights to a systems stability ~~and whether it is causal~~. Stability is demonstrated by the poles (roots of the denominator)of the network function $H(p)$ (that is, for a system to be stable, the poles must be in the left half of the complex plane (~~poles of $h[n]$ are within the unit circle~~) so the transient response, $h(t)$ (~~$h[n]$~~), goes to zero as t approaches infinity ~~and $h(t)$ ($h[n]$) must be bounded~~. ~~For causality, $h(t)$ ($h[n]$) must be zero for $t < 0$ ($n < 0$).~~

c) The unit impulse function, $\delta(t)$, also known as the Dirac delta function, is defined as:

$$\delta(t) = 0 \text{ for } t \neq 0;$$
$$= \text{undefined for } t = 0$$

and has the following special property:

$$\int_{-\infty}^{\infty} f(t)\delta(t-\tau)dt = f(\tau)$$

$$\therefore \int_{-\infty}^{\infty} \delta(t)dt = 1$$

The unit impulse function for discrete systems, $\delta[n]$, is defined as:

$$\delta[n] = 0 \text{ for } n \neq 0;$$
$$= 1 \text{ for } n = 0$$

Biomedical Signals and Systems Quiz #1 Answers

4. Derive the solution and its components for the following ODEs:

$a)\ \dot{y} + 3y = 12; y(0) = 4$

$(p+3)y = 12$

$p + 3 = 0;$ characteristic equation

$p = -3;$ eigenvalue

$y(t) = Ae^{-3t} + B;$ complete solution to the ODE

$0 + 3(B) = 12;$ subsitution of source solution into ODE

$B = 4$

$y(0) = A + 4 = 4;$ initial condition

$A = 0$

$y(t) = 4$

Check

$y(0) = 4$

Biomedical Signals and Systems Quiz #1 Answers

4. Derive the solution and its components for the following ODEs:

$b)\ \dot{y} + 3y = 12e^{-3t}; y(0) = 4$

$(p+3)y = 12e^{-3t}$

$p + 3 = 0;$ characteristic equation

$p = -3;$ eigenvalue

$y(t) = Ae^{-3t} + Be^{-3t};$ complete solution to the ODE

$(-3)Be^{-3t} + (3)(Be^{-3t}) = 12e^{-3t};$ subsitution of source solution into ODE

$-3B + 3B = 12$ THIS DOES NOT WORK!!!! TRY:

$y(t) = Ate^{-3t} + Be^{-3t};$ complete solution to the ODE

$y(t) = (At + B)e^{-3t}$

$\dot{y}(t) = Ae^{-3t} - 3(At + B)e^{-3t}$

$\dot{y} + 3y = 12e^{-3t}$

$Ae^{-3t} - 3(At + B)e^{-3t} + 3(At + B)e^{-3t} = 12e^{-3t}$

$Ae^{-3t} = 12e^{-3t}$

$A = 12$

$y(t) = (12t + B)e^{-3t}$

$y(0) = B = 4;$ initial condition

$y(t) = (12t + 4)e^{-3t}$

Check

$y(0) = (12 \times 0 + 4)e^{-3\times 0} = 4$

Biomedical Signals and Systems Quiz #1 Answers

4. Derive the solution and its components for the following ODEs:

$c)\ddot{y}+4\dot{y}+3y=9; y(0)=0; \dot{y}(0)=10$

$(p^2+4p+3)y=9$

$p^2+4p+3=0;$ characteristic equation

$p_{1,2}=-1,-3;$ eigenvalues are unequal negative real roots

$y(t)=Ae^{-1t}+Be^{-3t}+C;$ complete solution to the ODE

$0+4(0)+3(C)=9;$ subsitution of source solution into ODE

$C=3$

$\dot{y}(t)=-Ae^{-1t}-3Be^{-3t}+0$

$\ddot{y}(t)=Ae^{-1t}+9Be^{-3t}$

$y(0)=A+B+3=0;$ initial condition

$\dot{y}(0)=-A-3B=10;$ initial condition

$-2B+3=10$

$B=-3.5; A=0.5$

$y(t)=(0.5e^{-1t}-3.5e^{-3t}+3);$ total solution

Check

$y(0)=0.5+-3.5+3=0$

$\dot{y}(0)=-0.5+10.5+0=10$

Biomedical Signals and Systems Quiz #1 Answers

4. Derive the solution and its components for the following ODEs:

$d)\,\ddot{y}+2\dot{y}+y=e^{-10t};\,y(0)=10;\,\dot{y}(0)=10$

$(p^2+2p+1)y=e^{-10t}$

$p^2+2p+1=0;$ characteristic equation

$p_{1,2}=-1;$eigenvalues are equal roots

$y(t)=(A+Bt)e^{-1t}+Ce^{-10t};$complete solution to the ODE

$\dot{y}(t)=-(A+Bt)e^{-1t}+Be^{-t}-10Ce^{-10t}$

$=-(A+Bt-B)e^{-t}-10Ce^{-10t}$

$\ddot{y}(t)=(A+Bt-B)e^{-t}-Be^{-t}+100Ce^{-10t}$

$=(A+Bt-2B)e^{-t}+100Ce^{-10t}$

Substitute source solution into ODE.

