

ISLAMIC GIRIH TILES IN THEIR OWN RIGHT AS A HISTORY LESSON AND DESIGN EXERCISE IN THE CLASSROOM

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Abstract: *Fifteenth century Islamic architects and artisans may have presaged the twentieth century discovery of five-fold symmetry non-periodic tiling by their use of a set of polygon-shaped design templates called "girih tiles", but the historical use of the tiles by the Islamic artisans includes mostly periodic and hierarchial tilings. The idea that certain examples of Islamic tilings produced in the 15th century are true non-*

periodic tilings similar to those discovered by Penrose in the 20th century remains controversial. Non-periodic five-fold symmetry tilings are a useful analogy to 3D atomic structures called quasi-crystals. New ways to arrange girih tiles into novel random non-periodic designs may provide useful analogies as well. In the design classroom, the emphasis should be placed on all uses, not just non-periodic tiling, and design exercises should encourage students to look for new ways to tile and decorate girih tiles.

Keywords: Medieval Islamic non-periodic tiling quasi-crystal five-fold symmetry

INTRODUCTION

An article in Science by Lu and Steinhardt describes Lu's investigations into fifteenth century Islamic architectural tilings involving five-fold symmetry tiles used to decorate mosques, mausoleums and shrines. The tilings often have a complex interlacing of wide lines usually referred to as "strap patterns" and when looking at them it is not immediately apparent that there is an underlying or hidden tile layout pattern that governs their placement. Lu found that he could break up the complex tile patterns by overlaying tiles from a set of five equilateral polygon shapes, (Figure 1), that he termed "girih tiles". "Girih" is an Arabic term that refers to the patterns of ornate tilings that cover medieval Islamic mosques, palaces and other buildings. He plausibly showed how Islamic architects/artisans used them to more easily design and layout the construction of complex tilings involving 5-fold and 10-fold symmetries. Cromwell has added several others into this set that he calls Islamic prototiles.

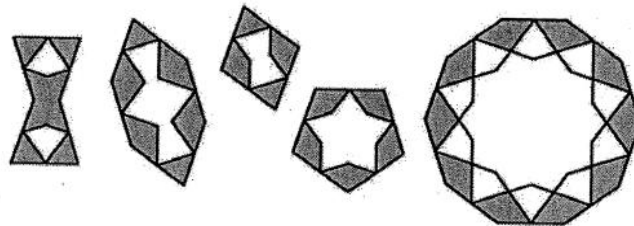


Figure 1: The set of five girih tiles: bowtie, bobbin, rhombus, pentagon, and decagon.

A BRIEF HISTORY OF ISLAMIC TILING TECHNIQUES

It was the Islāmī medieval Islamic religious buildings that used these techniques. The symmetries as well as the domes. Many of the edge and compartment designs create the design of the design method. The complex tile patterns to layout the mosque construction, a five-fold symmetry pattern otherwise result. He found evidence for the high level of similar tiles, with subdivisions, was generated. However, particularly that certainly possible process, or by derived from girih Islamic artists who they found no evidence. They recognized the framework and My opinion is that construction phase used or even complex subdivision process

A BRIEF HISTORY OF MEDIEVAL ISLAMIC TILING TECHNIQUES

It was the Islamic tradition of not depicting images of people or animals that led medieval Islamic artists to produce intricate decorative patterns in tile to decorate their religious buildings, some of the tile patterns requiring very sophisticated geometrical techniques. Their art evolved over three centuries and eventually included complex symmetries as well as precise methods for setting tiles on building walls, archways, and domes. Many of the designs are periodic and may have been drawn using only straight edge and compass, but more complex patterns suggests that another method was used to create the designs. Peter Lu, a Harvard graduate student in physics, recently discovered the design method used by the Islamic artisans for constructing some of their most complex tile patterns. The method uses templates of the five girih tiles mentioned above to layout the more complex patterns that define the actual glazed tiles positions. During construction, a few full-sized girih tile templates could be used by the artisans to scratch the overall design into the plaster base, thereby allowing a more rapid and accurate pattern generation and at the same time minimize the accumulation of error that would otherwise result. In several examples of tilings at the Darb-i Imam shrine, Isfahan, Iran, he found evidence of the use of a larger scale self-similar girih tiles which accounted for the high level of accuracy of the whole tile layout. This use of large-scale self-similar tiles, which contained non-periodic patterns of small-scale versions, called subdivisions, was found to be similar to the way non-periodic Penrose tilings are generated. However, this finding remains controversial in the view of others, particularly that of Makovicky and Cromwell. Cromwell concludes that, although it is certainly possible to construct quasi-periodic tilings from girih tiles by an inflation process, or by using point-in-contact methods in conjunction with matching rules derived from girih tiles filled with Penrose kites and darts, there is no evidence that Islamic artists were aware of these processes. Lu and Steinhardt said in their article that they found no evidence that the Islamic designers developed a matching rule approach. They recognized only that the Darb-i Imam tessellation is not imbedded in a periodic framework and can, "in principle", be extended into an infinite non-periodic pattern. My opinion is that a hierarchial method was used to draw the pattern during the construction phase and it is impossible to know whether or not the Islamic designers used or even conceived of the self-similar process of pattern generation that uses the subdivision process.

