CHAPTER 2

FUNCTIONS

2.1 FUNCTIONS

A *function* f is a relationship between an *input* and an *output* and a set of instructions as to how to obtain the output from a given input. For any input, there must be a *unique* output (only one.) If we call the input x and the output y then we use the notation: $y = f(x)$. Think of 'f 'as a machine that generates a unique value of y from the *range* of the function (possible outputs) for any x from the *domain* (possible inputs). Functions are very important in mathematics because they form the language in which math is encountered and conveyed to others. Just as architects use floorplans, or in recent years cad programs, the mathematician relates to others through functions.

Example 1: $y = f(x) = x^2$. The rule is: "The output number is the square of the input number."

$1 \rightarrow$	f	$\rightarrow f(1) = 1$
$0 \rightarrow$		$\rightarrow f(0) = 0$
$2 \rightarrow$		$\rightarrow f(2) = 4$
$1 \rightarrow$		$\rightarrow f(1) = 1$
$a \rightarrow$		$\rightarrow f(a) = a^2$
$2x \rightarrow$		$\rightarrow f(2x) = (2x)^2$

Domain: all real numbers
Range: all reals, $y \geq 0$

Example 2: $y = f(x) = \sqrt{x}$. The rule is : "the output number is the positive square root of the input number"

$0 \rightarrow$	f	$\rightarrow 0 = f(0) = 0$
$1 \rightarrow$		$\rightarrow 1 = f(1) = 1$
$4 \rightarrow$		$\rightarrow 2 = f(4) = 2$
$a+h \rightarrow$		$\rightarrow f(a + h) = \sqrt{a+h}$

Domain: $x \geq 0$
Range: $y \geq 0$

Example 3: $z = f(x,y) = x^2 + y^2 - 1$. The rule is : "The output number is 1 subtracted from the sum of the squares of the input numbers"

$(1,1) \rightarrow$	f	$\rightarrow 1$
$(1,0) \rightarrow$		$\rightarrow 0$
$(0,1) \rightarrow$		$\rightarrow 0$
$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \rightarrow$		$\rightarrow 0$

Domain: all pairs (x,y)
Range: $z \geq -1$

Remark 1: For the function of Example 3, an element from the domain is an ordered pair of numbers with the first number named x and the second y, and the output is a single number labeled z.

Remark 2: Notice that we don't have to place explicit numbers into a function. However, whatever is the input to the function we must apply the rule to get the output. Also it does not matter what letters we use for the input and output of the function. You can see that $s = f(t)$ or $u = f(v)$ gives the same outputs for given inputs.

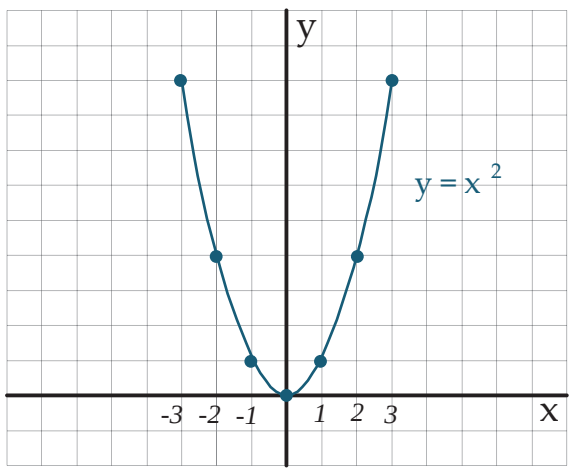
Another way to represent a function is with a table in which several key elements of the domain are placed on the top row and their corresponding elements from the range are in the row beneath it. For example, $f(x) = x^2$ is shown below.

Table 1

x	-3	-2	-1	0	1	2	3
y = f (x)	9	4	1	0	1	4	9

First of all notice that the same number appears twice in the second row whereas a number only appears once in the top row. This is because each element of the domain can have only one value in the range whereas several elements of the domain can map to the same element of the range. Such a function is called a *many-to-one function*.

From this table you can sketch the curve as shown below,



Notice that since each element in the domain maps to only one element in the range any vertical line intersects the curve only once. This is the *vertical line test* for functions. However, a horizontal line drawn on this graph intersects it more than once which is why the function is called many-to-one.

Next consider the linear function defined by the table,

Table 2

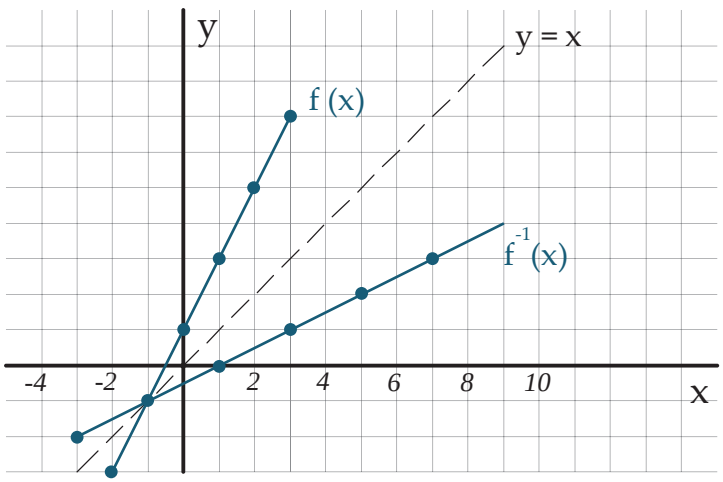
x	-2	-1	0	1	2	3
f(x)	-3	-1	1	3	5	7

Can you guess the rule of this function? Notice that there are no repeating elements in row 1 or row 2, therefore this is called a *one-to-one function*. Because there are no repeats in the second row (there are never repeats in the first row) we can define a new function, g(x) in which the rows are interchanged.

Table 3

x	-3	-1	1	3	5	7	x
g(x)	-2	-1	0	1	2	3	y

Notice that f(2) = 5 while g(5) = 2 and f(0) = 1 while g(1) = 0. In fact we find that if f(x) = y then g(y) = x for all values of x. Any function g that has this property is called the *inverse function* of f and written by the symbol f^{-1}. So I will rename $g = f^{-1}$. We will discuss inverse functions further in Section 2.7. The graphs of f and f^{-1} are drawn together in the figure below,



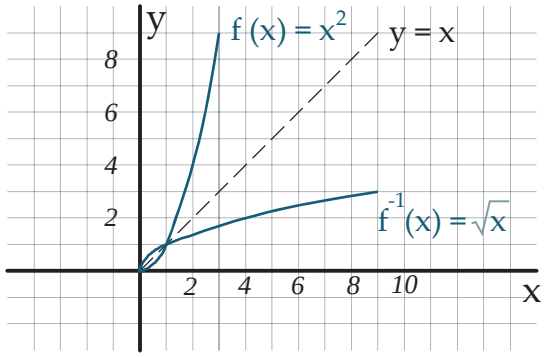
Notice that $y = f^{-1}(x)$ is the reflection of y = f(x) in a mirror placed on the diagonal y = x. This will always be true of a function and its inverse.

You will notice that the function $f(x) = x^2$ does not have an inverse because if you reverse the rows then $g(4) = \pm 2$ and $g(1) = \pm 1$ which cannot happen with a function which can only have a single value from the range for each element from the domain. So $f(x) = x^2$ does not have an inverse. However, if we limit the domain of f to only positive values then it does have an inverse,

Table 4

x	0	1	4	9
$f^{-1}(x)$	0	1	2	3

And the graphs are shown below. In fact we shall show in Section 2.7 that $f^{-1}(x) = \sqrt{x}$.

