Module 9: The Modulor of LeCorbusier

1. The golden mean

The standard Fibonacci series or F-sequence is defined as follows:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots \qquad (1)$$

Notice that each integer is the sum of the preceding two integers. This sequence has the property that the ratio of successive integers approaches the irrational number,

$$\phi = \frac{1+\sqrt{5}}{2} = 1.618...$$

called the golden mean, in the sense of a limit, i.e., 1/1, 2/1, 3/2, 5/3, 8/5, … approaches ϕ in terms of agreeing in more an more decimal places. How far out in the series do you have to go for ϕ to agree with the ratio of successive terms of the F-series to five decimal places?

Construction: Create a hierarchal pattern from the F-sequence. Your fundamental pattern can be dots, lines, or anything else of your choosing. Order your modules to give a geometrical rendering of the F- sequence. It might be useful to use graph paper to help organize your work at first. See pages 77 and 78 in Connections for examples.

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The golden mean has the property:

$$1+\phi = \phi^2 \qquad (2)$$

Verify Eq. 2 by showing, using algebra, that,

$$1+\left(\frac{1+\sqrt{5}}{2}\right) = \left(\frac{1+\sqrt{5}}{2}\right)^2$$

As a result of Eq. 2,

$$\ldots \frac{1}{\phi^2}+\frac{1}{\phi}=1,\quad \frac{1}{\phi}+1=\phi,\quad 1+\phi=\phi^2,\quad \phi+\phi^2=\phi^3, \ldots \qquad (3)$$

Therefore the double geometric series,

$$\ldots\frac{1}{\phi^2}, \frac{1}{\phi}, 1, \phi, \phi^2, \phi^3, \ldots$$

Is also a Fibonacci series called the ϕ – sequence.

It is because the $\phi -$ sequence is both geometric and additive that LeCorbusier used it as the basis of a system of architectural proportions called the Modulor (See Sec. 1.6.1 and 1.7 of Connections).

To see how the additive properties work from an algebraic standpoint consider the expression:

$$\phi^2 + 2\phi - 1 - \frac{1}{\phi}.$$

By using the Fibonacci properties of the $\phi -$ sequence this expression can be transformed as follows:

$$\begin{aligned} &\phi^2 + 2\phi - 1 - \frac{1}{\phi} \\ &= (\phi^2 + \phi) + (\phi - 1) - \frac{1}{\phi} \\ &= \phi^3 + \frac{1}{\phi} - \frac{1}{\phi} \\ &= \phi^3 \end{aligned}$$

1. Use the additive properties of the $\phi -$ sequence given by Eq. 3 to determine which pairs of the following expressions are equal. Do not use a calculator to solve this problem although you can check your results on a calculator.

Exercise 1:

a) $2\phi^2 + 1$
b) $\phi^3 - \phi + \phi^2$
c) $\phi + 3 + \frac{1}{\phi}$
d) $2\phi + 3$

Exercise 2:

a) $\phi + \phi^2 - \frac{1}{\phi} + 2$
b) $\phi^2 + 3\phi + 1$
c) $2\phi^2$
d) $2\phi + 2 + \frac{1}{\phi^2}$

2. One of the two diagrams in Fig. 1, A or B, is not consistent in its dimensions. Which is it and why? (Note: the diagrams are not drawn to scale)

3. Find a tiling of the rectangle in Fig. 2 into three tiles with two of them golden mean rectangles (sides in proportion $\phi:1$) and one square. The rectangle is not drawn to scale.

4. The $\phi-$series can be constructed using compass and straightedge as follows:

a) Divide a line segment into the golden section to obtain a pair of lengths 1 and ϕ units (see page 83 of Connections.)
b) Mark these lengths off on a straight line as indicated by AB and BC in Fig. 3a.
c) Place your compass point at C and pencil point at A and construct point D by sweeping out arc AD. From the Fibonacci properties of the $\phi-$ sequence, CD = ϕ^2.
d) Place your compass point at A and the pencil point at C and construct point E by sweeping out arc EC. $EA=\frac{1}{\phi}$ follows from the Fibonacci properties of the $\phi-$sequence.
e) Continue this procedure as indicated by Fig. 3 to obtain the entire $\phi-$ sequence shown in Fig. 3b.

2. The Modulor of LeCorbusier

1. The Modulor of LeCorbusier is a double series of scales referred to as the Red and Blue series. These scales are illustrated in Fig. 4a.

