BME 333 Biomedical Signals and Systems

Biomedical Signals and Systems Quiz #2

- Choose 4 out of 5
- Problems 2, 3, & 4 are mandatory

Biomedical Signals and Systems
Quiz #2

1. a) Sketch the convolution of $t^2u(t)$ with itself
   b) Convolve $tu(t)$ with $u(t)$ and sketch the result.
   NOTE THE ORDER.
   c) Sketch the convolution of $e^{-2t}u(t)$ with itself.

2. Compute the Fourier Transform for the following functions **without** integrating

   $NOTE: \Im[e^{-\alpha t} f(t)] = F(\alpha + j\omega)$

   a) $f(t) = e^{-t}u(t)$
   b) $f(t)=\cos(t)u(t)$
   c) $f(t)=e^{-t}\cos(t)u(t)$

3. Compute the Discrete Fourier Transform for the following function and sketch the FT for N = 10

   $$NOTE: \sum_{0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

   a) $x[n] = 10$ for $n = 0$
   $= 0$ for $n = 1, 2, \ldots, N-1$

   b) $x[n] = 10$ for $n = 0, 1, 2, \ldots, N-1$

   c) Describe why the results for both a) and b) makes sense. What is the relationship of these two problems?

Biomedical Signals and Systems Quiz #2

4. The following periodic signal is passed through an ideal bandpass filter with cutoff frequencies of *27.5 kHz* and *57.5 kHz*.

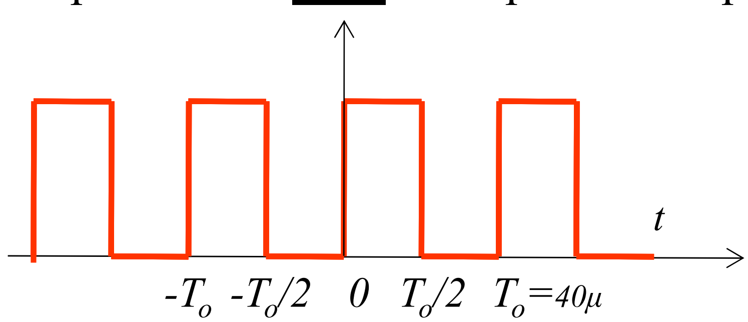
$$x(t) = \begin{cases} 1 & 0 \le t < 20\mu sec \\ 0 & 20\mu sec \le t < 40\mu sec \end{cases}$$

a) Find and sketch the spectrum of **both** the input and output signals.

b) What happens if the bandpass filter's cutoff frequencies are changed to *65 kHz* and *85 kHz?* Find and sketch the spectrum of **both** the input and output signals.

c) What happens if low pass filter's with cutoff frequency of *200 kHz* is used*?* Find and sketch the spectrum of **both** the input and output signals.

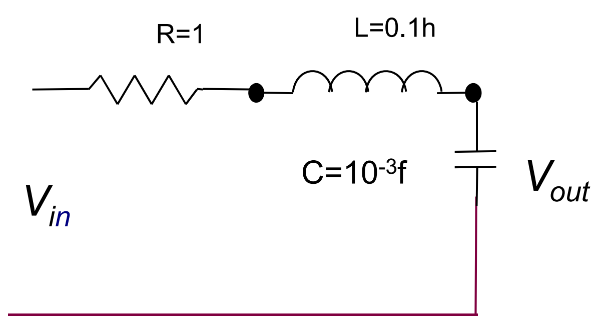
d) What happens if the band elimination filter with cutoff frequencies *210 kHz* and *230 kHz* is used*?* Find and sketch the spectrum of **both** the input and output signals.



Biomedical Signals and Systems Quiz #2

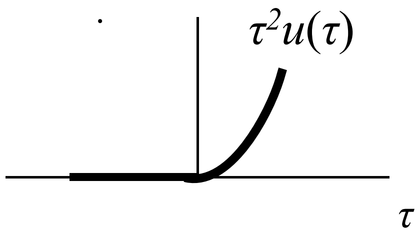
5. Apply the signal to the input of the circuit and calculate and plot the spectrum of the output signal. What happens if the carrier frequency is changed to 1kHz and the modulation signal to 10Hz?

