BME 333 Biomedical Signals and Systems

Biomedical Signals and Systems Quiz #3

- Grading:
  - All questions are rated at 25%
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Biomedical Signals and Systems Quiz #3

1. Consider N 5M Hz HD video signals which is to be quantized.

   a) Assuming the signal is sampled using 256^2 levels, what should the capacity (in Bytes/second) of the line be for N=1, 10, 100?
   b) How many messages can be transmitted if the channel capacity is 1MB/s.
   c) Repeat parts a) and b) for a channel capacity of 256 levels bits per second.
   d) At the channel capacity rate of 10MB/s, what is the maximum number levels the video signal can be coded?

2. Compute the Laplace Transforms for the following functions

$$a)\ f(t) = 6,\ 0 < t < 10$$
$$b)\ f(t) = 6t,\ 0 < t < 10$$
$$c)\ f(t) = 6te^{-t},\ 0 < t < 10$$

3. Find the inverse transforms for the following:

$$a)\ \frac{s^3 + s^2 + 6}{s(s+2)(s+3)};b)\ \frac{6s+6}{(s^2+9)(s+1)};\ c)\ \frac{s}{s^3(s+1)}$$

4. Find a) the impulse response and b) the output $y(t)$ for $x(t) = e^{-4t}u(t)$ using Laplace Transforms.

$$\ddot{y}(t) + 7\dot{y}(t) + 12y(t) = x(t)$$

Biomedical Signals and Systems Quiz #3 Answers

1. Consider N 5M Hz HD video signals which is to be quantized.

   a) Assuming the signal is sampled using 256^2 levels, what should the capacity (in Bytes/second) of the line be for N=1, 10, 100?

   b) How many messages can be transmitted if the channel capacity is 1MB/s.

   c) Repeat parts a) and b) for a channel capacity of 256 levels bits per second.

   d) At the channel capacity rate of 10MB/s, what is the maximum number levels the video signal can be coded?

   e) Each signal is sampled at 2x5M= 10M samples per second. Assuming 256^2=65536 = 2^{16} levels are used, 16 bits are needed for each sample. Therefore to send one (N=1) signal the line capacity should be 16x10M = 160Mb/s or 20MB/s are needed. To send N=10, line capacity of 1.6Gb/s or 200MB/s is needed and 16Gbs or 2GB/s is needed for N=100.

   f) 1MB/s = 8Mb/s. Since each channel is 160Mb/s, there are 0.05 messages or NO messages can fit into this channel.

   g) For 256 = 2^8 levels, 8 bits are used. 8x10Mb/s = 80Mb/s=10MB/s. For N=1, 10MB/s, N=10, 100MB/s and N=100, 1GB/s. 1MB/s, there 0.1 messages.

   h) Each signal requires 10Msamples/s. Since the channel capacity is 10MBytes/s or 80Mbits/s, then 80Mbits/s / 10Msamples/s or 8 bits/sample is available in the channel for the signal. Therefore, 2^8=256 levels is the maximum number of levels.

Biomedical Signals and Systems Quiz #3 Answers

2. Compute the Fourier and Laplace Transforms for the following function

a) f(t) = 6, 0 < t < 10
b) f(t) = 6t, 0 < t < 10
c) f(t) = 6te^{-t}, 0 < t < 10

$$a)\, f(t) = 6; 0 \le t \le 10$$
$$= 6u(t) - 6u(t-10)$$

$$£[u(t)] = \frac{1}{s}$$

$$£[f(t)] = £[6u(t) - 6u(t-10)]$$
$$= \frac{6}{s}(1 - e^{-s10})$$

$$b)\, f(t) = 6t; 0 \le t \le 10$$
$$= 6tu(t) - 6tu(t-10)$$
$$= 6tu(t) - 6(t-10)u(t-10) - 60u(t-10)$$

$$£[tu(t)] = \frac{1}{s^2}$$

$$£[(t-10)u(t-10)] = \frac{e^{-10s}}{s^2}$$

$$£[f(t)] = £[6tu(t) - 6(t-10)u(t-10) - 60u(t-10)]$$
$$= 6[\frac{1}{s^2} - \frac{e^{-10s}}{s^2} - \frac{10e^{-10s}}{s}]$$