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Girih tiles are decorated with a pattern of lines that intersect the midpoint of each edge of the tile, (Fig. 1), so that when they are tiled together a more complex pattern emerges defining the location of decagons, pentagons, and other tile shapes. Sometimes the lines show up in the final tiling as embossed overlapping wide lines called strap patterns or by a line of rectangular tiles with implied overlapping at intersections, or, they may be there only as a line of grout between tiles.

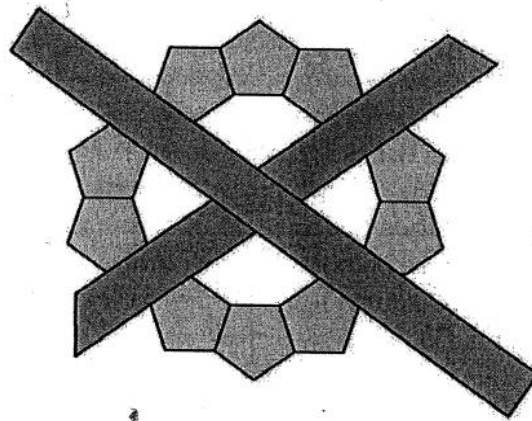


Figure 2: Exploded Decagonal leitmotif patch.

The large-scale strap line pattern in the designs at the Darb-i Imam shrine, (see Lu and Steinhardt Supporting Online Material, Fig. S7A), are in the design for at least two reasons. First they reveal a beautiful heirarchical structure that reflects the structure of the small-scale glazed tile pattern, but it should be noted that it is not scale invariant. Second, they may be necessary to cover up the seams of the construction process. The intersections of the strap lines always occur at the center of a decagon-shaped tile. The decagon, surrounded by ten pentagons, is the leitmotif of the large-scale pattern. Looking closely, you can see that the decagon shaped tiles have been cut and spread apart to accommodate the strap line tiles. This gives the decagon a distorted shape and it appears larger in diameter when seen from a distance when comparing them to decagons that have not been intersected. The distortions can be explained by using a graphic process in which parts of a drawing are exploded apart leaving an equal space between them as I have demonstrated Figure 2.

I think that the glazed tiles may have been set in plaster on a polygon shaped supporting panel in a horizontal position, perhaps on a large table or on the ground. The polygon shapes of the support panels are determined by the large-scale strap line pattern. Only

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three large-scale templates would be needed to transfer the tile pattern to a panel, a pentagon, a kite, and half of a ten-pointed polygon that is the outline of the leitmotif. It would be much easier to draw the pattern working on a horizontal surface and it would be far easier to set the glazed tiles. The tile laden panels would be fastened to the wall above the portal after they were fully bonded to the panels and the strap line tiles would be added later to hide the seams. This hierarchical method would ensure that the geometry would be accurate across the entire expanse of the wall. Even if the glazed tiles were set directly on the wall, one at a time, the pattern could have been transferred to the wall in a similar manner using the 3 large-scale templates. It would have been much more difficult to use a large-scale self-similar girih tile template because it would have had to provide space for the strap line tiles by repositioning the small-scale girih tile positions.

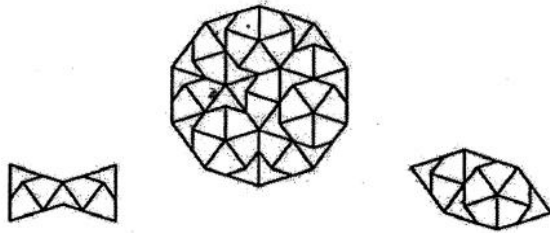


Figure 3: Penrose kite and dart tile shapes fill girih tiles.