Blue | $\frac{2}{\phi}$ 2 2ϕ $2\phi^2$ $2\phi^3$

Red | 1 ϕ ϕ^2 ϕ^3 ϕ^4

Fig. 4a The Red and Blue series

2. LeCorbusier conceived of the Modulor as embodying human scale so that 1 unit from the Red sequence was taken to be a six foot British policeman while $\frac{2}{\phi}$ units from the Blue sequence was the measure of the policeman with his hand raised over his head as in Fig. 4b.

3. Each scale is a $\phi-$sequence. However, each length of the Blue sequence is twice the length of a corresponding length of the Red sequence. Notice how the lengths of each scale intersperse the other. In fact, each Red length bisects the two lengths of the Blue sequence that brace it. However, each length of the Blue sequence divides the line segment of the two bracing Red lengths in the golden section ($\phi:1$). Show this by using the

algebraic properties of the $\phi-$ sequence given in Eq. 3 to prove the following identity:

$$\frac{2\phi-\phi^2}{\phi^3-2\phi}=\frac{1}{\phi}$$

The lengths from the Red and Blue series were used by LeCorbusier to form the lengths and widths of a modular set of rectangular tiles shown in Fig. 5. In this figure there are three types of rectangles. Type 1 is made up of lengths and widths from the Red series, type 2 uses lengths and widths from the Blue sequence, while type 3 uses a length from the Red and a with from the Blue or vice versa. These tiles can be used to partition any rectangular space such as a room or the façade of a building.

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4. Fig. 6 has been taken from LeCorbusier's book, Modulor. Each rectangle has been subdivided into Red and Blue sequence rectangles coded according to the rectangles on the upper right. Use the Fibonacci properties of the $\phi-$ sequence to show that the sum of the lengths across the top edges of the rectangles in Fig. 6 agree with the sum of the lengths across the bottom edges. Also check the left and right edges for agreement.

Example: For the rectangle on the upper left hand corner of Fig. 6:

$$L_6+L_3+L_1=L_{11}+L_4$$
$$1+2\phi+2\phi^2=2\phi^2+\phi^3$$
$$\phi^2+\phi+2\phi^2=2\phi^2+\phi^3$$

Therefore, $$\phi^3+2\phi^2=2\phi^2+\phi^3$$

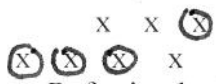
5. Because of the additive properties of the $\phi-$ sequence, the Modulor is a versatile system of proportionality. Fig. 7 illustrates how the tiles of one particular subdivision of a $\phi^2\times 2\phi$ rectangle can be rearranged to tile the rectangle in 48 other ways. The original subdivision is not of interest from a design standpoint. However, several of the subdivisions are aesthetically pleasing.

The six tiles shown in Fig. 8 with listed dimensions can be combined in many different ways to form a square of dimensions $\phi^2\times\phi^2$. Two ways are shown. Cut out several sets of the 6 tiles and combine them in at least 8 other combinations to form squares of dimension, $\phi^2\times\phi^2$.

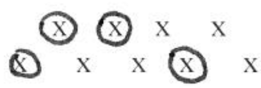
6. There are many identities hidden in the Red and Blue sequences. It is sometimes helpful to create an approximation to this double series with the F-series and its double shown as follows:

	2		4		6		10		16
1	2	3		5		8		13	

We see that 1 + 2 + 3 = 6 which has a pattern given by the circles,



. Referring back to the Red and Blue sequences in Fig. 4 this is equivalent to $1+\phi+\phi^2=2\phi^2$ which has the same pattern. As another example, 3+4+6=13 with the pattern,



which has the same pattern as : $\phi+2+2\phi=\phi^4$.

3. A Workshop on the Modulor

Material needed:

a) Enlargements of the rectangles in Fig.5.
b) Scissors
c) Rubber cement
d) Magic markers

Work in a group of three or four students and carry out the following steps:

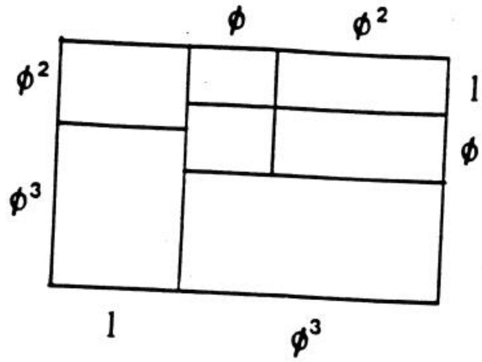
1. Make several copies of each enlarged rectangle from Fig. 5.
2. Draw three squares of dimensions $2\phi^3\times2\phi^3$ at the scale of our rectangles.
3. Tile a square of dimensions $2\phi^3\times2\phi^3$ using tiles from the Red and Blue sequences.
4. For each tiling, use the $\phi-$ sequence to show that the sum of the lengths along the top equals the sum of the lengths along the bottom of the tiling and the same for the lengths along the left and right sides.
5. Rearrange the same tiles to get other tilings of the $2\phi^3\times2\phi^3$ square.
6. When you get an interesting tiling, use rubber cement to make it permanent and color it with magic markers.