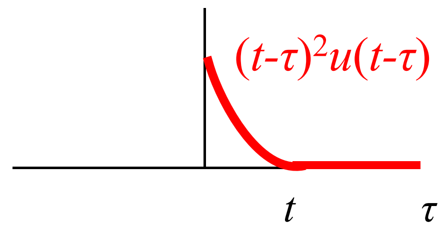
$$v_{in}(t)=(1+m_o\cos t)\cos 100t$$

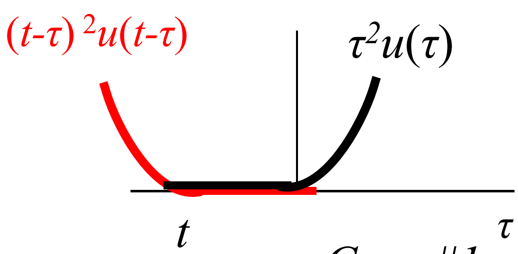


Biomedical Signals and Systems
Quiz #2

1. a)) Sketch the convolution of $t^2u(t)$ with itself

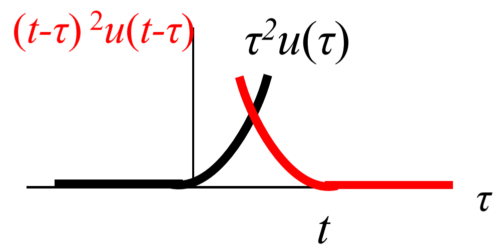






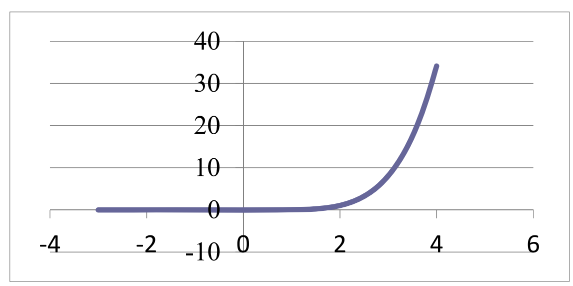
Case #1

t<0



Case #2

t>0



Cast #1

$$C = \int_{-\infty}^{\infty} \tau^2 u(\tau)(t-\tau)^2 u(t-\tau)d\tau = 0 \text{ since}$$

$u(\tau) = 0$ for τ & $t < 0$

Cast #2

$$C = \int_{-\infty}^{-\infty} \tau^2 u(\tau)(t-\tau)^2 u(t-\tau)d\tau \quad \text{for } t \geq 0$$

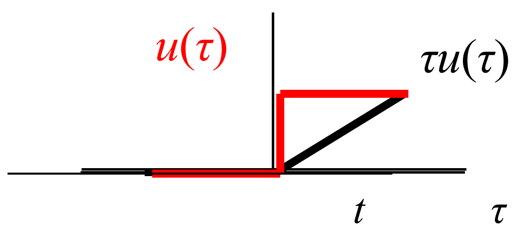
$$C = \int_0^t \tau^2(t-\tau)^2 d\tau = \int_0^t \tau^2(t^2 - 2t\tau + \tau^2)d\tau = \int_0^t (\tau^2 t^2 - 2t\tau^3 + \tau^4)d\tau =$$

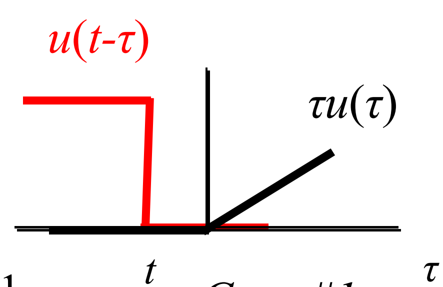
$$= \left(\frac{\tau^3 t^2}{3} - \frac{2t\tau^4}{4} + \frac{\tau^5}{5}\right)\Bigg|_0^t = \frac{t^5}{3} - \frac{t^5}{2} + \frac{t^5}{5} = \frac{t^5}{30}; \text{ for } t > 0$$

$$C = \frac{t^5}{30}u(t)$$

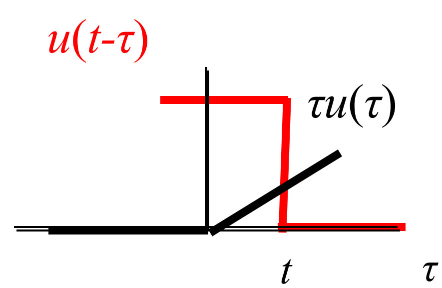
Biomedical Signals and Systems Quiz #2

b) Convolve $tu(t)$ with $u(t)$ and sketch the result. *NOTE THE ORDER*
THIS IS THE WRONG ORDER





Case #1
t<0



Case #2
t>0

Case #1

$$C = \int_{-\infty}^{\infty} \tau u(\tau)u(t-\tau)d\tau = 0 \text{ for } t < 0 \text{ since}$$

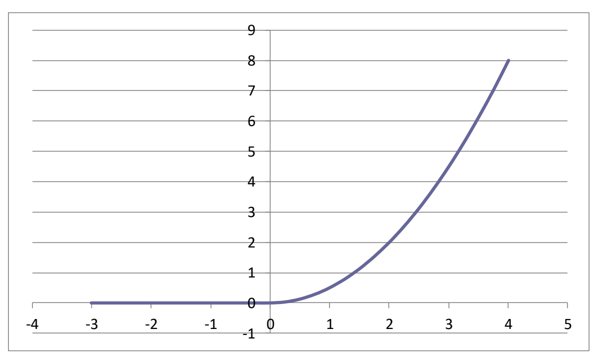
$$u(\tau) = 0 \text{ for } \tau \ \& \ t < 0$$