Biomedical Signals and Systems
Quiz #3 Answers

2. Compute the Fourier and Laplace Transforms for the following function: $f(t) = 6te^{-t}$, $0 < t < 10$

$$c)\, f(t) = 6te^{-t}; 0 \le t \le 10$$

$$= 6te^{-t}u(t) - 6te^{-t}u(t-10) = 6te^{-t}u(t) - \frac{6te^{-t}e^{10}}{e^{10}}u(t-10)$$

$$= 6te^{-t}u(t) - \frac{6te^{-(t-10)}}{e^{10}}u(t-10)$$

$$= 6te^{-t}u(t) - \frac{6(t-10)e^{-(t-10)}}{e^{10}}u(t-10) - \frac{60e^{-(t-10)}}{e^{10}}u(t-10)$$

$$f(t) = 6te^{-t}u(t) - \frac{6(t-10)e^{-(t-10)}}{e^{10}}u(t-10) - \frac{60e^{-(t-10)}}{e^{10}}u(t-10)$$

$$£[f(t)] = 6[\frac{1}{(s+1)^2} - \frac{e^{-10(s+1)}}{(s+1)^2} - \frac{10e^{-10(s+1)}}{(s+1)}]$$

OR

$$f(t) = 6te^{-t}; 0 \le t \le 10$$

$$= g(t)e^{-t}; \text{ where } g(t) = 6tu(t) - 6(t-10)u(t-10) - 60u(t-10)$$

From Part b)

$$G(s) = 6[\frac{1}{s^2} - \frac{e^{-10s}}{s^2} - \frac{10e^{-10s}}{s}]$$

$$£[f(t)] = £[g(t)e^{-t}] = G(s+1) = 6[\frac{1}{(s+1)^2} - \frac{e^{-10(s+1)}}{(s+1)^2} - \frac{10e^{-10(s+1)}}{(s+1)}]$$

Biomedical Signals and Systems Quiz #3 Answers

$$3a)\ £^{-1}\left[\frac{s^3+s^2+6}{s(s+2)(s+3)}\right]=£^{-1}\left[1+\frac{K_1}{s}+\frac{K_2}{s+2}+\frac{K_3}{s+3}\right]$$

$$K_1=\frac{s^3+s^2+6}{(s+2)(s+3)}\Big|_{s=0}=\frac{6}{(2)(3)}=1$$

$$K_2=\frac{s^3+s^2+6}{s(s+3)}\Big|_{s=-2}=\frac{-8+4+6}{(-2)(1)}=\frac{2}{-2}=-1$$

$$K_3=\frac{s^3+s^2+6}{s(s+2)}\Big|_{s=-3}=\frac{-27+9+6}{-3(-1)}=\frac{-12}{3}=-4$$

$$£^{-1}\left[\frac{s^3+s^2+9}{s(s+3)(s+5)}\right]=\delta(t)+[1-1e^{-2t}-4e^{-3t})]u(t)$$

A Check

$$1+\frac{1}{s}+\frac{-1}{s+2}+\frac{-4}{s+3}=\frac{s(s+2)(s+3)+(s+2)(s+3)-s(s+3)-4s(s+2)}{s(s+2)(s+3)}$$

$$=\frac{(s^3+5s^2+6s)+(s^2+5s+6)-(s^2+3s)-4(s^2+2s)}{s(s+2)(s+3)}$$

$$=\frac{s^3+s^2+6}{s(s+2)(s+3)}$$

Biomedical Signals and Systems
Quiz #3 Answers

$$3b)\ [\frac{6s+6}{(s^2+9)(s+1)}] = \frac{6(s+1)}{(s^2+9)(s+1)} = \frac{6}{(s^2+9)}$$

$$= \frac{\mathbf{K_1}}{s - j3} + \frac{\mathbf{K_1}^*}{s + j3}$$

$$\mathbf{K_1} = \frac{6}{(s+j3)}\Big|_{s=j3} = \frac{6}{(j6)} = -j = 1\angle -\frac{\pi}{2}$$

$$\mathbf{K_1}^* = 1\angle\frac{\pi}{2}$$

$$£^{-1}[\frac{6s+6}{(s^2+9)(s+1)}] = £^{-1}[\frac{6}{(s^2+9)}] = £^{-1}[\frac{1\angle -\frac{\pi}{2}}{s - j3} + \frac{1\angle\frac{\pi}{2}}{s + j3}] = [2\cos(3t - \frac{\pi}{2})]u(t) = 2\sin(3t)u(t)$$

OR

$$£^{-1}[\ [\frac{6s+6}{(s^2+9)(s+1)}] = \frac{\mathbf{K_1}}{s - j3} + \frac{\mathbf{K_1}^*}{s + j3} + \frac{K_2}{(s+1)}]$$

$$K_2 = \frac{6s+6}{(s^2+9)}\Big|_{s=-1} = \frac{-6+6}{(1+9)} = 0$$

$$\mathbf{K_1} = \frac{6s+6}{(s+j3)(s+1)}\Big|_{s=j3} = \frac{18j+6}{(j6)(3j+1)} = \frac{6(3j+1)}{(j6)(3j+1)} = \frac{6}{(j6)} = -j = 1\angle -\frac{\pi}{2}$$

