Module 5:   A Sacred Cut Tiling Design

The Sacred Cut is gotten by taking a circular arc through the middle of a square from a vertex (see Fig. 1a). Four circular arcs in Fig. 1b leads to a regular octagon and creates a natural subdivision of the square into three rectangles (see Fig. 2): a square (S) 1:1; a square root rectangle (SR) $\sqrt{2}:1$; and a Roman Rectangle (RR) $\theta:1$ where $\theta = 1+\sqrt{2}$. The architect historians Donald and Carole Watts have discovered, by studying the city of Ostia the port city of the Roman Empire and the ruins a Pompeii and Hertzegovia, that this system, based on the sacred cut, was used to create much of the architecture of ancient Rome. A tapestry from the Garden Houses of Ostia is shown in Fig. 3 suggests that the sacred cut was used.

We will describe the algebra behind this system of proportions in Module 11, A Unified Theory of Proportions. Here we focus on the geometry of the three species of rectangle: S, SR, and RR as shown in Fig. 4. Notice that if a double square (DS) is removed from a RR it leaves behind another RR at a smaller scale. In a similar way DS can be added to RR to yield an RR at a larger scale. We refer to the original rectangle as a *unit U* and the portion of the rectangle removed to yield the unit at a smaller scale as a *gnomon G* so that U = U + G. In this case G = DS (double square). This is an example of the Law of Repetition of Ratios that will be discussed in greater detail in Module 10.

What is the gnomon of an SR rectangle? If an SR is cut in half it gives rise to a pair of SR rectangles at a smaller scale, so that we can say G = SR and SR = SR + SR. Likewise, an SR can be added to another SR to give rise to an SR at a larger scale.

Fig. 4 also shows that if S is either removed from or added to an SR rectangle the result is an RR at either a smaller or larger scale.

We see from Fig. 4 that the Roman system of proportions has many additive properties, one of the essential elements of a good system of proportions. Use this system to carry out the following construction.

Construction: Use the geometric properties of the Roman system to create a number of S,SR, and RR rectangles at three different scales and use them to create an interesting tiling design. Add color to your design. One response to this construction, shown in Fig. 5, was carried out by Mark Bac.
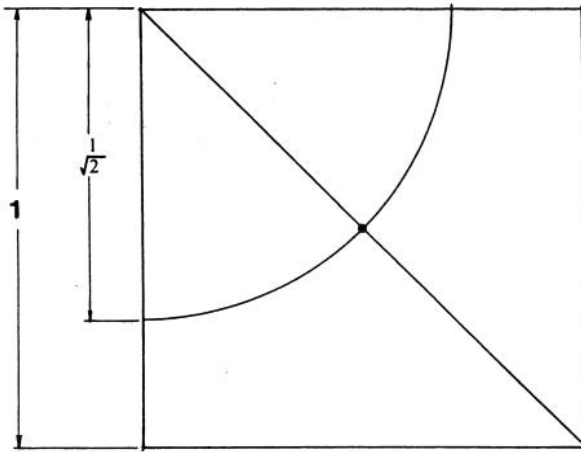
a) The sacred cut.
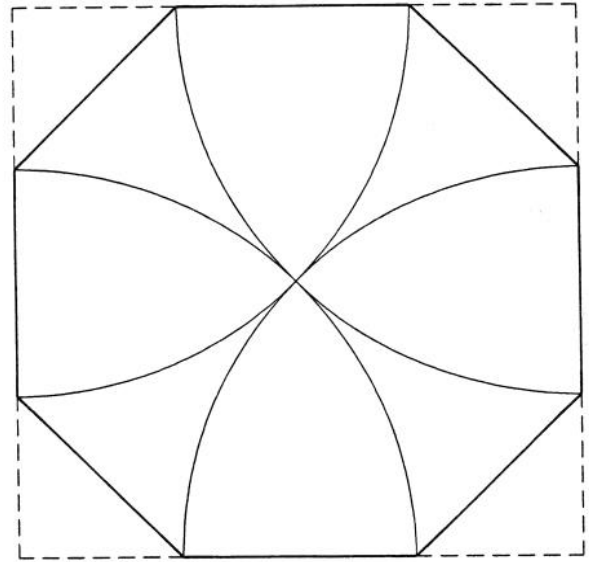
b) Construction of a regular octagon from four sacred cuts.
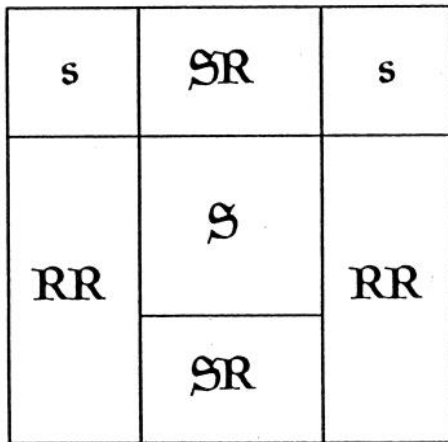
FIG. 1