Case #2

$$C = \int_{0}^{t} \tau u(\tau)u(t-\tau)d\tau \quad \text{for } t > 0$$

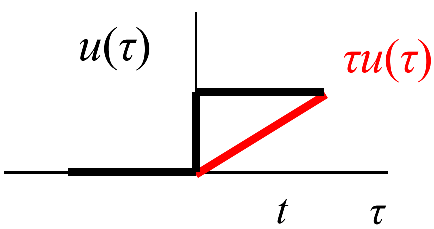
$$C = \int_{0}^{t} \tau d\tau = \frac{\tau^2}{2}\Big|_0^t = \frac{t^2}{2}; \text{ for } t > 0$$

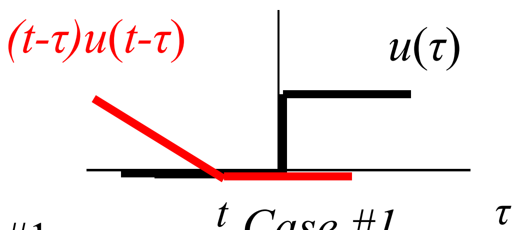
$$C = \frac{t^2}{2}u(t)$$



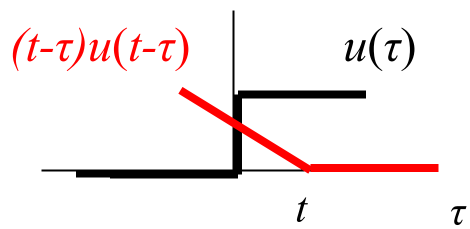
Biomedical Signals and Systems Quiz #2

b) Convolve $tu(t)$ with $u(t)$ and sketch the result. *NOTE THE ORDER*
THIS IS THE CORRECT ORDER





Case #1
t<0



Case #2
t>0

Case #1

$$C = \int_{-\infty}^{\infty} (t-\tau)u(\tau)u(t-\tau)d\tau = 0 \text{ for } t<0 \text{ since}$$

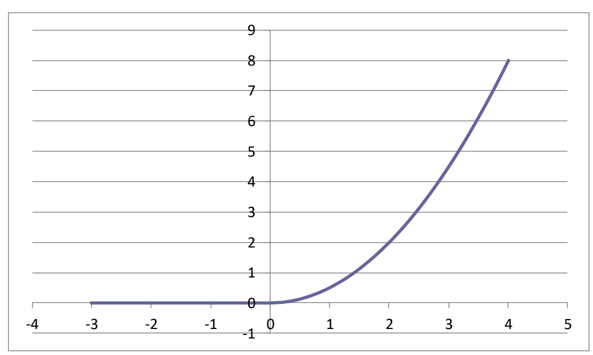
$$u(\tau) = 0 \text{ for } \tau \ \& \ t<0$$

Case #2

$$C = \int_{0}^{t} (t-\tau)u(\tau)u(t-\tau)d\tau \quad \text{for } t>0$$

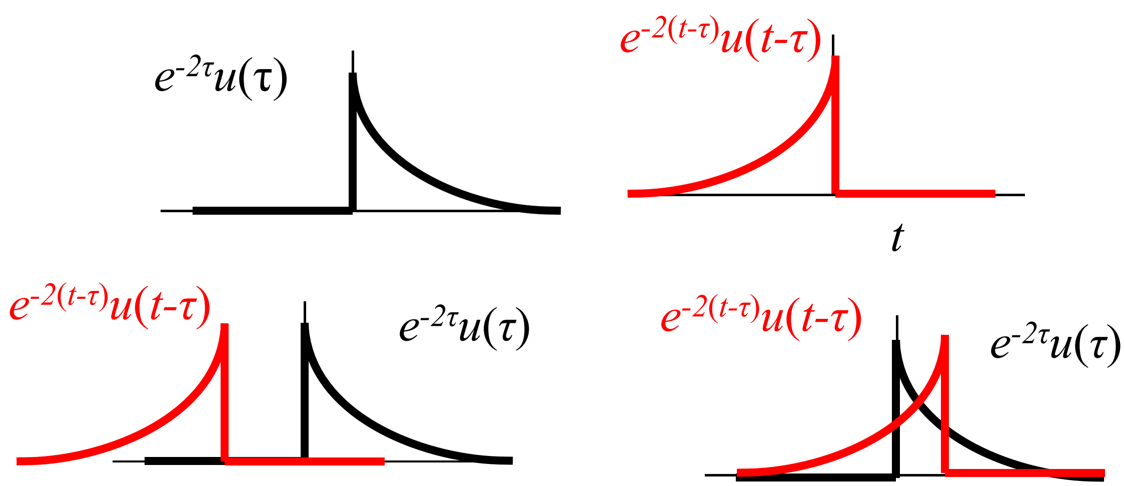
$$C = \int_{0}^{t} (t-\tau)d\tau = -\frac{(t-\tau)^2}{2}\Big|_0^t = \frac{t^2}{2}; \text{ for } t>0$$

$$C = \frac{t^2}{2}u(t)$$



Biomedical Signals and Systems
Quiz #2

1. b) Sketch the convolution of $e^{-t}u(t)$ with itself.