3b) A Check

$$[\frac{1\angle -\frac{\pi}{2}}{s - j3} + \frac{1\angle\frac{\pi}{2}}{s + j3}] = \frac{-j}{s - j3} + \frac{j}{s + j3} = \frac{-j(s+j3) + j(s - j3)}{(s - j3)(s+j3)}$$

$$= \frac{-js + 3 + js + 3}{(s - j3)(s + j3)} = \frac{6}{(s^2+9)}$$

Biomedical Signals and Systems
Quiz #3 Answers

$$4c)\pounds^{-1}[\ \frac{s}{s^3(s+1)}] = \pounds^{-1}[\ \frac{1}{s^2(s+1)}] = \frac{K_1}{s+1} + \frac{M_0}{s^2} + \frac{M_1}{s}$$

$$K_1 = \ \frac{1}{s^2}|_{s=-1} = \frac{1}{1} = 1$$

$$M_0 = \frac{1}{(s+1)}|_{s=0} = 1$$

$$M_1 = \frac{d\frac{1}{(s+51}}{ds}|_{s=0} = \frac{d(s+1)^{-1}}{ds}|_{s=0} = -1(s+1)^{-2}|_{s=0} = -\frac{1}{(s+1)^2}|_{s=0}$$

$$= -1$$

$$\pounds^{-1}[\ \frac{1}{s^2(s+1)}] = \pounds^{-1}[\frac{1}{s+1} + \frac{1}{s^2} - \frac{1}{s}]$$

$$= [e^{-t} + t - 1]u(t)$$

3c)A Check

$$[\frac{1}{s+1} + \frac{1}{s^2} - \frac{1}{s}] = \frac{s^2 + (s+1) - s(s+1)}{s^2(s+1)}$$

$$= \frac{1}{s^2(s+1)}$$

Biomedical Signals and Systems
Quiz #3 Answers

$$4)a)\ \ddot{h}(t)+7\dot{h}(t)+12h(t)=\delta(t)$$

$$(s^2+7s+12)H(s)=1$$

$$H(s)=\frac{1}{(s^2+7s+12)}=\frac{1}{(s+3)(s+4)}$$

$$=\frac{K_1}{(s+3)}+\frac{K_2}{(s+4)}$$

$$K_1=\frac{1}{(s+4)}|_{s=-3}=\frac{1}{(-3+4)}=1$$

$$K_1=\frac{1}{(s+3)}|_{s=-4}=\frac{1}{(-4+3)}=-1$$

$$h(t)=(e^{-3t}-e^{-4t})u(t)$$

CHECK

$$[\frac{1}{(s+3)}-\frac{1}{(s+4)}]=[\frac{s+4-(s+3)}{(s+3)(s+4)}]$$

$$=\frac{1}{(s+3)(s+4)}$$

$$4)\ b)\ \ddot{y}(t)+7\dot{y}(t)+12y(t)=x(t)=e^{-4t}u(t)$$

$$(s^2+7s+12)Y(s)=(s+3)(s+4)Y(s)=\frac{1}{s+4}$$

$$Y(s)=\frac{1}{(s+4)^2(s+3)}=\frac{K_1}{s+3}+\frac{K_2}{(s+4)^2}+\frac{K_3}{(s+4)}$$

$$K_1=\frac{1}{(s+4)^2}|_{s=-3}=\frac{1}{(1)^2}=1$$

$$K_2=\frac{1}{(s+3)}|_{s=-4}=\frac{1}{-1}=-1$$

$$K_3=\frac{d\frac{1}{(s+3)}}{ds}|_{s=-4}=-\frac{1}{(s+3)^2}|_{s=-4}=-1$$

$$Y(s)=[\frac{1}{s+3}+\frac{-1}{(s+4)^2}+\frac{-1}{(s+4)}]$$

$$y(t)=(e^{-3t}-te^{-4t}-e^{-4t})u(t)$$

CHECK

$$\frac{1}{s+3}+\frac{-1}{(s+4)^2}+\frac{-1}{(s+4)}=\frac{(s+4)^2-(s+3)-(s+3)(s+4)}{(s+3)(s+4)^2}$$

$$=\frac{s^2+8s+16-s-3-s^2-7s-12}{(s+3)(s+4)^2}=\frac{1}{(s+3)(s+4)^2}$$