

MODULE 18: ZONOGONS

Transformation of regular tilings

Starting with a polygon or aggregation of polygons in the form of a tiling of the plane, and applying a set of rules of transformations, there are several ways in which new polygons or tilings of the plane can be generated. Robert Williams [] has considered a number of different classes of transformations. In this section we will consider three of these classes, namely, distortions, vertex motion, and augmentation-deletion.

a. Distortion

The distortion operation consists of expanding, contracting, twisting, flattening, and stretching polygons either in isolation or in aggregation. One special type of distortion operation involves n-zonagons. An n-zonagon is a $2n$ sided polygon with opposite sides equal and parallel. For example,

the parallelograms and hexagons that combine to tile the plane regularly are 2-zonagons and 3-zonagons respectively.

An n -zonagon can be constructed by specifying a star of n -directed line segments (vectors) representing the orientation and length of its sides. For example a star of 3 vectors and the resulting 3-zonagon is shown in Figure 3.25a and b.

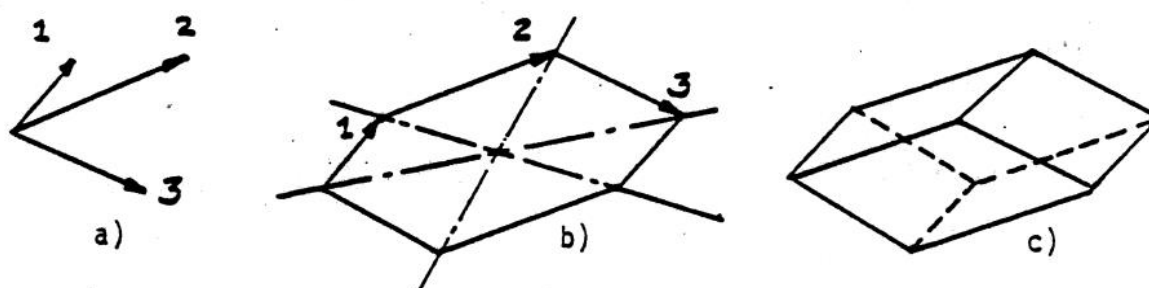


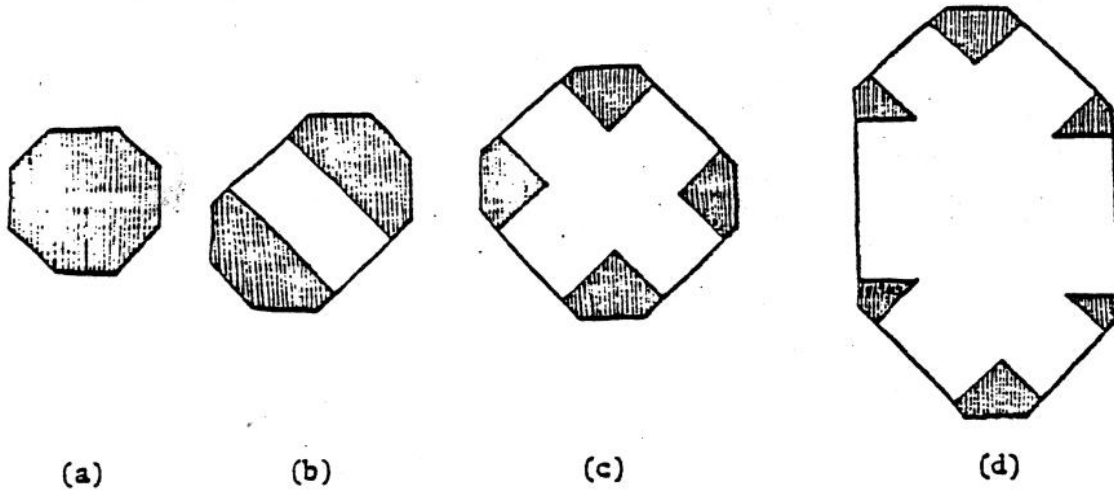
Figure 3.25 a) A star of three vectors; b) A 3-zonagon illustrating central symmetry; c) Dividing a 3-zonagon into parallelograms in two ways.