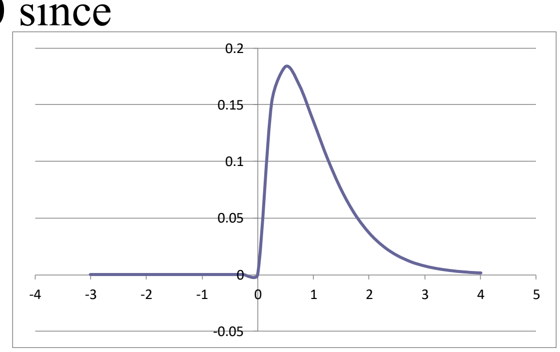
Case #1
t<0

Case #2
t>0

Case #1

$$C = \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)e^{-2(t-\tau)}u(t-\tau)d\tau = 0 \text{ for } t<0 \text{ since}$$

$$u(\tau) = 0 \text{ for } \tau \ \& \ t<0$$



Case #2

$$C = \int_{0}^{t} e^{-2\tau}u(\tau)e^{-2(t-\tau)}u(t-\tau)d\tau \quad \text{for } t>0$$

$$= \int_{0}^{t} e^{-2\tau}u(\tau)e^{-2(t-\tau)}u(t-\tau)d\tau = \int_{0}^{t} e^{-2\tau}e^{-2(t-\tau)}d\tau$$

$$= \int_{0}^{t} e^{-2\tau}e^{-2t+2\tau}d\tau = \int_{0}^{t} e^{-2\tau-2t+2\tau}d\tau = \int_{0}^{t} e^{-2t}d\tau = e^{-2t}\int_{0}^{t} 1d\tau$$

$$= e^{-2t}t; \text{ for } t>0$$

$$= e^{-2t}tu(t)$$

Biomedical Signals and Systems Quiz 2

2. Compute the Fourier Transform for the following functions without integrating

   a) $f(t) = e^{-t}u(t)$
   b) $f(t)=\cos(t)u(t)$
   c) $f(t)=e^{-t}\cos(t)u(t)$

a) $\Im[u(t)] = \dfrac{1}{j\omega};$

using the frequency displacement property

$$\Im[e^{-\alpha t}u(t)] = \frac{1}{\alpha + j\omega}; \Im[e^{-t}u(t)] = \frac{1}{1+j\omega}$$

b) $\Im[\cos(t)u(t)] = \Im[\dfrac{e^{jt}+e^{-jt}}{2}u(t)] = \dfrac{1}{2}\Im[e^{jt}u(t)] + \dfrac{1}{2}\Im[e^{-jt}u(t)]$

using the frequency displacement property

$$= \frac{1}{2}[\frac{1}{-j+j\omega} + \frac{1}{j+j\omega}]$$

$$= \frac{1}{2}[\frac{j+j\omega+-j+j\omega}{(-j+j\omega)(j+j\omega)}] = \frac{j\omega}{(1-\omega^2)}$$

c) $\Im[e^{-t}\cos(t)u(t)] = \Im[e^{-t}\dfrac{e^{jt}+e^{-jt}}{2}u(t)] = \dfrac{1}{2}\Im[e^{(-1+j)t}u(t)] + \dfrac{1}{2}\Im[e^{(-1-j)t}u(t)]$

using the frequency displacement property

$$= \frac{1}{2}[\frac{1}{1-j+j\omega} + \frac{1}{1+1j+j\omega}]$$

$$= \frac{1}{2}[\frac{1+j+j\omega+1-1j+j\omega}{(1-j+j\omega)(1+j+j\omega)}]$$

$$= \frac{1+j\omega}{(2-\omega^2+j2\omega)}$$

Biomedical Signals and Systems Quiz 2

3. Compute the Discrete Fourier Transform for the following function and sketch the FT for N = 10

a) $x[n] = 10$ for $n = 0$
$= 0$ for $n = 1, 2, \ldots, N\text{-}1$

b) $x[n] = 10$ for $n = 0, 1, 2, \ldots, N\text{-}1$

c) Describe why the results for both a) and b) makes sense. What is the relationship of these two problems?

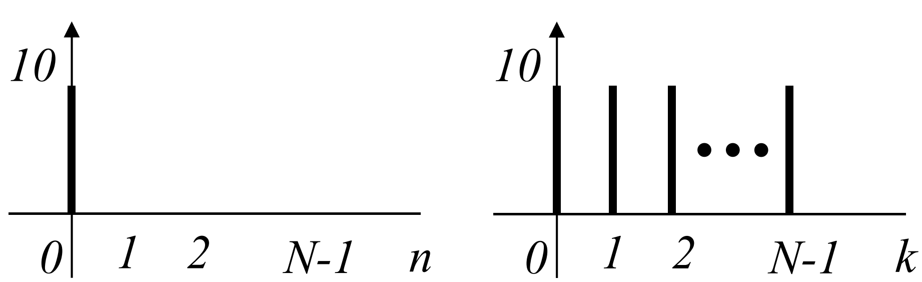
$$\mathbf{X}(k) = \sum_{n=0}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$NOTE: \sum_{0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$x(n) = \sum_{n=0}^{N-1} \mathbf{X}(k) e^{jk2\pi n/N}$$

$$a) x(n) = 10 \text{ for } n = 0$$
$$= 0 \text{ for } n = 1, 2, \ldots N$$

$$\mathbf{X}(k) = \sum_{n=0}^{N-1} x(n) e^{-jk2\pi n/N}$$
$$= x(0)e^{-jk2\pi 0/N} + x(1)e^{-jk2\pi 1/N} + \cdots + x(N)e^{-jk2\pi N/N}$$
$$= 10$$