$100Ce^{-10t}+2(-10Ce^{-10t})+Ce^{-10t}=e^{-10t}$

$100C+2(-10C)+C=1$

$81C=1$

$C=\frac{1}{81}=0.012$

$y(0)=A+C=10;$initial condition

$A=\frac{810}{81}-\frac{1}{81}=\frac{809}{81}=9.99$

$\dot{y}(0)=-(A-B)-10C=10;$initial condition

$-\frac{809}{81}+B-\frac{10}{81}=10$

$B=\frac{1629}{81}=20.1$

$y(t)=(\frac{809}{81}+\frac{1629}{81}t)e^{-t}+\frac{1}{81}e^{-10t};$total solution

Check

$y(0)=(\frac{809}{81}+\frac{1629}{81}\times 0)e^{-\times 0}+\frac{1}{81}e^{-10\times 0}=\frac{809}{81}+\frac{1}{81}=\frac{810}{81}=10$

$\dot{y}(0)=-(\frac{809}{81}+\frac{1629}{81}\times 0)e^{-\times 0}+\frac{1629}{81}e^{-\times 0}-\frac{10}{81}e^{-10\times 0}=\frac{1629}{81}-\frac{809}{81}-\frac{10}{81}=\frac{810}{81}=10$

Biomedical Signals and Systems Quiz #1 Answers

5. Calculate the Transfer Function $V_{out}(j\omega) / V_{in}(j\omega)$ for this circuit and sketch the Bode plot.

$$v_{out}(t) = L\frac{di(t)}{dt} \Rightarrow v_{out}(t) = pLi(t)$$

$$v_{in}(t) = \frac{1}{C}\int i(t)\,dt + L\frac{di(t)}{dt} \Rightarrow \frac{dv_{in}(t)}{dt} = \frac{1}{C}i(t) + L\frac{d^2i(t)}{dt^2}$$

$$pv_{in}(t) = \frac{1}{C}i(t) + Lp^2i(t) = (\frac{1}{C} + Lp^2)i(t)$$

$$v_{in}(t) = (\frac{1}{pC} + Lp)i(t)$$

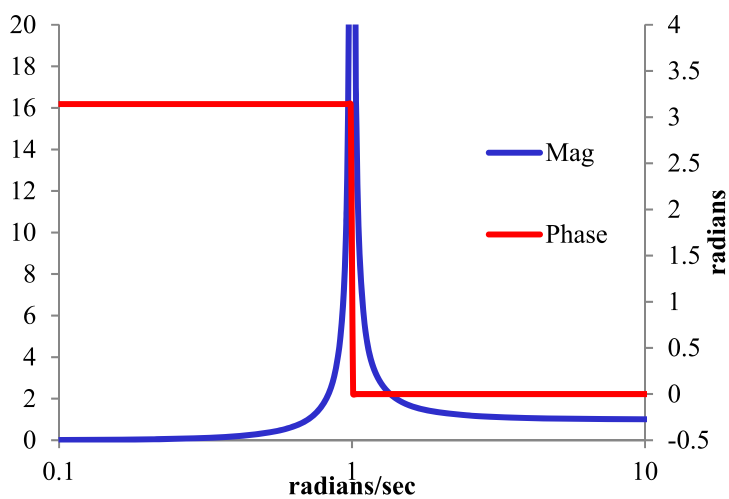
$$\frac{v_{out}(t)}{v_{in}(t)} = \frac{pLi(t)}{(\frac{1}{pC} + Lp)i(t)} = \frac{p^2LC}{1 + p^2LC}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{(j\omega)^2LC}{1 + (j\omega)^2LC} = \frac{-\omega^2LC}{1 - \omega^2LC}$$

DEFINE :

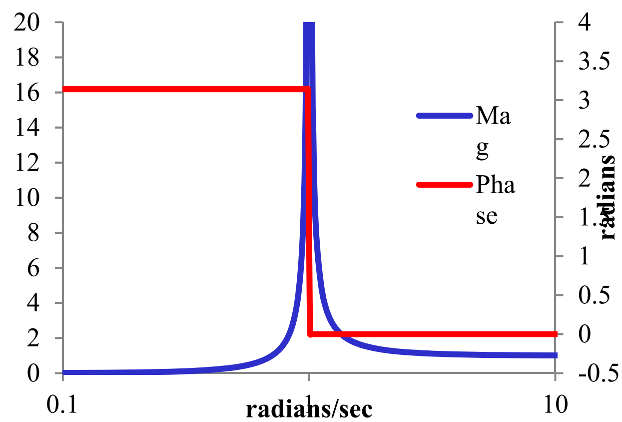
$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{-(\frac{\omega}{\omega_o})^2}{1 - (\frac{\omega}{\omega_o})^2}$$