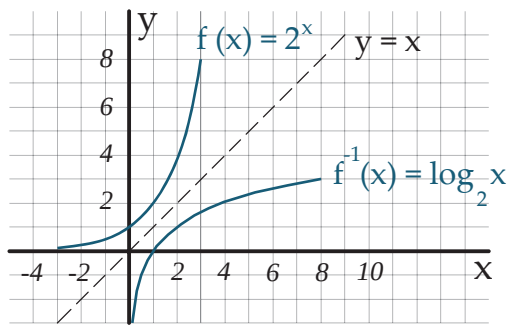


Next consider the function, $f(x) = 2^x$,

Table 5

x	-3	-2	-1	0	1	2	3	4	5	$x = \log_2 2^x$	$\log_2 y$
$f(x) = 2^x$	1/8	¼	½	1	2	4	8	16	32	2^x	$y = 2^{\log_2 y}$

Notice that neither row has any repeating values so the rows can be reversed and f has an inverse function, f^{-1}. In fact the inverse function is better known as $f^{-1}(x) \equiv \log_2 x$. And if you draw the functions $y = 2^x$ and $y = \log_2 x$ on the same graph you will notice again how they reflect in a mirror at y = x.



Also notice that if you add two values from the domain such as 2 and 3 to get 5, the corresponding range values 4 and 8 multiply to give 32. In other words,

$$5 = \log_2 32 = \log_2 4 \times 8 = \log_2 4 + \log_2 8 = 2 + 3$$

Before hand calculators were invented, this property of logarithms enabled integers to be multiplied by adding their logarithms. Since addition is less time consuming that multiplication, this shortened many computations. In fact, logarithms have the following properties which you can verify by choosing examples from the above table. The corresponding properties of exponentials are listed beside them. Note that when the base of the logarithm is not stated, the logarithm property holds for any base.

1.	$\log 1 = 0$	$a^0 = 1$
2.	$\log_b b = 1$	$a^1 = a$
3.	$\log(a \times b) = \log a + \log b$	$a^b a^c = a^{b+c}$
4.	$\log \frac{b}{a} = \log a - \log b$	$\frac{a^b}{a^c} = a^{b-c}$
5.	$\log a^b = b \log a$	$(a^b)^c = a^{bc} = (a^c)^b$
6.	$\log(a+b) \neq \log a + \log b$	$(a+b)^c \neq a^c + b^c$

7. $\log_b b^x = x$ $\qquad$ $b^{\log_b x} = x$

8. $\log_b a = \dfrac{\log_{10} a}{\log_{10} b} = \dfrac{\log_e a}{\log_e b}$

Remark 3: If base b is the irrational number e = 2.718…, then $\log_e a$ is generally written as $\ln a$ where ln is the abbreviation for 'natural" logarithm. We postpone a discussion of why base e is considered to be "natural" for later.

Remark 4: Table 5 enables you to find $\log_2 a$ where a is any number that is an integral power of 2. But what if you wanted to compute $\log_2 5$? You cannot do this with your calculator since it is only able to compute logs to the base 10 and base e. However Property 8 of logarithms enables you compute logs to the base 2 in terms of logs to the base 10 or e. For example,

$$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2} = \frac{\ln 5}{\ln 2} = 2.32$$

Problem 1: For the following two functions, draw the graphs of the function and its inverse on the same graph.

a)

x	-2	-1	0	1	2	3
f(x)	-4	-1	2	5	8	11

b)

x	1/4	1/3	½	1	2	3	4
f(x)	4	3	2	1	½	1/3	¼

Consider Expression 1. This expression is very important to the study of calculus and will be discussed in Lesson 4.

$$\boxed{\frac{f(a+h)-f(a)}{h}} \qquad (1)$$

Problem 2:
For the following functions compute Expression 1and simplify it by algebra:

a) $f(x) = x^2$; b) $f(x) = \sqrt{x}$; c) $f(x) = x^3$; d) $f(x) = 3x + 4$

Problem 3:
If $f(x) = x^2 + 1$, find and simplify,

a) $f(t+1)$; b) $f(t^2 + 1)$; c) $2\,f(t)$; d) $(f(t))^2 + 1$; e) $f(2)$

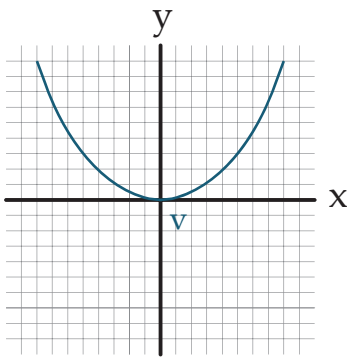
2.2 GRAPHS AND FUNCTIONS

Any curve in the plane can be represented by an equation of the form : $f(x,y) = 0$

Example 4a: $f(x,y) = y - x^2 = 0$ or $y = x^2$, a parabola cupped up. Notice that the parabola is characterized by its vertex, v.

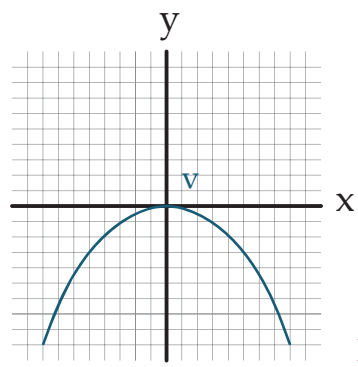
This curve passes the vertical line test in that any vertical line cuts the curve only once. Therefore, we say that this is the graph of a function, i.e., $y = f(x)$ where $f(x)= x^2$

Example 4b : $f(x,y) = y + x^2 =0$ or $y = - x^2$, a parabola cupped down.



$y = x^2$

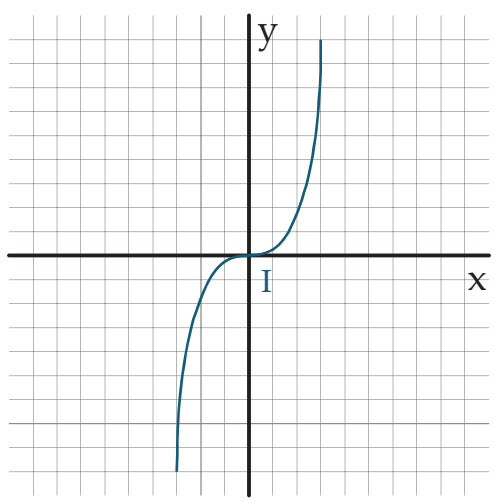
Fig. 4a



$y = -x^2$

Fig. 4b

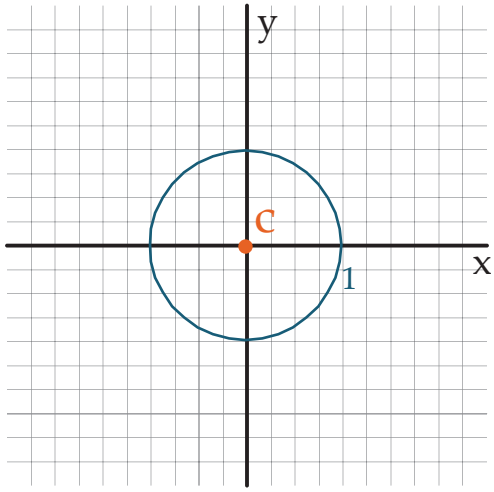
Example 5: $f(x,y) = y - x^3 = 0$ or $y = x^3$ is a cubic. This curve is characterized by its inflection point I , the place on the curve where its curvature changes from concave down to concave up.