Lu was intrigued by the possibility that some of the patterns were produced in the same manner as the tilings of the famous kites and darts invented by Penrose, i.e., by using the self-similar transformation method. By filling three of the girih tiles, (Figure 3), with arrangements of Penrose kites and darts he was able to show that he could overlay them on the girih tile pattern found in a spandrel from the Darb-i Imam shrine in Isfahan, Iran. Figure 4 is a drawing of the subdivision rules transforming the large bowtie and decagon girih-tile pattern into the small girih-tile patterns found on the spandrel. This demonstrated that a perfect Penrose tiling could be overlaid on the girih tile pattern of the spandrel after a few minor corrections to the girih tile pattern of the spandrel. Parts of the overlay are strikingly similar to the cartwheel pattern shown in Martin Gardner's 1977 on the theory of Penrose tiling. But does this really demonstrate that the Islamic architects/artisans presaged the discovery of non-periodic tiling that came five centuries later? Although the Penrose tiling overlay is perfect within the spandrel boundary it would be impossible to expand it very far beyond the spandrel

area using this large-scale girih tile pattern. This is one of the most common girih tile patterns, (Fig. 5c), and it is obviously periodic. Since Penrose kites and darts come with matching rules that force them to be tiled non-periodically, by extension the small-scale girih tiles also have matching rules. Sooner or later, the matching rules will be violated if they are put into a periodic pattern. Why did the Islamic designers choose the periodic pattern of Figure 5c over a self-similar pattern? The periodic pattern is better suited to the architectural space for the design. A self-similar pattern would require a much larger space to generate an interesting design, i.e., too little of it would show up in the spandrel area to make sense of it. Penrose points out in his paper on aesthetics that a symmetrical configuration allows one to appreciate the whole by examining only a portion of it. If the Islamic designers had used the self-similar pattern only a very small portion of it that would fit into the spandrel and it would have been too small to notice or understand it. The designers may have understood the concept of self-similar design and subdivision, but rejected it as being out of scale with their architectural project.

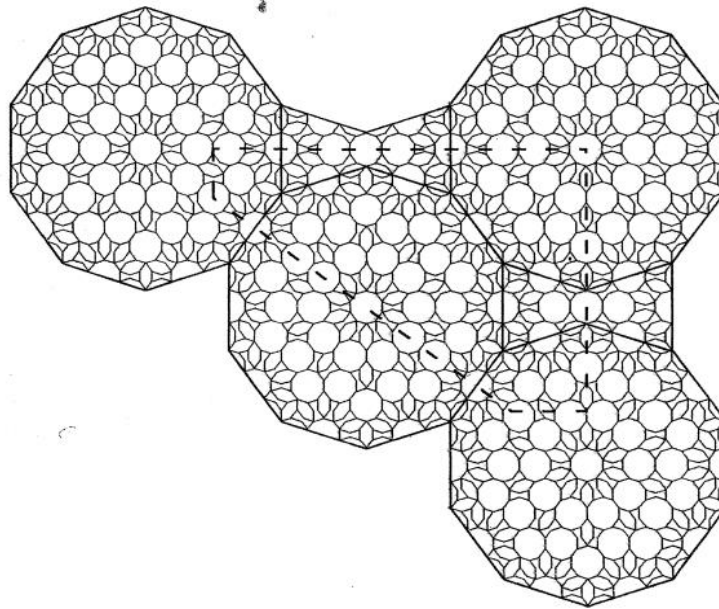
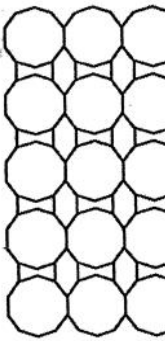
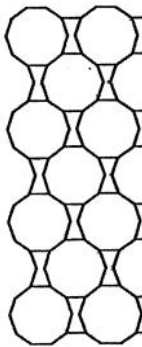


Figure 4. Girih-tile subdivision pattern rules. The area of the spandrel from the Darb-i Imam shrine, Isfahan, Iran is indicated by the dashed lines.

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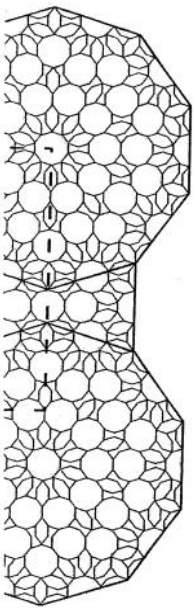
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Many of the tilings found in Islamic architecture have girih tile patterns with periodic 2-fold symmetry like the examples shown in Figure 5. Others are non-periodic but have bilateral symmetry. The most complex are non-periodic and have self-similar patterns at two different scales like those found at the Darb-i Imam shrine. But there are many other ways to arrange girih tiles that were probably never used by Islamic artisans. With five or more girih tiles to work with there are endless possibilities for new periodic and non-periodic designs.