Fig 2. Subdivision of a square into
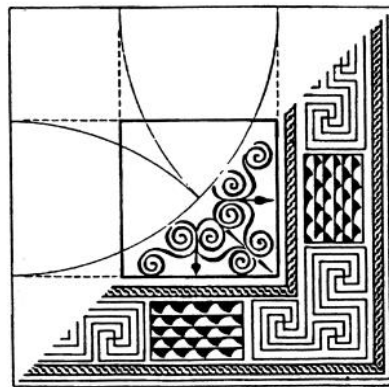S, SR, and RR rectangles

Fig. 3. A Floor mosaic found in the one of
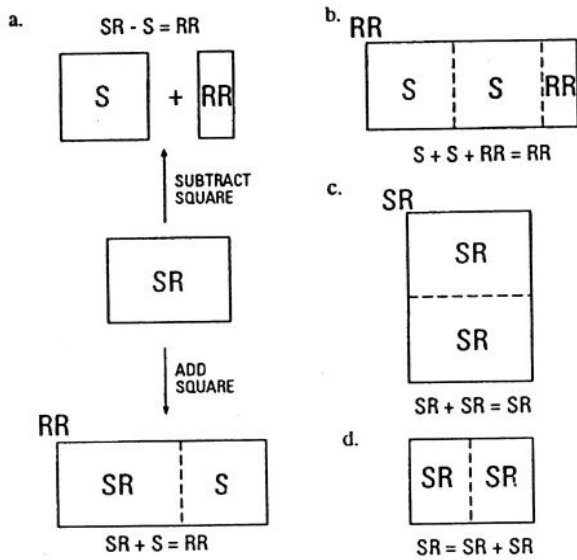Garden Houses of Ostia using the
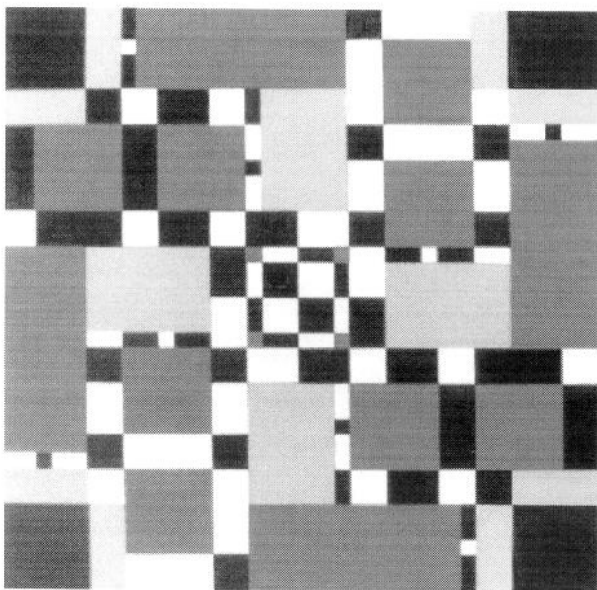sacred cut

Figure 4. Geometric relationships between S, SR, and RR.

Fig. 5 A sacred cut design by Mark Bak