$y = x^3$

It passes the vertical line test so we can say : $y = f(x)$ where $f(x) = x^3$

Example 6: $f(x,y) = x^2 + y^2 - 1 = 0$ or $x^2 + y^2 = 1$ is a unit circle (radius = 1). The circle is characterized by its center, c.

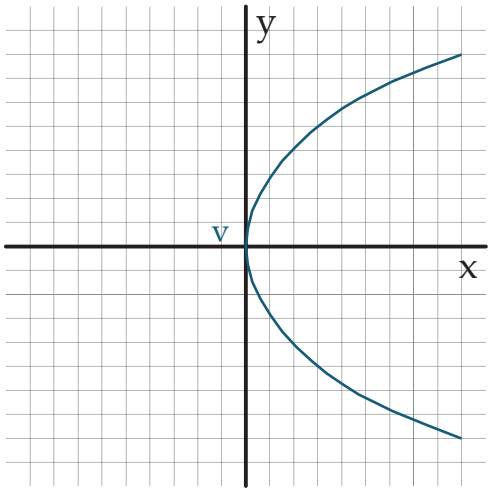


$x^2 + y^2 = 1$

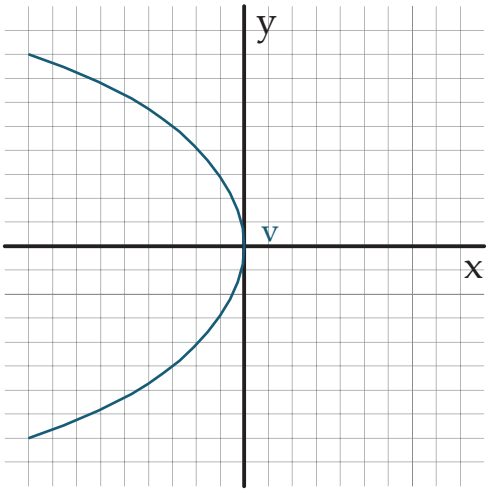
Notice this curve does not pass the vertical line test (why?) so it cannot be expressed as the graph of a function.

Example 7: $f(x,y) = x \mp y^2 = 0$ or $x = \pm y^2$ are parabolas turned on their side.

$y^2 = x$

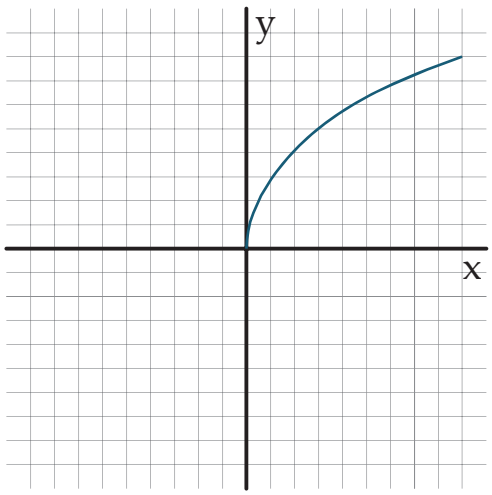


$x = -y^2$

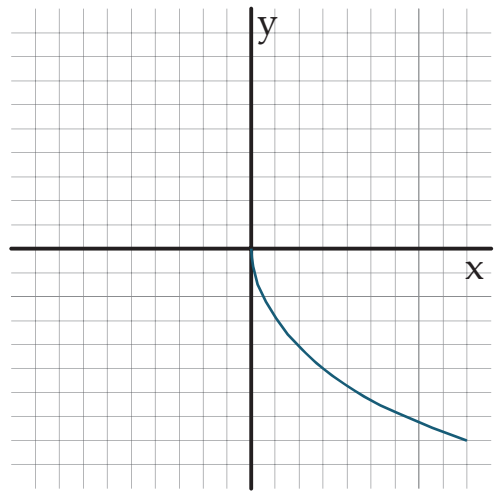


This curve is not the graph of a function (why?).

Example 8: f(x,y) = $y \mp \sqrt{x} = 0$ or $y = \pm\sqrt{x}$
Is this the graph of a function? Notice its domain and range on the graph. What are they?



$y = \sqrt{x}$



$y = -\sqrt{x}$

Our vocabulary of curves now includes: $y = x^2$, $x = y^2$, $y = \sqrt{x}$, $y = x^3$, $x^2 + y^2 = 1$

2.3 NEW GRAPHS FROM OLD ONES

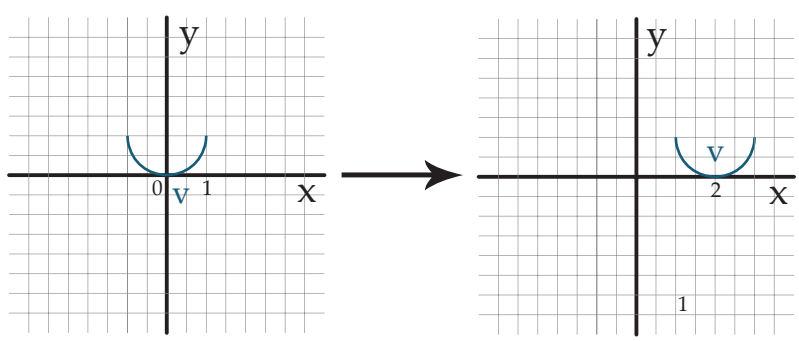
In what follows we will assume that 'a' represents a real number and that a > 0.

a)Translations

i) f(x–a, y) = 0 . This takes the curve and translates it 'a' units to the right.

Example 9: $y = (x - 2)^2$

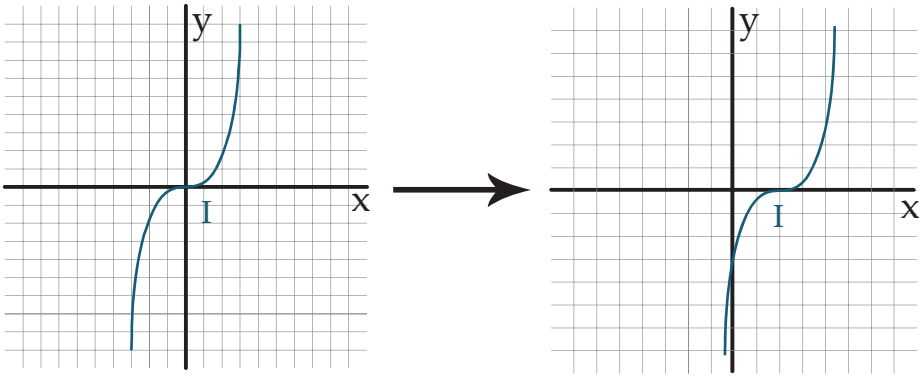
$y = x^2$



$y = (x - 2)^2$

Notice that the vertex v is translated 2 units to the right and so is the rest of the curve.