Project: Create three different breakdowns of a $2\phi^3\times2\phi^3$ square and tile the square in three different ways using each breakdown. Finally tile a 5 inch by 5 inch square to within a quarter inch tolerance using rectangles from the Red and Blue sequences. One example of this project is shown in Fig. 9.

Figure 10 shows the façade of a building in Paris, Unite' Habiliatation in which LeCorbusier incorporated the Modulor.

A

B



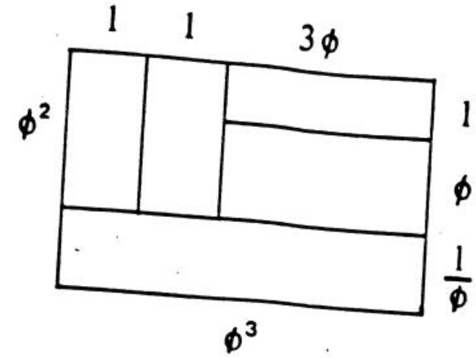


FIG. 1

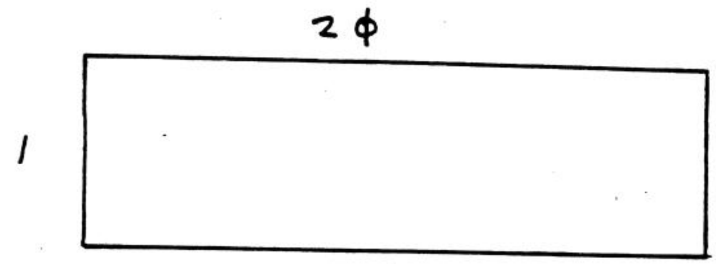


FIG. 2

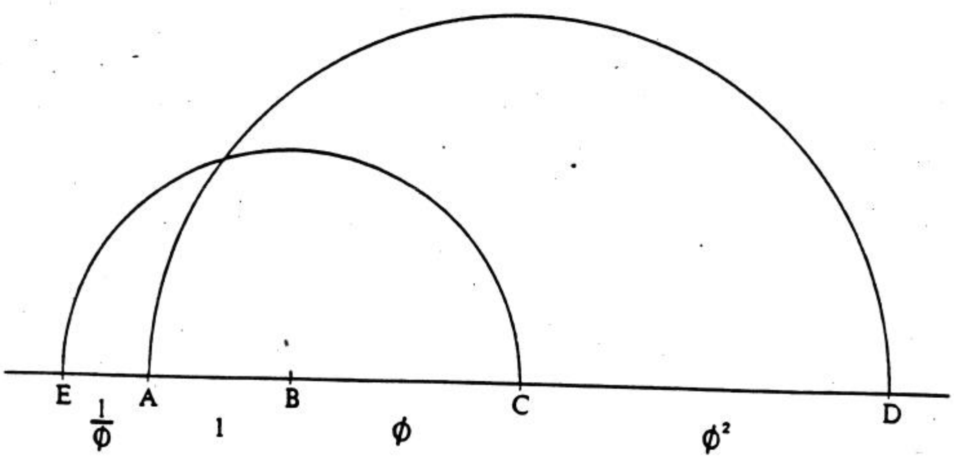
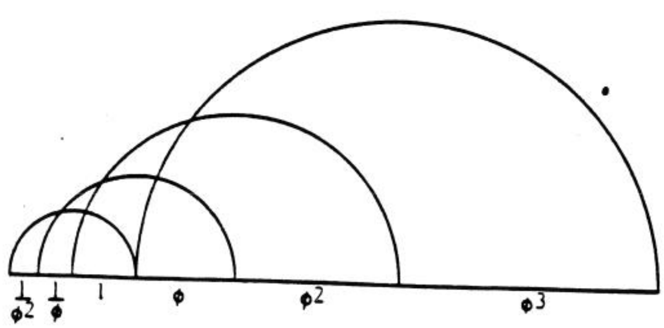


FIG. 3a



Construction of a φ series with compass and straightedge beginning with lengths 1 and φ.