the Darb-i Imam shrine, Isfahan,

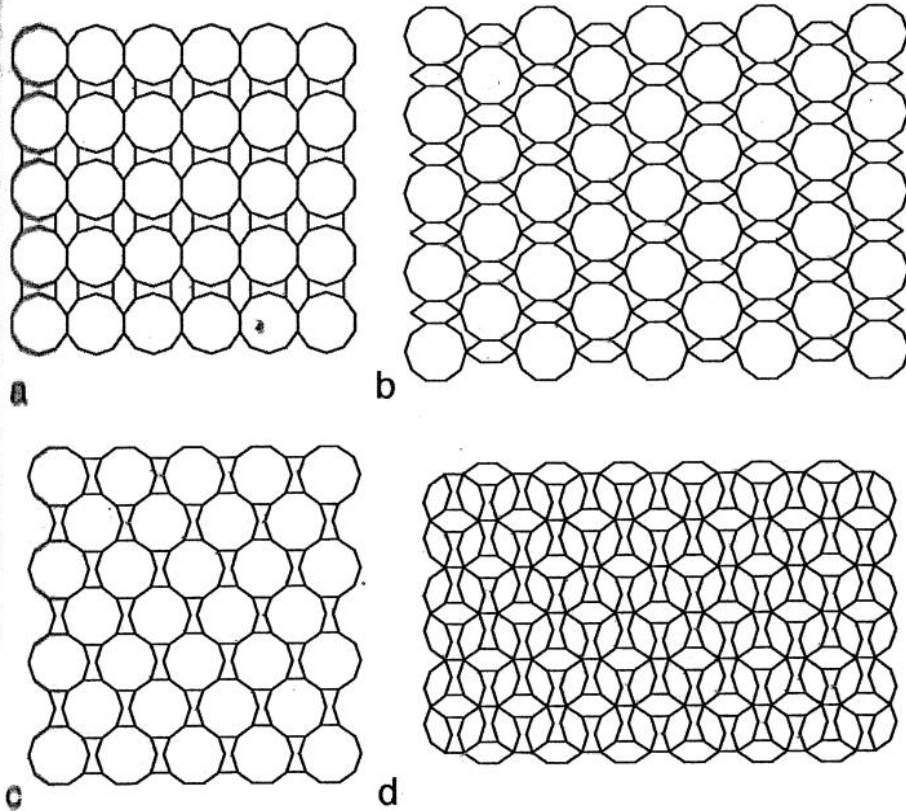


Figure 5: Typical Islamic girih tiling patterns having periodic symmetry: (a) from the Timurid shrine of Khwaja Abdullah Ansari at Gazargah in Herat, Afghanistan. (b) from the Abbasid Al-Mustansiriyya Madrasa in Baghdad, Iraq. (c) from a portal at the Friday Mosque in Isfahan, Iran. (d) from the Seljuk Mama Hatun Mausoleum in Tercan, Turkey.

I found a CAD-like drawing from a 15th century scroll, now in the Topkapi Palace Museum in Istanbul, Turkey, that contained faint lines of the girih tile patterns, which

gave him the clue as to how they were used to generate a tile design. Figure 6 is my own drawing of it copied from the original scroll drawing in which I made the girih tile lines bolder so they can be seen more easily. The Topkapi scroll drawing contains five girih tiles and has no apparent unit-cell of a periodic pattern, but it has lines, (dashed lines in Figure 6), showing a self-similar pattern to the strap lines but at a larger scale. However, the large pattern is not scale invariant with the strap line pattern. It may be that the larger pattern is used only to break up the whole tile design into manageable sections or panels for construction purposes, or it may have been the outline of the actual template used to make this drawing. Using this scroll panel and several others from the Topkapi scroll, Cromwell deduced yet another set of large-scale templates with subdivisions that could be used to generate hierarchical but not scale invariant designs similar to styles found on buildings in Isfahan, Iran.

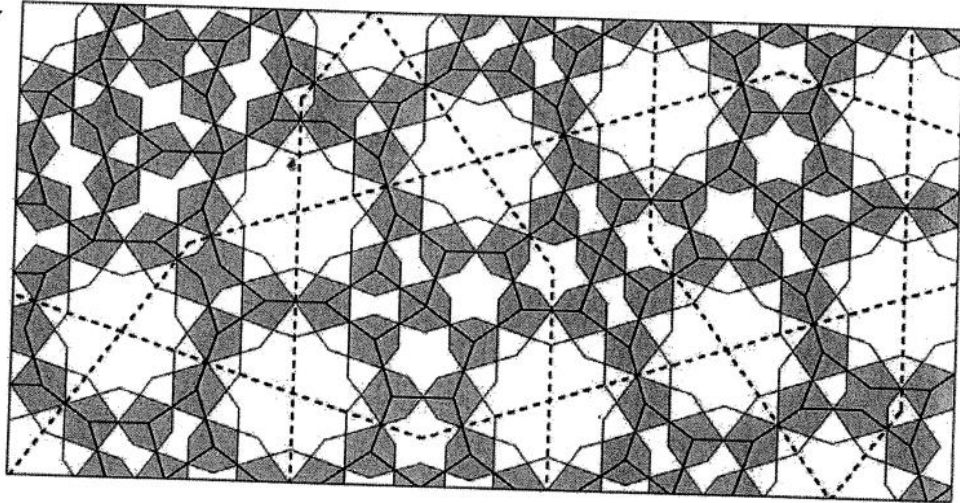


Figure 6: Girih tile pattern in panel 28 of the Topkapi Palace Museum scroll.