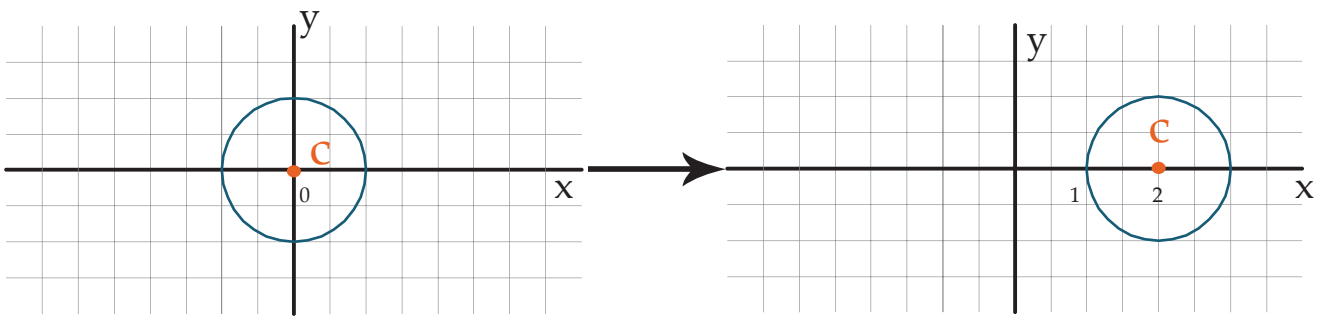
Example 10: $y = (x - 2)^3$



$y = x^3$ $\qquad$ $y = (x - 2)^3$

Notice that the point of inflection I is translated 2 units to the right and so is the rest of the cubic.

Example 11: $(x - 2)^2 + y^2 = 1$



$x^2 + y^2 = 1$ $\qquad$ $(x - 2)^2 + y^2 = 1$

Notice that the center c is translated 2 units to the right and so is the rest of the circle.

ii) $f(x+a,y) = 0$ This translates a curve 'a' units to the left (remember that $a > 0$).

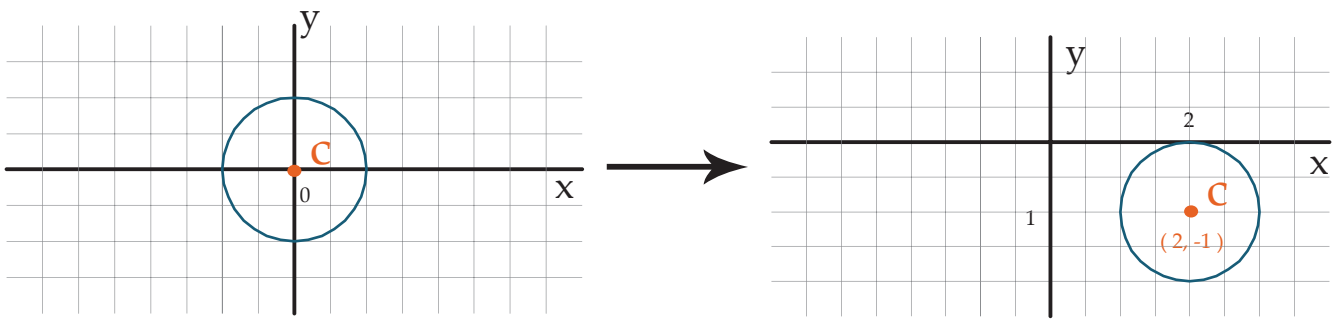
Use this to sketch the graphs of : $y =(x +2)^2$, $y = (x+2)^3$; $(x + 2)^2 + y^2 = 1$

iii) and iv). $f(x, y - a) = 0$ and $f(x, y+a)=0$. These translate the curve up and down by 'a' units respectively.

Sketch the graphs of : $y +2 = x^2$, $(y + 2)^2 = x$, and $x^2 + (y+2)^2 = 1$

v) $f(x \pm a, y \pm b)$ combines the above translations so that now a curve can be translated any combination of a and b units to the left, right , up or down.

Example 12: $(x-2)^2 + (y+1)^2 = 1$



$\boxed{x^2 + y^2 = 1}$ $\boxed{(x-2)^2 + (y+1)^2 = 1}$

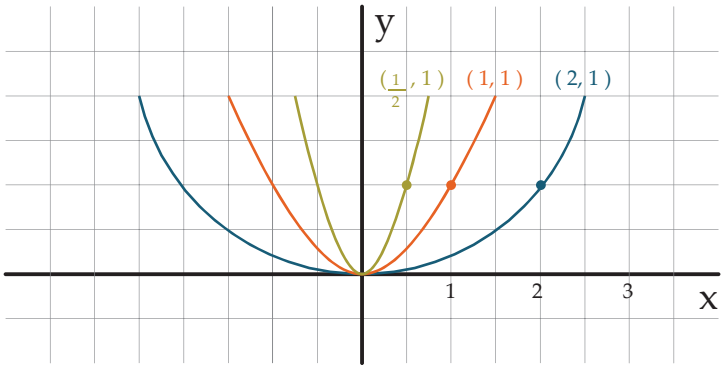
Problem 4 :

Sketch : a) $y - 1 = (x+2)^2$; b) $(y+1)^2 = x + 2$; c) $(x+1)^2 + (y-1)^2 = 1$; d) $y+2 = \sqrt{x-1}$

b. Squeezing or expanding a curve

i) f(ax,y) = 0. This squeezes the curve in the direction of the y-axis by a factor of 'a.'

Example 13: $y = (2x)^2$ and $y = (x/2)^2$



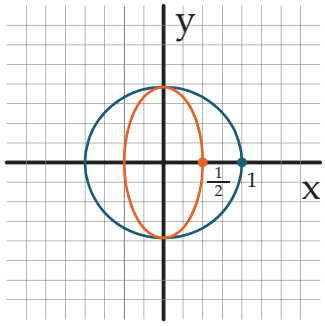
$\boxed{y = (2x)^2}$

$\boxed{y = x^2}$

$\boxed{y = (x/2)^2}$

Notice that the point (1,1) is squeezed to (1/2 , 1), a factor of 2, in the first curve and expanded to (2,1) in the second, expanded to a factor of ½, and so is every other point on these curves.

Example 14: $(2x)^2 + y^2 = 1$



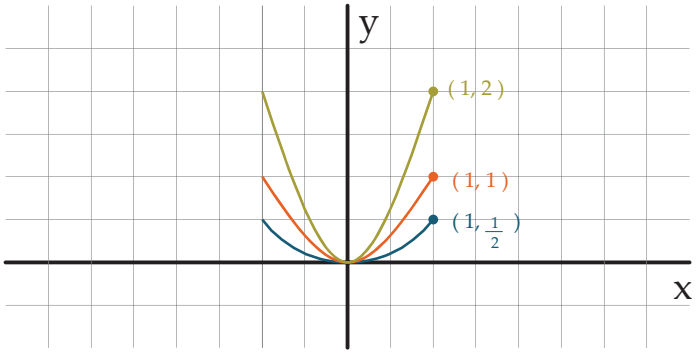
$x^2 + y^2 = 1$

$(2x)^2 + y^2 = 1$

Notice that (1,0) is squeezed to (1/2,0), a factor of 2, and so are all of the other points. The circle becomes an ellipse.

ii) f(x, ay) = 0. This squeezes the curve in the direction of the x-axis by a factor of 'a'.