Biomedical Signals and Systems Quiz 2

3. Compute the Discrete Fourier Transform for the following function and sketch the FT for N = 10

b) $x[n] = 10$ for $n = 0, 1, 2, \ldots, N\text{-}1$

c) Describe why the results for both a) and b) makes sense. What is the relationship of these two problems?

$$NOTE: \sum_{0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$b)\, x[n] = 10$$

$$\mathbf{X}(k) = \sum_{n=0}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$= \sum_{n=0}^{N-1} 10 e^{-jk2\pi n/N} = 10 \sum_{n=0}^{N-1} (e^{-jk2\pi/N})^n$$

$$= 10 \frac{1 - e^{-j(k2\pi/N)N}}{1 - e^{-j(k2\pi/N)}}$$

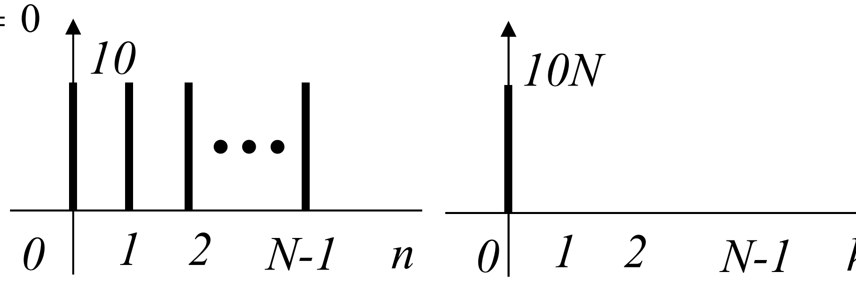
$$= 10 \frac{1 - e^{-jk2\pi}}{1 - e^{-j(k2\pi/N)}} = 0; k \neq 0$$

For $k = 0$, use L'Hopital's Rule

$$\lim_{k \to 0} 10 \frac{\frac{d}{dk} 1 - e^{-jk2\pi}}{\frac{d}{dk} 1 - e^{-j(k2\pi/N)}} = \lim_{k \to 0} 10 \frac{-j2\pi e^{-jk2\pi}}{-j \frac{2\pi}{N} e^{-j(k2\pi/N)}} = 10 \frac{-j2\pi}{-j \frac{2\pi}{N}} = 10N = 100$$

$$\mathbf{X}(k) = 10N = 100; k = 0$$

$$\mathbf{X}(k) = 0; k \neq 0$$



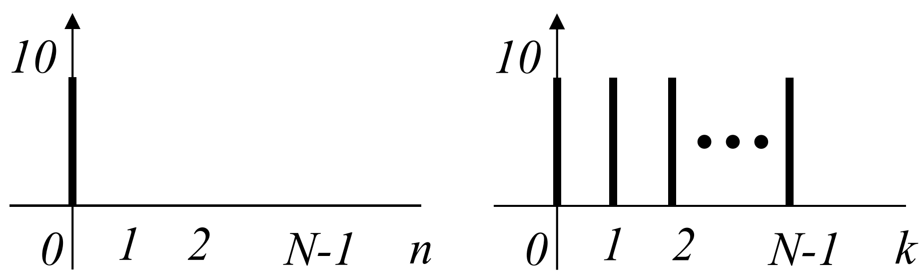
Biomedical Signals and Systems Quiz 2

3. Compute the Discrete Fourier Transform for the following function and sketch the FT for N = 10

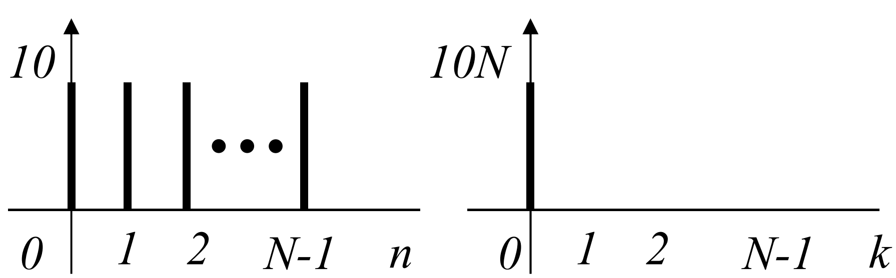
c) Describe why the results for both a) and b) makes sense. What is the relationship of these two problems?

c) They are duals of each other.