A similar girih tile design was extrapolated by Lu, (Figure 7), from the tiled panels at the Gunbad-i Kabud tomb tower in Maragha, Iran, but with more randomness. Notice that the girih rhombuses are always grouped into a star form. Other groupings occur as well such as a five bobbins around a point and 2 bowties end-to-end. The panel design, delineated by dashed lines, is basically a nonperiodic tiling that is reflected at the vertical edge of each panel, but if two panels are considered at once a cartwheel pattern shows up that is similar to those found at the Darb-i Imam shrine. The tower has eight sides with a panel on each side. Three of the panels are shown here to reveal the bilateral symmetry created by the reflections.

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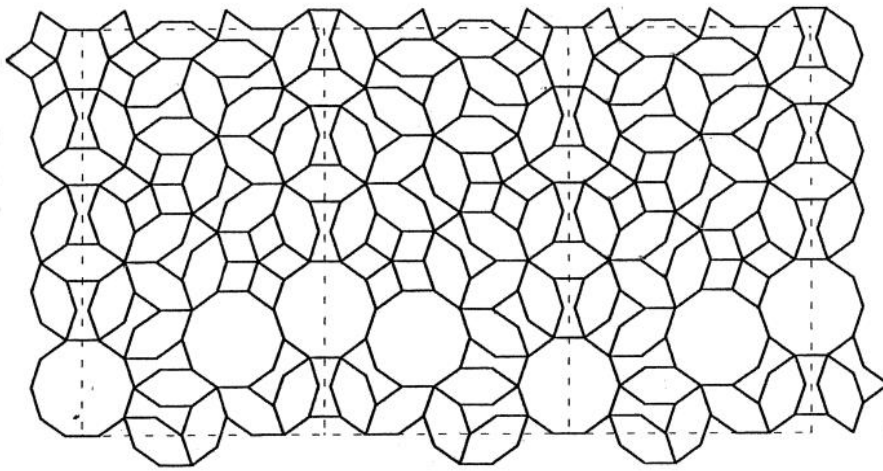
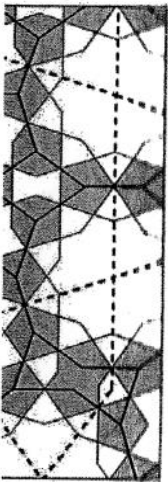


Figure 7: Girih tile extrapolation by Lu of 3 panels at the Gunbad-i Kabud tomb tower.

CLASSROOM DESIGN EXERCISE

There are many creative possibilities for the student using the girih tiles in their own right, i.e., to find ways to arrange them periodically or non-periodically to produce a tile design. I explored this idea by making dozens of drawings of periodic and non-periodic girih tilings. I started using a shorthand method of drawing by converting a patch of several girih tiles into a "super girih tile" shape. For example, a star-shaped patch of 2 bowties and 1 bobbin tiles, as seen in Figure 8, becomes a star-shaped super girih tile. Another interesting super girih tile shape can be made from 2 bobbins and 1 bowtie. It has an s-shaped form that I call a "link", (because it reminds me of shapes found in tire treads that are meant to instill the feeling of a chain link), and its mirror image. Figure 8 shows two of my drawings: a non-periodic spiral of decagons, stars, and bobbins, and a non-periodic tiling of decagons, stars, links, and bobbins. Both of these drawings are more interesting to look at using the super girih tile shapes than they would otherwise look if I deconstructed them into bobbins and bowties.

There are many different ways to create spirals depending on how the drawing is started. A two or three arm spiral can be made in the same manner, or it can start with an s-shape of decagons with the ends of the "s" continuing to spiral around each other like the tails of a spiral galaxy. The intervening spaces can be filled with patterns of bobbins and bowties. On the other hand, the non-periodic drawing of Figure 8 (right) is



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much more difficult to draw. The pattern never repeats except for small local formations and there is no rule for setting the next tile except to not allow voids in the pattern and to keep the density of the different tile shapes more or less the same. This pattern can be enlarged but it is a real puzzle to find arrangements of tiles that fill all the space without voids. Sometimes it is necessary to deconstruct an area to start again to find a workable arrangement. I think this pattern has a lot of visual appeal, i.e., it has a certain flair and unpredictability about it, especially when compared to the spiral drawing.