Example 15: $2y = x^2$ or $y = ½ x^2$ and $y/2 = x^2$ or $y = 2x^2$



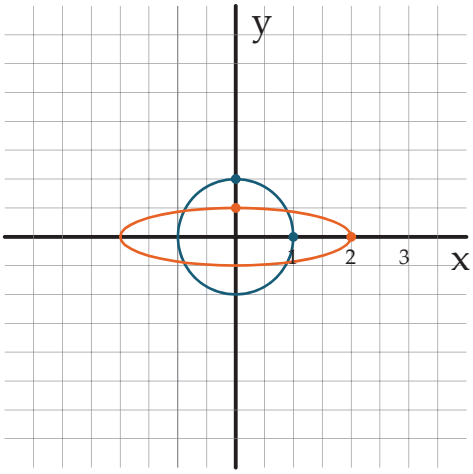
$y/2 = x^2$

$y = x^2$

$2y = x^2$

Notice that the point (1,1) squeezes to (1,1/2) in the first curve and (1,2) in the second and so do all the other points.

Example 16: sketch the curve: $(x/2)^2 + (2y)^2 = 1$



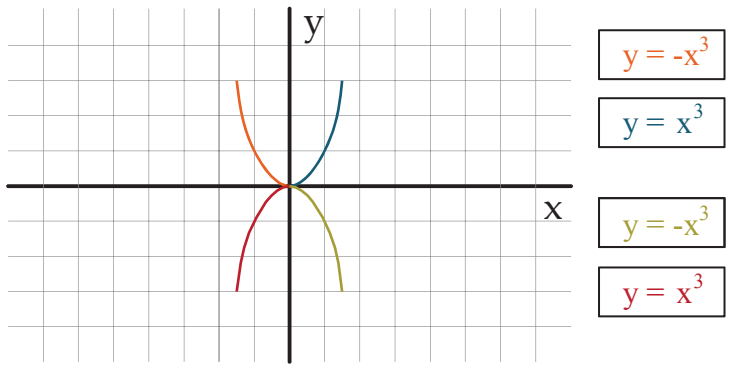
$x^2 + y^2 = 1$

$(x/2)^2 + (2y)^2 = 1$

c. Reflections

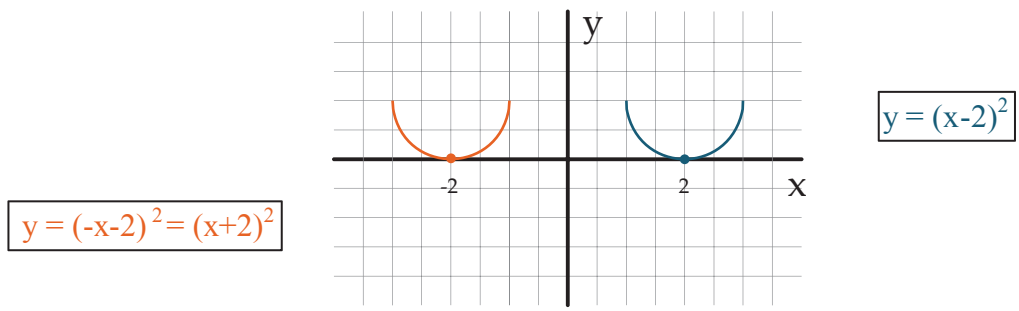
i) f(-x,y) = 0. This reflects a curve as if the y-axis was a mirror.

Example 17: $y = (-x)^3$ or $y = -x^3$



Notice that the cubic reflects across the y-axis

Example 18: Consider the curve: $y = (x-2)^2$ what does $y = (-x-2)^2$ look like? Well $y = (-x-2)^2$ is the same curve as $y = (x+2)^2$ (why?)



Notice that the parabola reflects across the y-axis

But notice that $y = (-x)^2$ is the same curve as $y = x^2$ so nothing changes. Why?

ii) $f(x,-y) = 0$. This reflects the curve as if the x-axis is a mirror.

Example 19: Apply this to the curves : $y = x^2$ and $y = x^3$, i.e., sketch $-y = x^2$ (or $y = -x^2$) and $-y = x^3$ (or $y = -x^3$). Notice that if this rule is applied to $x^2 + y^2 = 1$ the circle maintains its position and is unchanged. Why?

iii) $f(-x,-y) = 0$. This reflects the curve first in the y-axis and then in the x-axis. It has the effect of reflecting the curve across the origin in the respect that any point (x,y) on the original curve maps to the point (-x,-y) on the transformed curve.

Apply this to $y = x^3$. This gives $-y = (-x)^3$ or $y = x^3$ so that the curve is unchanged.

Problem 5: For the functions in the graphs below apply the rules of translation, reflections, and squeezing to these transformed functions: a) y–2 = f(x); b) y/3 = f(x); c) y = f(x-1); d) y = f(-2x); e) y = -f(-x) ; f) y–2 = f(x-1)