Problem a) is an impulse in the time domain which yields a constant in the frequency domain (needs all frequencies).



Problem b) is a constant in the time domain which yields an impulse in the frequency domain (single frequency at DC).



Biomedical Signals and Systems
Quiz 2

4. The following periodic signal is passed through an idea bandpass filter with cutoff frequencies of

$$x(t) = \begin{cases} 1 & 0 \le t < 20\mu sec \\ 0 & 20\mu sec \le t < 40\mu sec \end{cases}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

$$a_k = \frac{1}{T_o}\int_{-T_o/2}^{0} x(t)e^{-jk\omega_o t}dt = \frac{1}{T_o}\int_{0}^{T_o/2} 1e^{-jk\omega_o t}dt$$

$$= \frac{1}{T_o(-jk\omega_o)}e^{-jk\omega_o t}\Big|_0^{T_o/2} = \frac{1}{-j2k\pi}[e^{-jk\frac{2\pi}{T_o}\frac{T_o}{2}} - e^{-jk\frac{2\pi}{T_o}0}]$$

$$= \frac{1}{j2k\pi}[1 - e^{-jk\pi}]$$

Recall that $e^{-jk\pi} = \cos k\pi - j\sin k\pi = 1$ for even values of k

$= -1$ for odd values of k

or $e^{-jk\pi} = (-1)^k$

$$a_k = \frac{1}{j2k\pi}[1 - e^{-jk\pi}]$$

$$= \frac{1}{j2k\pi}[1 - (-1)^k]$$

$$= \frac{1}{jk\pi} \quad \text{for odd values of } k$$

$$= 0 \quad \text{for even values of } k$$

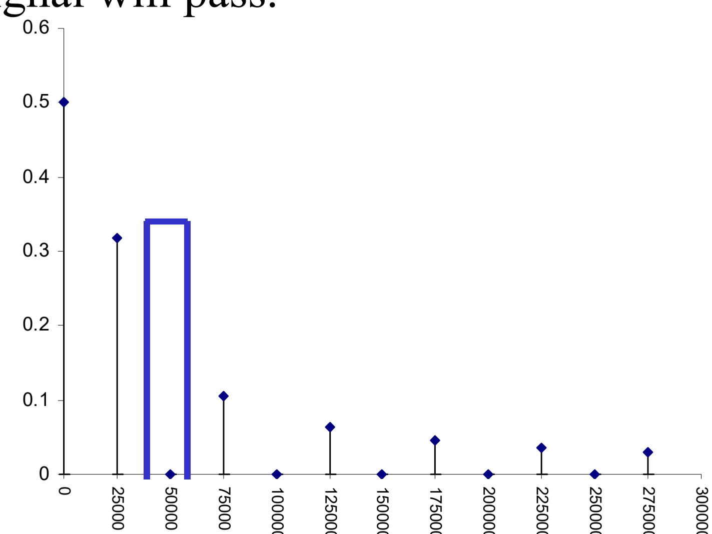
$$a_0 = \frac{1}{T_o}\int_{-T_o/2}^{T_o/2} x(t)dt = \frac{1}{T_o}\int_{0}^{T_o/2} 1dt = \frac{1}{2}$$

Biomedical Signals and Systems Quiz 2

4. The following periodic signal is passed through an idea bandpass filter with cutoff frequencies of

$$x(t) = \begin{cases} 1 & 0 \leq t < 20\mu sec \\ 0 & 20\mu sec \leq t < 40\mu sec \end{cases}$$

a) Since $T_o = 40\mu\, sec$ then $f_o = 25$kHz, then output signal will have components of 25 kHz, 75 kHz, 125 kHz, etc. (odd values of k). Since the bandpass filter's cutoff frequencies are 27.5 kHz and 57.5 kHz only the even spectral component, k =2 will pass but it has an amplitude of zero and so no signal will pass.



Biomedical Signals and Systems
Quiz 2

4. The following periodic signal is passed through an idea bandpass filter with cutoff frequencies of

$$x(t) = \begin{cases} 1 & 0 \le t < 20\mu sec \\ 0 & 20\mu sec \le t < 40\mu sec \end{cases}$$

b) Since the bandpass filter's cutoff frequencies are *65 kHz* and *85 kHz*, spectral component k =3 passes.

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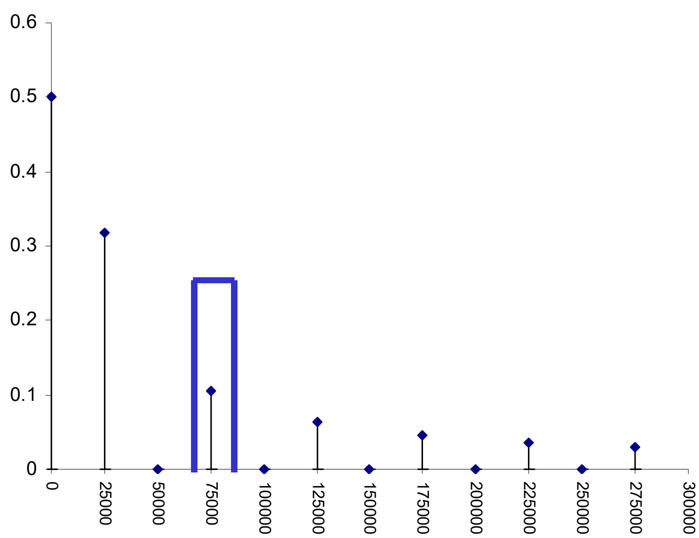