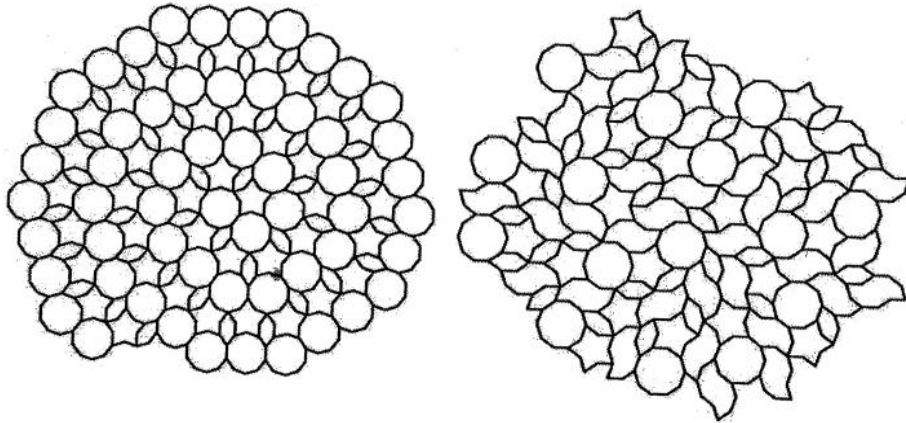


Figure 8: Two drawings made with super girih tiles: (left) a spiral pattern of decagons, stars, and bobbins, (right) a non-periodic tiling with decagons, stars, links, and bobbins.

There are several reasons why girih tiles may be just as interesting to mathematical researchers as Penrose tiles proved to be. The ratio of the area of the kite to the dart is the same as the golden ratio, $\tau = 1.618\dots$, and this is also true of the ratio of the area of the bobbin to the bowtie. An infinite non-periodic tiling of kites and darts contains $1.618\dots$ times as many kites as darts. This may also prove to be true in certain types of random non-periodic tilings of bobbins and bowties such as the one shown in Figure 9.

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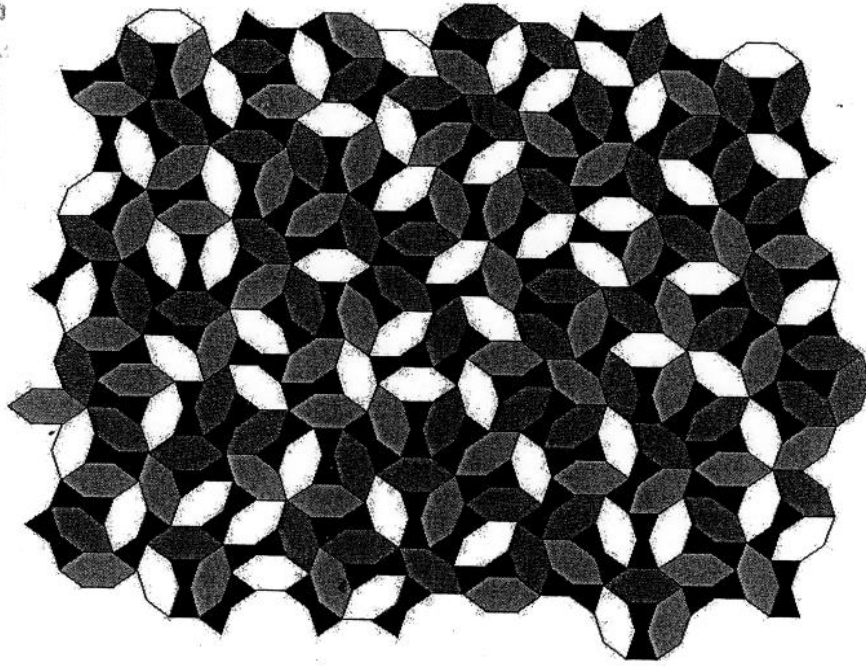
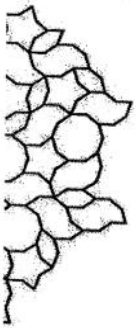


Figure 9: Random non-periodic tiling of bobbins and bowties.

This naturally raises the possibility that random non-periodic girih tilings may be related to the quasi-periodicity found in quasi-crystals. Roger Penrose had no way of knowing that the non-periodic tiling of pentagons, 5-pointed stars, 3-pointed half-stars and diamonds that he presented in his 1974 paper would someday help explain the structure of quasi-crystals. He just felt that it had a certain aesthetic appeal in the way certain regularities in the pattern jumped out optically in your mind as you looked at it. It was the aesthetic appeal of it that motivated him to study it. I feel the same way about making random patterns using girih tiles. As an artist, I have always been attracted to randomness in design. There is something about the way the girih tiles don't line up that feels more natural. It is challenging and fun to make patterns with them. It is like working on an infinitely large jigsaw puzzle, especially when working with tiles derived from patches of bobbins and bowties.

Figure 10 gathers several examples of super girih tiles but it is by no means inclusive of all the possibilities. These shapes can be useful in finding new girih tile patterns or just

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to speed up the drawing process when using a computer aided drafting program. I find them interesting as design elements in their own right. It should be noted that the decagon is also a super girih tile when it represents a patch of 3 bobbins and 1 bowtie.