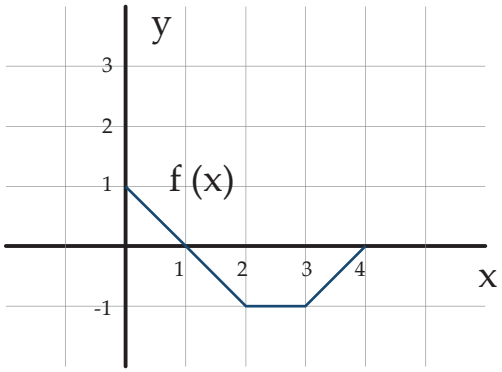


Fig. 1

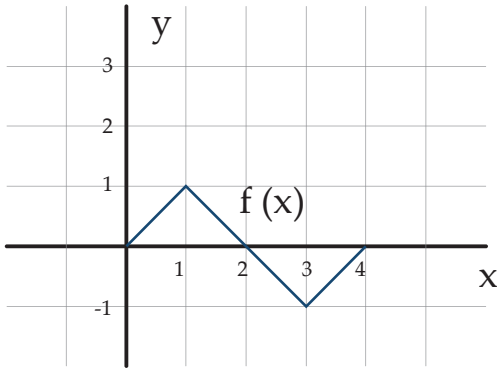


Fig. 2

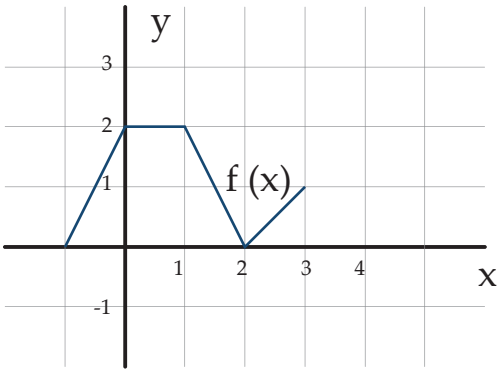


Fig. 3

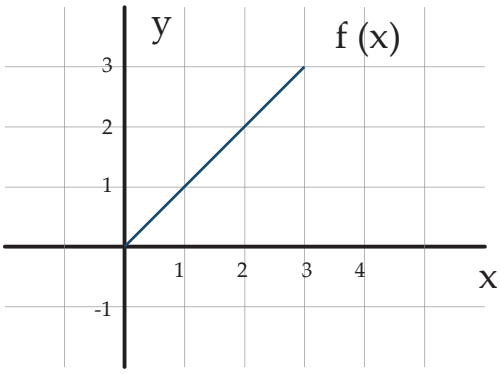


Fig. 4

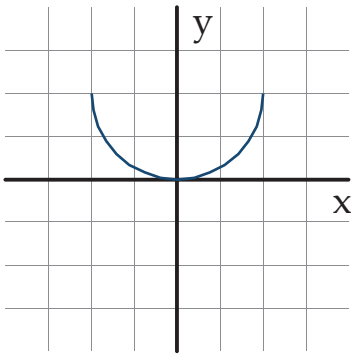
2.4 SYMMETRIC CURVES

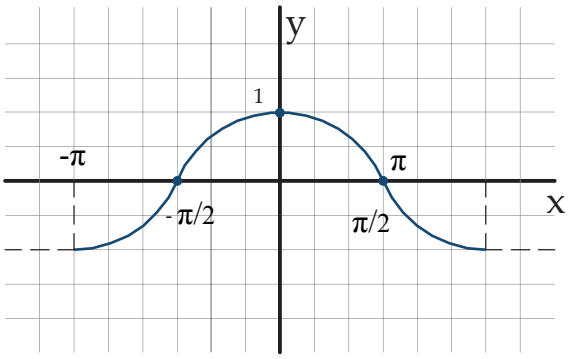
If the graph of a function has symmetry, this symmetry can be used to simplify certain math problems as we shall see in Chapters 11 and 13 in the semester. There are two kinds of symmetry that you should learn about.

i. The graph of any function that has the property, f(-x) = f(x) is symmetric with respect to the y-axis, i.e., the y-axis acts like a mirror. Such functions are called "even."

Example 20: The graphs of $f(x) = x^2$ or cos x are even as shown on the graph below.

$y = x^2$



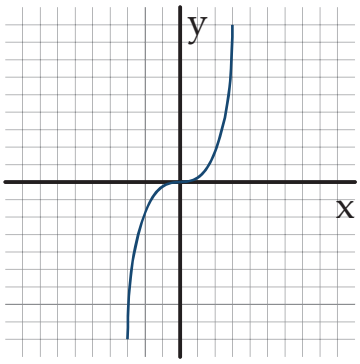


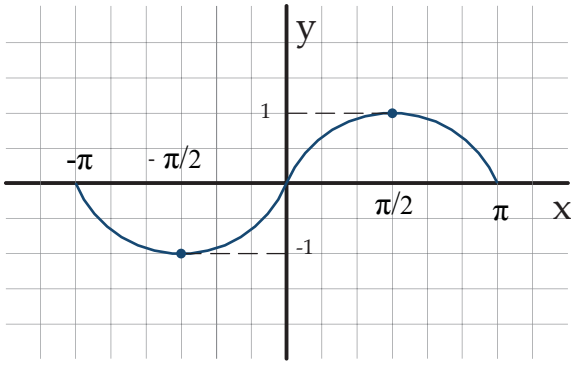
$y = \cos x$

ii. The graph of any function that has the property, f(-x) = -f(x) is symmetric with respect to the origin of the coordinate system, i.e., any value (x,y) maps to (-x,-y). Also if you reflect the curve first in the y-axis and then in the x-axis its trace is unchanged. Such functions are called "odd."

Example 21: The graphs of $f(x) = x^3$ and sin x are odd as shown on the graph below.

$y = x^3$





$y = \sin x$

Problem 6:

Which of the following functions are even, odd, or neither?

a) $x^4 - x^2$; b) $x^3 - 3x$; c) $x^2 + 5$; d) $2x^2 - 3x$

2.5 COMPLETING THE SQUARE

What do the parabolas, $y = x^2 + 2x + 4$ or $y = -2x^2 + 4x - 6$ look like? We can find out by doing something that mathematicians call *completing the square*. Here is how it works:

a) First consider the case where the coefficient of the x^2 term is +1, e.g., $y = x^2 + 2x + 4$.
b) Group the x^2 term and the x term together, i.e., $(x^2 + 2x)$
c) Write these two terms as a square, i.e., $x^2 + 2x = (x + 1)^2 - 1$. Note that the number in parentheses, +1, will always be half of the coefficient of the x term, (2 in this example) and since the square of the constant term in the parentheses (1 in this case) was not there originally you have to subtract out its square.
d) Now add the last number, 4 to get: $y = x^2 + 2x + 4 = (x + 1)^2 + 3$.
e) Rewrite this expression as $y - 3 = (x + 1)^2$ and draw the graph using the translation rules.

What if the coefficient of the x^2 term is not +1? For example what does $y = -2x^2 + 4x - 6$ look like? Follow this procedure:

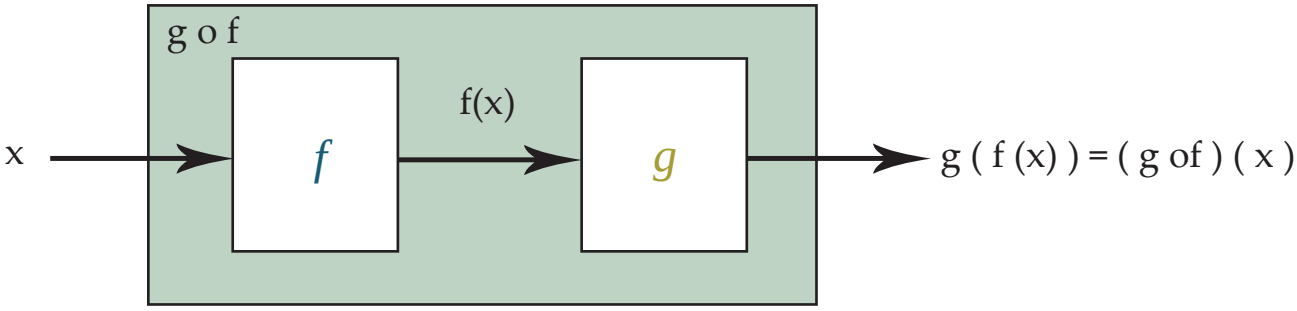
a) Factor out the coefficient of the x^2 term, i.e, $y = -2(x^2 - 2x + 3)$
b) Take the term in parentheses and apply the above procedure for the case where the coefficient of x^2 is +1, i.e, $y = -2([x - 1]^2 - 1 + 3)$ or $y = -2(x + 1)^2 - 4$
c) Rewrite the last expression as : $y + 4 = -2(x + 1)^2$ and use the translation and the squeeze rules of a function to draw the graph.

Problem 7: Complete the square for the following parabolas and sketch.

a) $y = x^2 - 4x + 2$; b) $y = -x^2 + 2x - 3$; c) $2x^2 - 4x + 2$; d) $y = x^2 + x + 1$

2.6 COMPOSITE FUNCTIONS

Sometimes the output of a function f is placed into the input of another function g as shown in the figure below. Since f(x) is the output of the f function if x is the input, then g(f(x)) is the output of the g function if f(x) is the input. The new function g(f(x)) is called a composite function and sometimes written, $g \circ f$. The domain of this function is the domain of the f function and the range is the range of the g function.



Problem 8:

1. For the following functions f and g where :

a) $f(x) = x^2$, $g(x) = x + 1$; b) $f(x) = 1/x$, $g(x) = 3x + 4$; c) $f(x) = \sqrt{x+4}$, $g(x) = x^2$

find and simplify: f(g(1)); g(f(1)); f(g(x)); g(f(x))

2.7 INVERSE FUNCTION

Consider the functions: $f(x) = 2x + 1$ and $g(x) = \frac{x-1}{2}$. Check to see that f(g(x)) = g(f(x)) = x. In this case we say that g is the inverse function of f and write $g \equiv f^{-1}$. It has the property that,

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

It is easy to find the formula for an inverse functions. If you want to find the inverse function of $f(x) = 2x + 1$ just set it equal to y and solve for x, e.g.,

$$y = 2x + 1 \quad \text{and } x = (y - 1)/2$$

Then change the y back to x to get $f^{-1}(x) = (x - 1)/2$.