FIG. 3b

The 'trade mark' of Le Corbusier's proportional system, the *Modulor*. 'A man-with-arm-upraised provides, at the determining points of his occupation of space – foot, solar plexus, head, tips of fingers of the upraised arm – three intervals which give rise to a series of golden sections, called the Fibonacci series.' (See pages 234–7 and Figure 9.13.)

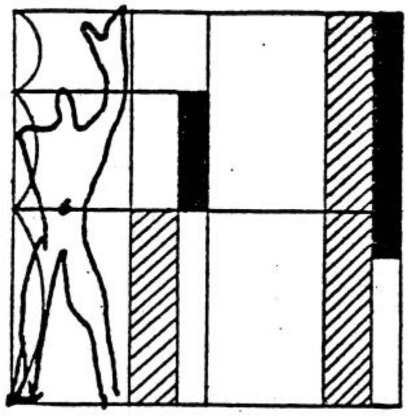


FIG. 4b

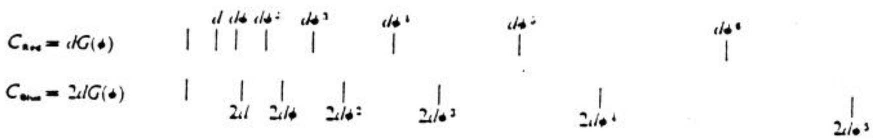


FIG. 4a

The Modulor red and blue scale of lengths measured from a common origin.

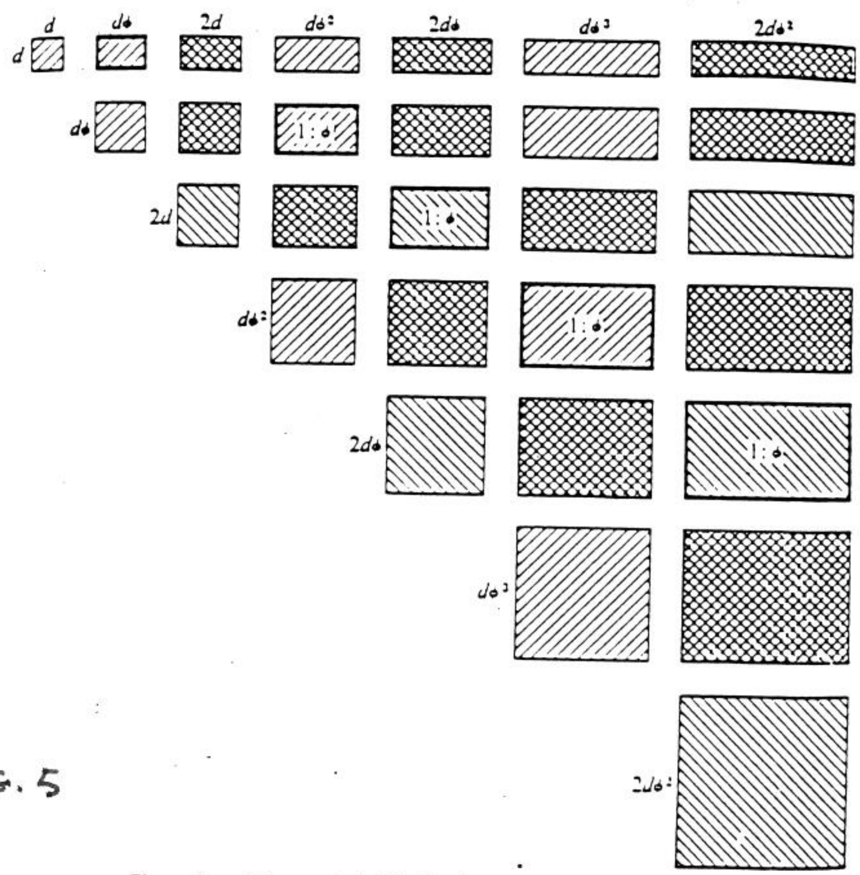


FIG. 5

The red and blue series of Le Corbusier's Modulor and the tiles of various proportions which the system gives rise to. There are three groups: rectangles whose side lengths are drawn only from the blue series (shaded in one direction), rectangles whose side lengths are drawn only from the red series (shaded in the opposite direction), and those rectangles produced from pairs of dimensions, one red and one blue (both shadings superimposed).