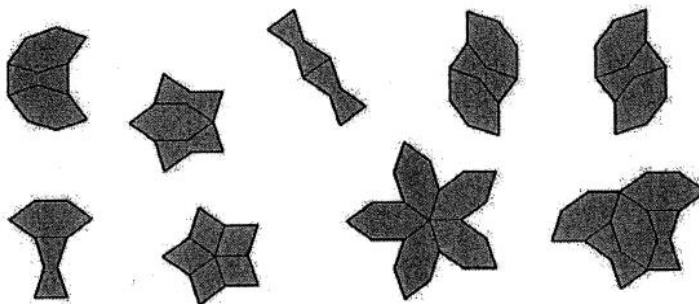


Figure 10: Several examples of super girih tiles.

In another drawing, (Figure 11), I used an "anti-crowding" rule to grow a pattern of decagons and bowties in a random manner that would, as a result, be non-periodic and have no particular center and no rotational symmetry. The same technique can be used to make drawings with perfect rotational symmetry but I find them not as interesting.

This drawing is made by adding a new decagon to an existing one with a bowtie serving as a connector. This is done randomly by trial and error until a complete circuit of decagons and bowties is made. The void inside the circuit is then completely filled with girih bobbins and bowties. My conjecture is that any area enclosed by a circuit of decagon-bowties contains an integral number of girih bobbins and bowties with no unfilled space left over. I don't know how to prove it but I have yet to find an exception.

The anti-crowding rule for this drawing is that two bowtie tiles may not touch each other on the perimeter of a decagon. This rule prevents the decagon tiles from getting too close to each other. Thus there is room for as many as three bowties to connect to any given decagon in the drawing.

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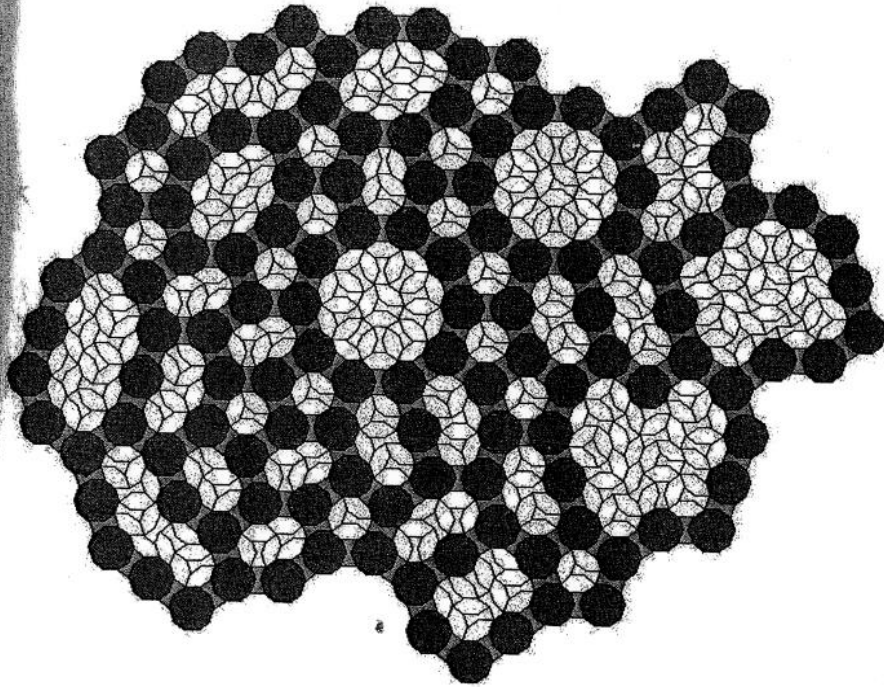


Figure 11: Random non-periodic pattern of girih tiles with no rotational symmetry.

After I made this drawing, I realized that it was a 2-dimensional analog of my most recent sculpture maquette, (Figure 12), in which the decagons represent icosahedrons and the bowties represent octahedrons. The main structure of this sculpture is grown with a similar anti-crowding rule, i.e., any two octahedra may not touch each other on the surface of a particular icosahedron. Up to four octahedra can connect to any given icosahedron. Connections can be made in any random direction and there is always a way to complete a circuit by repeating every offset with an offset in the opposite direction somewhere else in the circuit. All of the icosahedra in the sculpture have the same spatial orientation giving the sculpture a visual sense of order but overall the sculpture has an amorphous form with no rotational symmetry.