As we showed in Section 2.1, there will be no inverse function f^{-1} unless f is *one-to-one*, in other words for each output from the f function there is only one input that gives you that value. Therefore the function must pass the *horizontal line test: any horizontal line should cut the graph of t he function no more than once.* For example, $f(x) = 2x + 1$ has that property but $f(x) = x^2$ does not since $f(1) = f(-1) = 1$, e.g., $x = 1$ and $x = -1$ both give the output value 1 (a horizontal line will cut the graph of $y = x^2$ twice). But if we limit the domain of f to only positive values then f(x) for $x \geq 0$, does have an inverse. To find the inverse set $y = x^2$, for $x \geq 0$, and solve for x:

$$y = x^2 \quad \text{and } x = \sqrt{y} \quad \text{(the positive square root of y)}$$

so $f^{-1}(x) = \sqrt{x}$. You can check that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

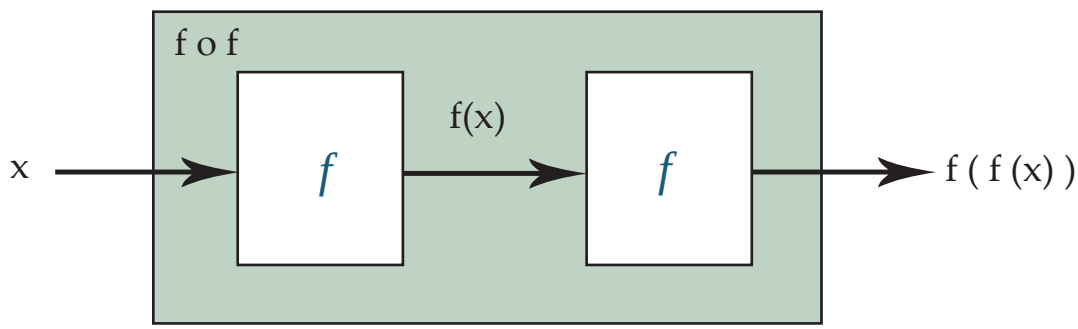
Problem 9:

Find the inverse of the following functions:

a) f(x) = 3x + 2; b) f(x) = 1/x; c) $f(x) = \dfrac{x+1}{x-1}$ where $x \neq 1$.

2.8 ITERATIVE FUNCTIONS

Sometimes the output of a function is fed back to itself so that the result is f(f(x)) as shown in the figure below. This is what is done in a new field of mathematics called chaos theory where a function is fed back to itself over and over again to get f(f(f(f…(x))))). A whole new theory about science and mathematics has evolved to deal with this possibility.



Problem 10:

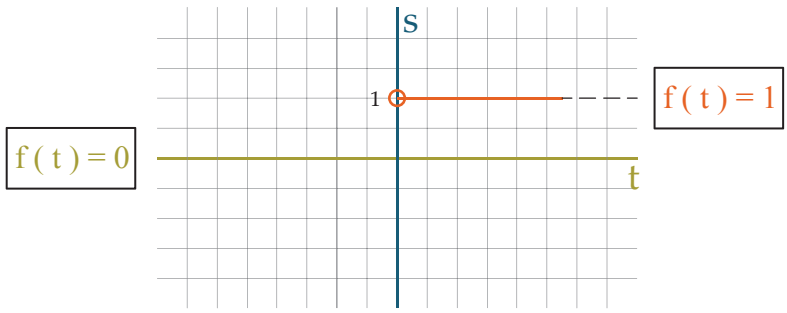
I recently made a discovery about the simple function: $f(x) = x^2 - 2$. Take the value x = 1.2469 and find f(1.2469), f(f(1.2469)) , f(f(f(1.2469))), and f(f(f(f(1.2469)))). What do you observe?

2.9 FUNCTIONS DEFINED BY MORE THAN ONE FORMULA

In our calculus and structures course, most of the functions that we will encounter will have to be defined by several formulas because they behave differently over different intervals. Sometimes these function have values that jump as you move your finger along the curve. Such curves are said to be discontinuous.

Example 22: An example of a discontinuous function is a switch that sends a signal s = 1 after time 0 and is off until t = 0 where s = 0 as shown below. Its formula is:

$$s = f(t) = \begin{cases} 0 & \text{for } t \le 0 \\ 1 & \text{for } t > 0 \end{cases}$$

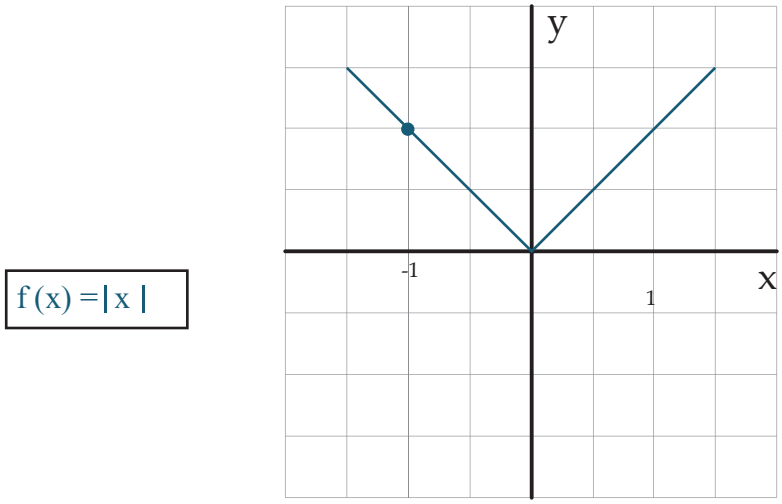


Note that a circle has been drawn around the endpoint of the portion of the curve t > 0 because the value of the function at t = 0 is 0 not 1 so the point in the circle is not part of the graph

Other curves have kinks in them. The simplest example of such a curve is the absolute value function, f(x) = |x|. It is defined as follows:

$$|x| = \begin{cases} -x \text{ if } x \le 0 \\ x \text{ if } x > 0 \end{cases}$$

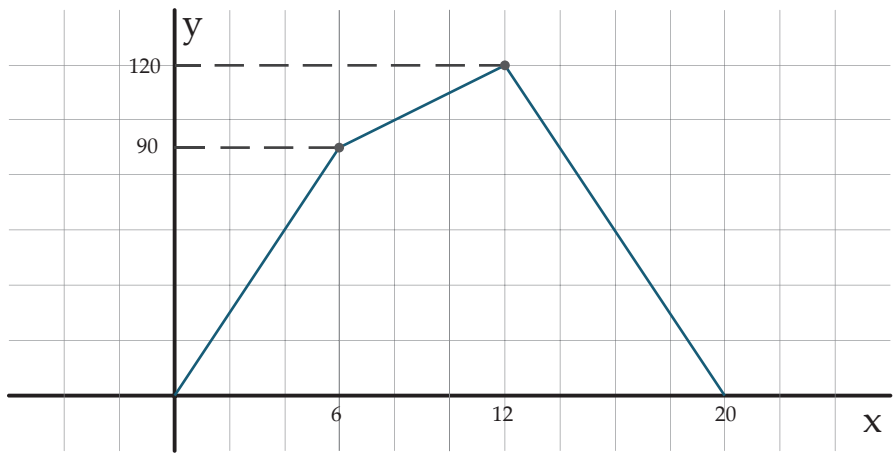
So | -2| = 2 while | 2 | = 2.