Biomedical Signals and Systems Quiz #1 Answers

5. Calculate the Transfer Function $V_{out}(j\omega)/V_{in}(j\omega)$ for this circuit and sketch the Bode plot.



$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{-(\frac{\omega}{\omega_o})^2}{1-(\frac{\omega}{\omega_o})^2}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=0} = \frac{-(\frac{\omega}{\omega_o})^2}{1-(\frac{\omega}{\omega_o})^2}\Big|_{\omega=0} = \frac{-(0)}{1-0} = 0\angle\pi;$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega\to\infty} = \frac{-(\frac{\omega}{\omega_o})^2}{1-(\frac{\omega}{\omega_o})^2}\Big|_{\omega\to\infty} \to \frac{-(\frac{\omega}{\omega_o})^2}{-(\frac{\omega}{\omega_o})^2}\Big|_{\omega\to\infty} = 1\angle 0$$

$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=\omega_0} \to \infty\angle$ ACTUALLY WE MUST BE CAREFUL TO FIND THE ANGLE

Just before ω_0 that is ω_0^-

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=\omega_0^-} = \frac{-(\frac{\omega_0^-}{\omega_o})^2}{1-(\frac{\omega_0^-}{\omega_o})^2} = \frac{\text{negative number less than but close to 1}}{1-\text{ number less than but close to 1}}$$

$$= \frac{\text{negative number less than but close to 1}}{\text{small positive number}} = \text{large negative number} = \text{large number}\angle\pi \to \infty\angle\pi$$

Just after ω_0 that is ω_0^+

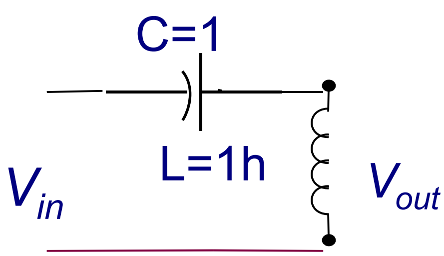
$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=\omega_0^+} = \frac{-(\frac{\omega_0^+}{\omega_o})^2}{1-(\frac{\omega_0^+}{\omega_o})^2} = \frac{\text{negative number greater than but close to 1}}{1-\text{ number greater than but close to 1}}$$

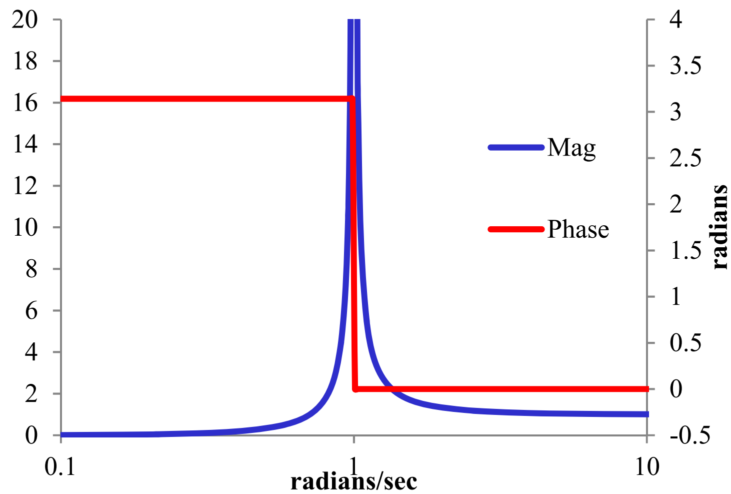
$$= \frac{\text{negative number greater than but close to 1}}{\text{small negative number}} = \text{large negative number when negative sign cancels}$$

$= \text{large number}\angle 0 \to \infty\angle 0$

Biomedical Signals and Systems
Quiz #1 Answers

5. Calculate the Transfer Function $V_{out}(j\omega)/V_{in}(j\omega)$ for this circuit and sketch the Bode plot.





$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}|_{\omega=0} = 0\angle\pi$; ω=0 capacitor open and inductor shorted; output=0

$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}|_{\omega\to\infty} = 1\angle 0$; $\omega\to\infty$ capacitor short and inductor open and output=input

Note roots of $H(p)$ are $\pm j\omega_0$

and are on the imaginary axis. This is undamped and unstable.