Biomedical Signals and Systems
Quiz 2

4. The following periodic signal is passed through an idea bandpass filter with cutoff frequencies of

$$x(t) = \begin{cases} 1 & 0 \leq t < 20\mu sec \\ 0 & 20\mu sec \leq t < 40\mu sec \end{cases}$$

c) Since the low filter's cutoff frequency is 200 kHz, the spectral component for $k = 0$ (DC), 1 (25kHz), 3 (75kHz), and 5 (125kHz) 7 (175kHz) pass.

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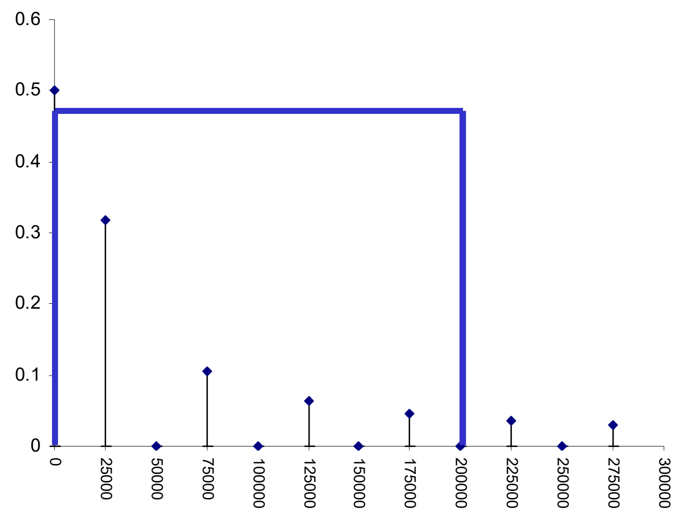
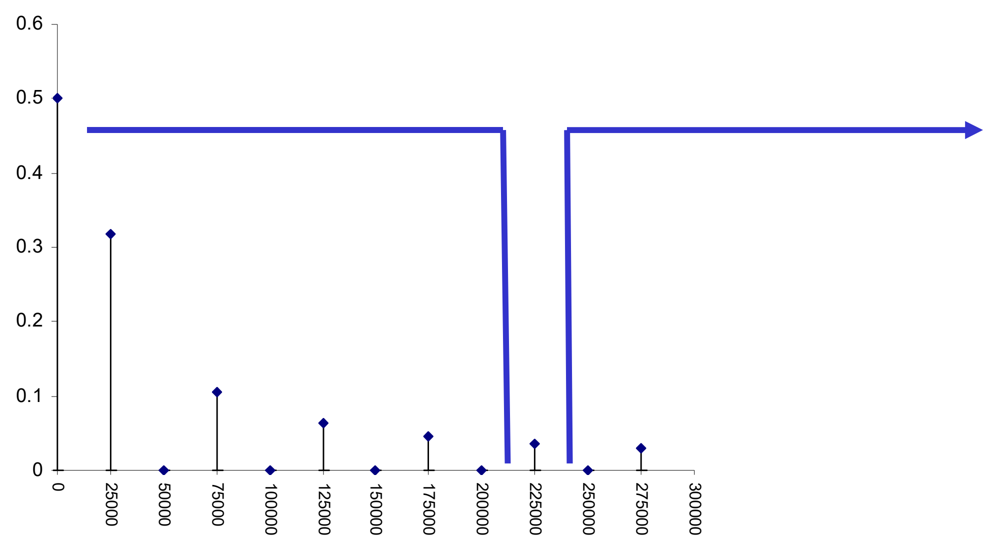


Final Answers

4. The following periodic signal is passed through an idea bandpass filter with cutoff frequencies of

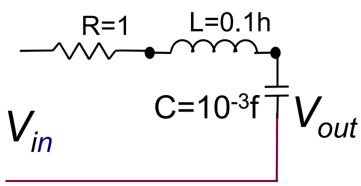
$$x(t) = \begin{cases} 1 & 0 \leq t < 20\mu sec \\ 0 & 20\mu sec \leq t < 40\mu sec \end{cases}$$

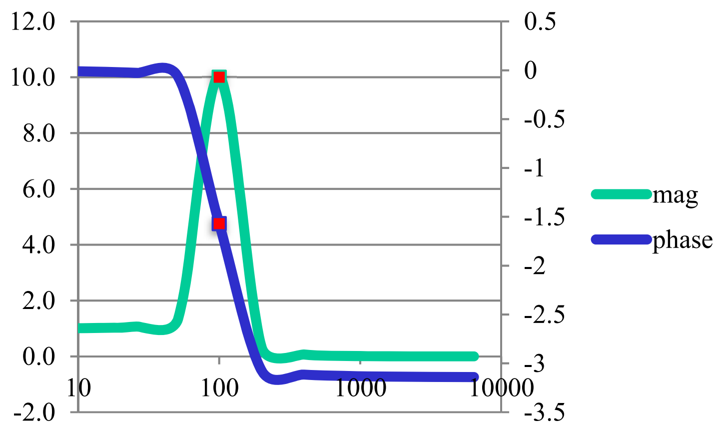
d) Since the band elimination filter's cutoff frequencies are 210 kHz and 230 kHz, only the spectral component for k=9 (225 kHz) is blocked.