A function that we will soon encounter in our course is:

$$f(x) = \begin{cases} 15x, & 0 \le x \le 6 \\ 60 + 5x\ , & 6 \le x \le 12 \\ 200 - 15x\ , & 12 \le x \le 20 \end{cases}$$

This curve, shown below, is made up of three line segments and it is continuous, the curve has no breaks. Compute: f (2), f(6) , f(12) and f(15).



Problem 11:

Sketch the graph of the discontinuous function:

$$f(x) = \begin{cases} 15\ , & 0 \le x \le 6 \\ 5, & 6 < x \le 12 \\ -15\ , & 12 < x \le 20 \end{cases}$$

Find: f(3), f(6) , f(12), f(15)

Problems

1. The time T in minutes that it takes Dan to run x kilometers is a function $T = f(x)$. Explain the meaning of the statement $f(5) = 23$ in terms of running.

2. The population of a city, P, in millions, is a function of t, the number of years since 1970, so $P = f(t)$. Explain the meaning of the statement $f(35) = 12$ in terms of the population of this city.

3. Let $W = f(t)$ represent wheat production in Argentina, in millions of metric tons, where t is years since 1990. Interpret the statement $f(12) = 9$ in terms of wheat production.

4. The number of sales per month, S, is a function of the amount, a, (in dollars) spent on advertising that month, so $S = f(a)$.

(a) Interpret the statement $f(1000) = 3500$.
(b) Which of the graphs in Figure 1.4 is more likely to represent this function?
(c) What does the vertical intercept of the graph of this function represent, in terms of sales and advertising?

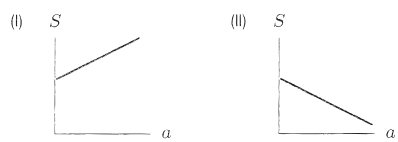


Figure 1.4

5. Describe what Figure 1.5 tells you about an assembly line whose productivity is represented as a function of the number of workers on the line.

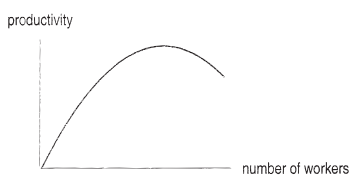


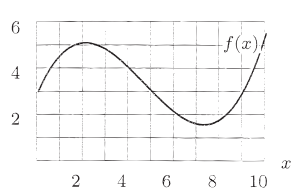
Figure 1.5

For the functions in Problems 6–10, find $f(5)$.

6. $f(x) = 2x + 3$

7. $f(x) = 10x - x^2$

8.



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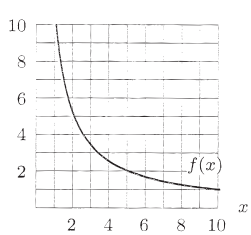

10.

x	1	2	3	4	5	6	7	8
$f(x)$	2.3	2.8	3.2	3.7	4.1	5.0	5.6	6.2

11. A filter-based clean-up program is adopted for a very polluted lake. The filters gradually clog, impairing their effectiveness. This problem is dealt with by removing the filters once a week for two days for cleaning after which they are replaced as good as new. Graph the quantity of pollution in the lake as a function of time for a three week time period.

12. Financial investors know that, in general, the higher the expected rate of return on an investment, the higher the corresponding risk.

(a) Graph this relationship, showing expected return as a function of risk.
(b) On the figure from part (a), mark a point with high expected return and low risk. (Investors hope to find such opportunities.)

13. In tide pools on the New England coast, snails eat algae. Describe what Figure 1.6 tells you about the effect of snails on the diversity of algae.[2] Does the graph support the statement that diversity peaks at intermediate predation levels?

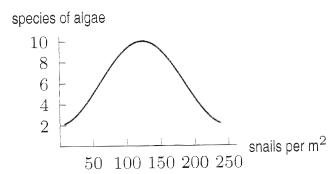

Figure 1.6

14. An object is put outside on a cold day at time $t = 0$. Its temperature, $H = f(t)$, in °C, is graphed in Figure 1.7.

(a) What does the statement $f(30) = 10$ mean in terms of temperature? Include units for 30 and for 10 in your answer.
(b) Explain what the vertical intercept, a, and the horizontal intercept, b, represent in terms of temperature of the object and time outside.

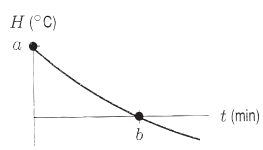

Figure 1.7

15. In the Andes mountains in Peru, the number, N, of species of bats is a function of the elevation, h, in feet above sea level, so $N = f(h)$.

(a) Interpret the statement $f(500) = 100$ in terms of bat species.
(b) What are the meanings of the vertical intercept, k, and horizontal intercept, c, in Figure 1.8?

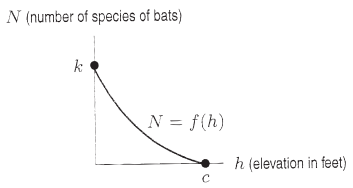

Figure 1.8

16. After an injection, the concentration of a drug in a patient's body increases rapidly to a peak and then slowly decreases. Graph the concentration of the drug in the body as a function of the time since the injection was given. Assume that the patient has none of the drug in the body before the injection. Label the peak concentration and the time it takes to reach that concentration.

17. Figure 1.9 shows the amount of nicotine, $N = f(t)$, in mg, in a person's bloodstream as a function of the time, t, in hours, since the person finished smoking a cigarette.

(a) Estimate $f(3)$ and interpret it in terms of nicotine.
(b) About how many hours have passed before the nicotine level is down to 0.1 mg?
(c) What is the vertical intercept? What does it represent in terms of nicotine?
(d) If this function had a horizontal intercept, what would it represent?

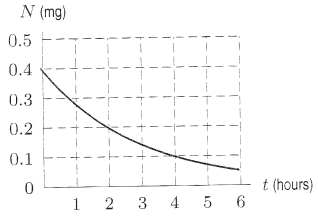

Figure 1.9

18. **(a)** A potato is put in an oven to bake at time $t = 0$. Which of the graphs in Figure 1.10 could represent the potato's temperature as a function of time?
(b) What does the vertical intercept represent in terms of the potato's temperature?

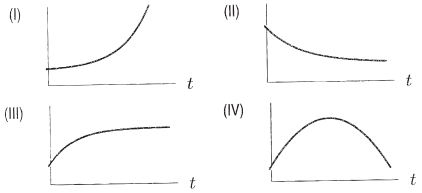

Figure 1.10

19. A deposit is made into an interest-bearing account. Figure 1.11 shows the balance, B, in the account t years later.

(a) What was the original deposit?
(b) Estimate $f(10)$ and interpret it.
(c) When does the balance reach \$5000?

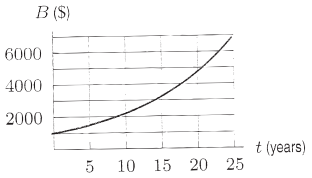

Figure 1.11