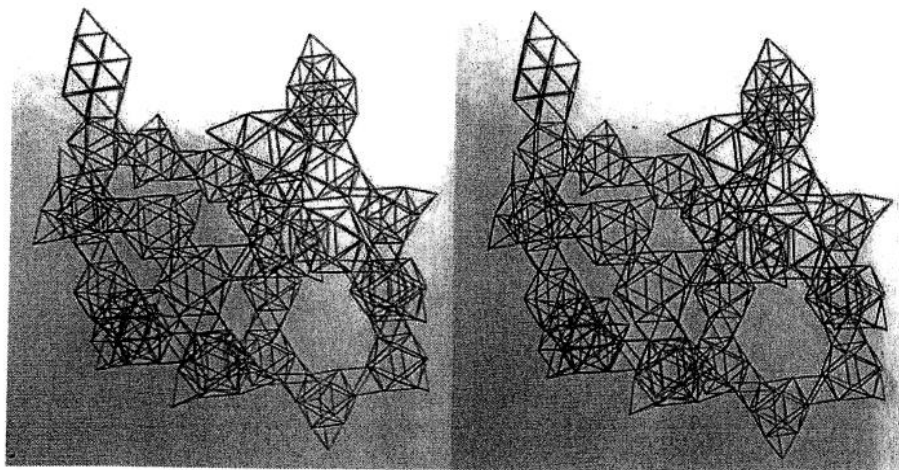


Figure 12: Cross-eyed stereo photo of Sculpture maquette, 80 cm x 70 cm by 75 cm.

CONCLUSION

Whether or not medieval Islamic artisans actually made a true non-periodic tiling of girih tiles that have self-similar subdivisions in a similar manner to the way Penrose tilings are constructed is a question that remains controversial. They may have been able to visualize it in their minds but rejected it as not suitable to their architectural requirements because the large-scale factor between self-similar tiles at different levels would require too many tiles to make the design understandable. The Islamic artisans kept their trade secrets to themselves and very little is known about their methods. If they didn't achieve it, they nevertheless came very close to it at the Darb-i Imam shrine. Even Cromwell concludes that it is possible to construct quasi-periodic tilings of girih tiles. What is important is the discovery out of history of another set of tiles that can be used to study non-periodic five-fold symmetry. Girih tiles should be studied in their own right and perhaps new mathematical principles not related to Penrose tiles will be found that may help explain quasi-periodicity in quasi-crystals.

There are many ways to tile the basic girih tile shapes, both periodically and non-periodically, and there may be new ways to decorate them that go beyond the girih lines or overlapping strap patterns used by the Islamic artisans. Replacing them with Penrose kites and darts is a case in point. Making tilings of tile shapes derived from patches of girih tiles may become an interesting pastime like working on a jigsaw puzzle.

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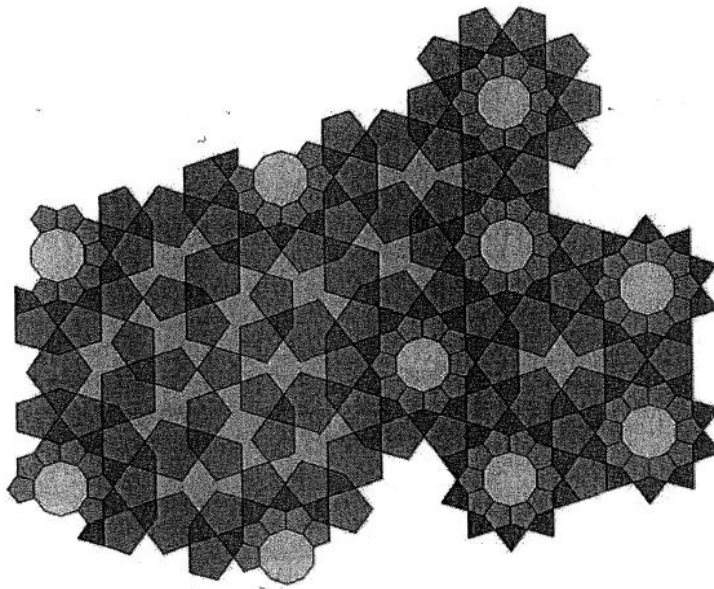
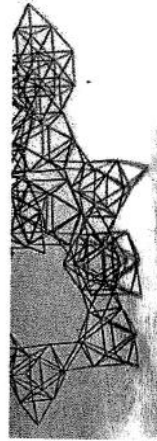


Figure 13: Hypothetical student rendering of girih tiles.

As a classroom exercise, the student would design a girih tile pattern and then decorate the girih tiles with traditional Islamic strap lines, Penrose tiles, or some other invention of the student. Or they may choose to create a non-periodic tiling of super girih tiles. Figure 13 illustrates how a student might render a girih tiling, adding color to further enhance the art. There would likely be a large variety of project results that would enliven the creative atmosphere, stimulate the imaginations of the students, and garner a lasting appreciation for the historic achievements of the Islamic architects and artisans.

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