Biomedical Signals and Systems Quiz 2

5. Calculate the Transfer Function $V_{out}(j\omega)/V_{in}(j\omega)$ for these circuits and sketch the Bode plots:





$$v_{in}(t) = (1 + m_0 \cos t)\cos 100t$$

$$v_{in}(t) = \cos 100t + \frac{m_0}{2}\cos 101t + \frac{m_0}{2}\cos 99t$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{(1-\omega^2 LC) + j\omega CR}$$

$$= \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega CR)^2}} \angle -\tan^{-1}\left(\frac{\omega RC}{(1-\omega^2 LC)}\right); \omega_o = \sqrt{\frac{1}{LC}} = 10^2; RC = 10^{-3}$$

$$= \frac{1}{\sqrt{[(1-\omega^2 10^{-4})]^2 + \left(\omega 10^{-3}\right)^2}} \angle -\tan^{-1}\left(\frac{\omega 10^{-3}}{(1-\omega^2 10^{-4})}\right)$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=0} = \frac{1}{(1-0LC) + j0CR} = 1\angle 0$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=\frac{1}{\sqrt{LC}}=100} = \frac{1}{(1-1) + j100\times 10^{-3}} = 10\angle -\frac{\pi}{2}$$

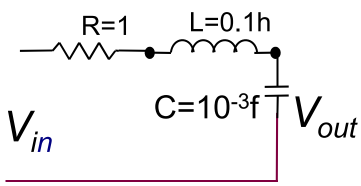
$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega\to\infty} \to \frac{1}{-\omega^2 LC} = 0\angle -\pi$$

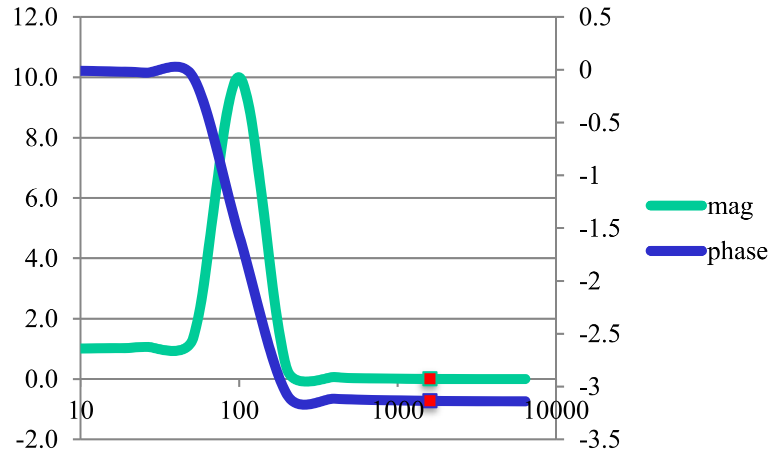
$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=99} \approx 9.9\angle -1.4\angle 0; \frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=100} = 10\angle -1.57; \frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=101} \approx 9.7\angle -1.77$$

$$v_{out}(t) \approx 10\cos(100t - 1.57) + \frac{9.7m_0}{2}\cos(101t - 1.77) + \frac{9.9m_0}{2}\cos(99t - 1.4)$$

Biomedical Signals and Systems Quiz 2

5. Calculate the Transfer Function $V_{out}(j\omega)/V_{in}(j\omega)$ for these circuits and sketch the Bode plots:





$$v_{in}(t) = (1 + m_0 \cos 2\pi 10t)\cos 2\pi 1000t$$

$$v_{in}(t) = \cos 2\pi 1000t + \frac{m_0}{2}\cos 2\pi 990t + \frac{m_0}{2}\cos 2\pi 1010t$$

$$v_{in}(t) = \cos 6283.2t + \frac{m_0}{2}\cos 6220.4t + \frac{m_0}{2}\cos 6246t$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=6346} \approx 2.5\times10^{-5}\angle-\pi$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=6283.2} \approx 2.5\times10^{-4}\angle-\pi$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\Big|_{\omega=6220.4} \approx 2.5\times10^{-4}\angle-\pi$$

$$v_{out}(t) \approx 2.5\times10^{-4}[\cos(6283.2t-\pi) + \frac{m_0}{2}\cos(6346.1t-\pi) + \frac{m_0}{2}\cos(6220.4t-\pi)]$$

None of the signal gets through the filter.