Also zonagons have the property of being centrally symmetric as shown in Figure 3.25b. In addition an n -zonagon can always be decomposed into $\frac{n(n-1)}{2}$ parallelograms, the total number of ways in which two vectors can be chosen from a set of n vectors. This is demonstrated in Figure 3.25c for a 3-zonagon in two different ways.

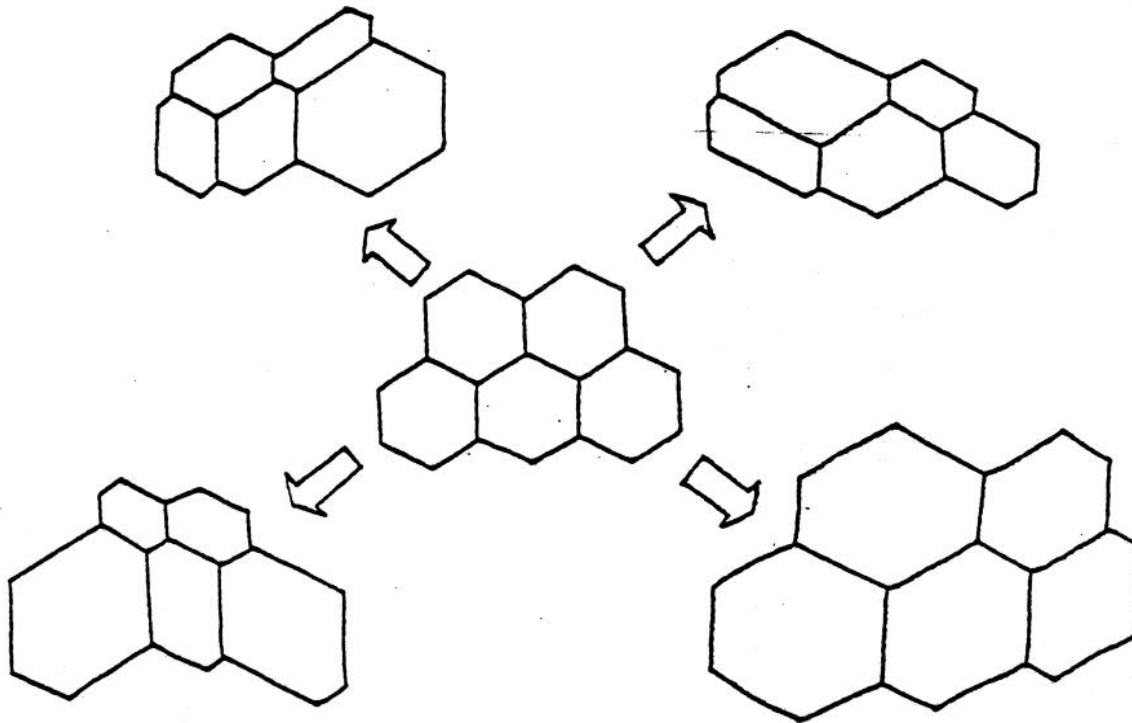
What is important about zonagons is that they can be contracted or expanded in a direction parallel to any pair of opposite sides as shown in Figure 3.25d, without altering the angles between adjacent sides. Thus any space filling aggregate of zonagons will remain space filling after distorting any individual zonagon in this way and then adjusting adjacent zonagons of the tiling accordingly, as shown in Figure 3.25d.

Construction 3.2:

Construct an interesting aggregate of 3-zonagons starting with a star of 3 vectors of your choosing. Your aggregate should illustrate the ability



Examples of stretching an individual zonagon.



Examples of stretching aggregated zonagons.

Figure 3.25d