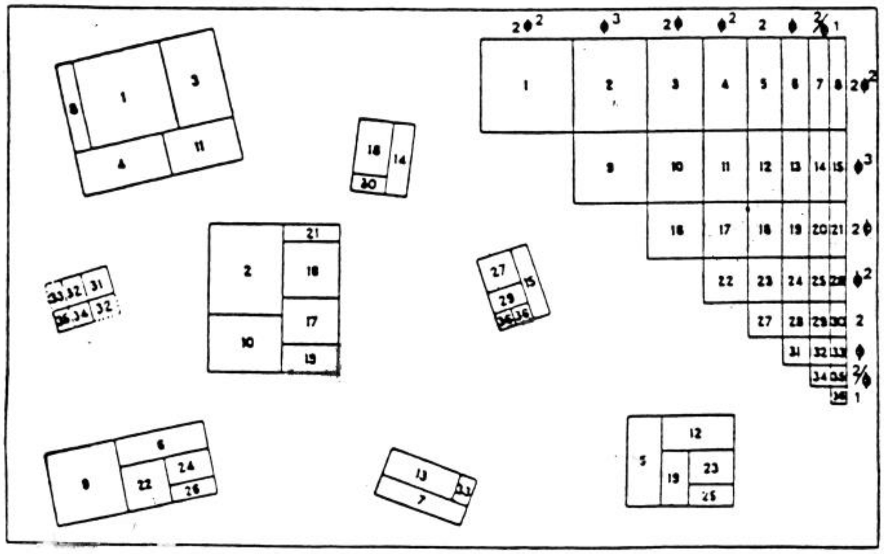
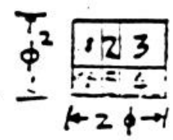


FIG. 6

A Modulor exercise by Le Corbusier. Eight rectangles are subdivided by Modulor rectangles and coded according to the table in the upper right-hand corner.



DIMENSIONS OF TILES:

1 – $\phi : 1$ 4 – $1 : 1$

2 – $\phi : \frac{1}{\phi}$ 5 – $1 : \frac{1}{\phi}$

3 – $\phi : \phi$ 6 – $1 : \phi$

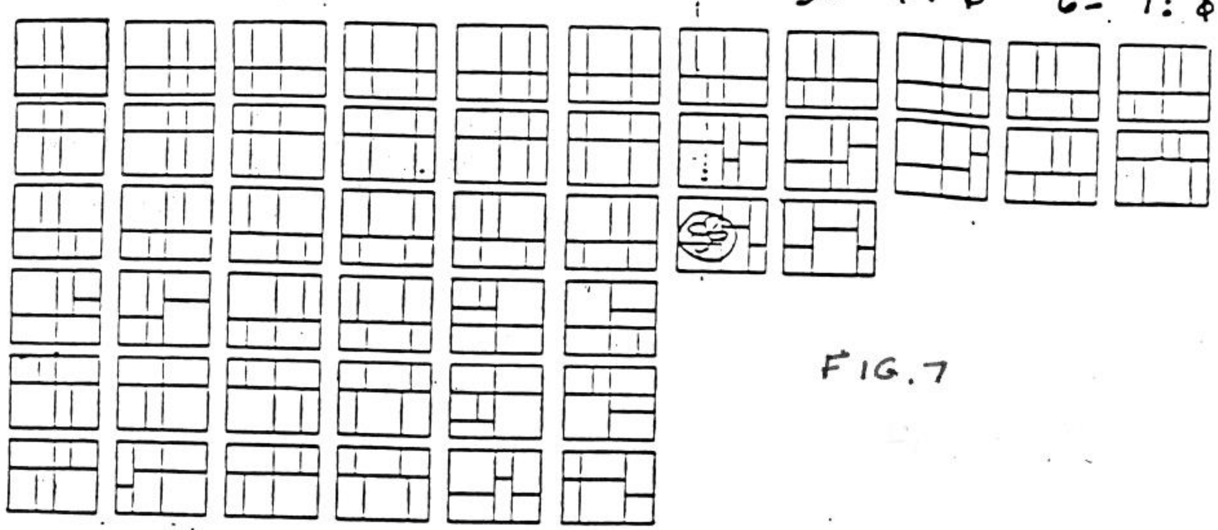
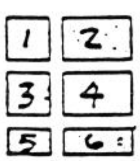


FIG. 7



DIMENSIONS OF TILES:

1,3 – $1 : 1$ 6 – $\frac{1}{\phi} : \phi$

2,4 – $1 : \phi$. Figure 2.25d

5 – $\frac{1}{\phi} : 1$

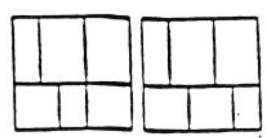
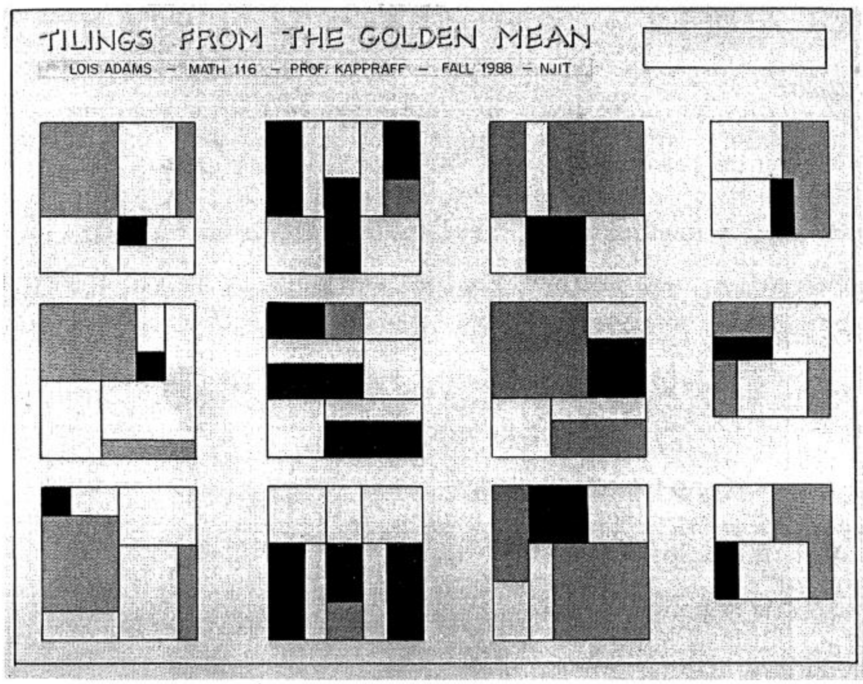


FIG. 8



A Modulor tiling by Allison Baxter. In the first three columns a $2\phi^3$ by $2\phi^3$ square is subdivided into three different sets of of tilings. Each set uses the same tiles but is arranged in three different ways. The last column presents the tiling of a 5- by 5-inch square to within 1/4-inch tolerance by the same tiles arranged in three different ways.

FIG. 9

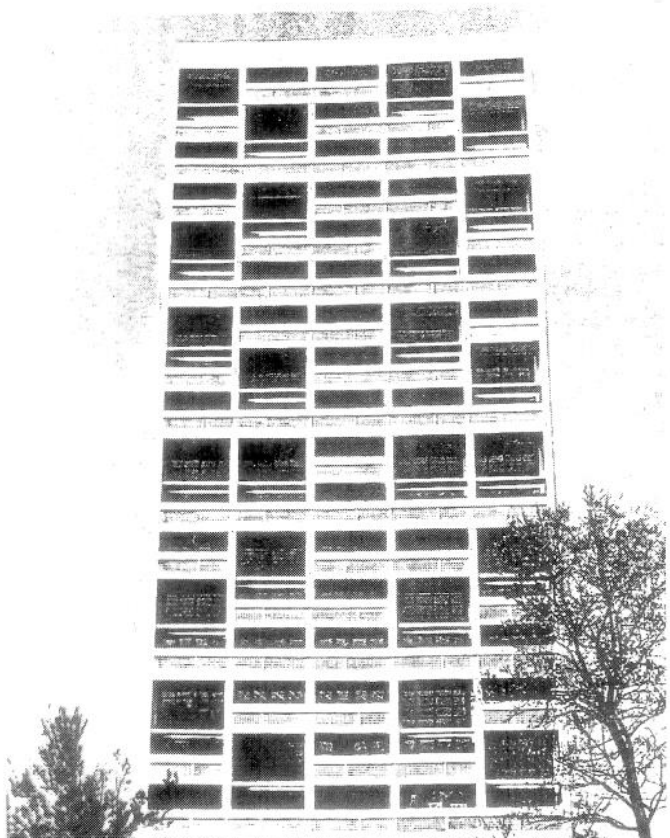


FIG. 10