

Common exam #1: 8:30-9:45am on Feb. 9th(Fri.) at 205 Kupfrian Hall
(arrive by 8:15am, Proctor: Yixuan Li)

Chapters 1, 2, 3. Bring scientific calculators

HW#3 due Feb. 9

Quiz#3 today

<http://geocities.com/kenahn7/>

Today in this class...

Chapter 4. Motion in Two and Three Dimensions

Position and Displacement

Average Velocity and Instantaneous Velocity

Average Acceleration and Instantaneous Acceleration

Examples:

Projectile Motion → Partly, on Monday

Uniform circular motion → On Monday

Very brief review of Ch.1, 2, 3

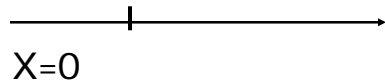
Quiz for Ch.3

Motion along the straight line + Vectors

One dimension (1D)

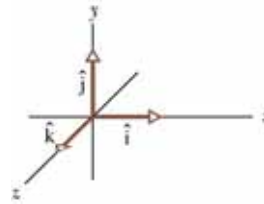
Position: $x(t)$ m
 Velocity: $v(t)$ m/s
 Acceleration: $a(t)$ m/s²

All are **vectors**: have direction and magnitude.



Three dimension (2D)

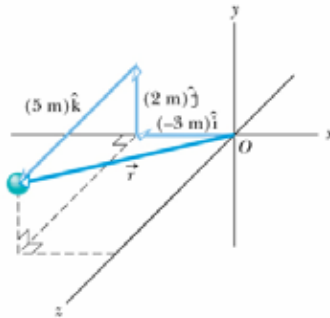
Position: $\vec{r}(t)$ m
 Velocity: $\vec{v}(t)$ m/s
 Acceleration: $\vec{a}(t)$ m/s²



Motion in 3D: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

- > $t = 0$ beginning of the process
- > $\vec{r} = 0$ is arbitrary; can set where you want it
- > $\vec{r}_0 = \vec{r}(t=0)$ position at $t=0$;

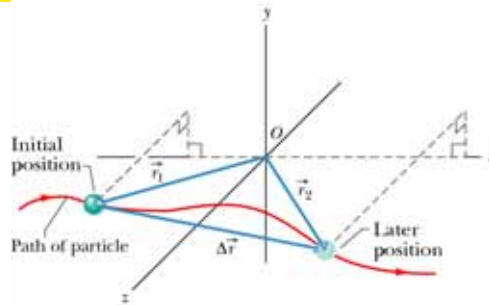
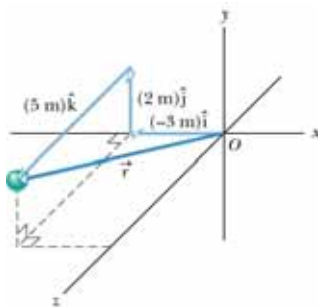
$$\vec{r} = \vec{r}_x + \vec{r}_y + \vec{r}_z = x \cdot \hat{i} + y \cdot \hat{j} + z \cdot \hat{k}$$



$$\vec{r}_1 = (-3.0m)\hat{i} + (2.0m)\hat{j} + (5.0m)\hat{k}$$

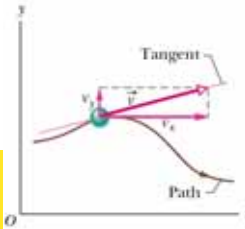
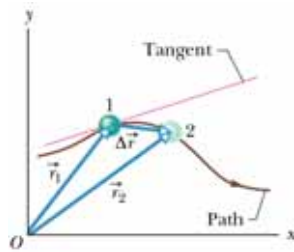
Position and Displacement

$$\begin{aligned} \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ \Delta\vec{r} &= \vec{r}_2 - \vec{r}_1 \\ \Delta\vec{r} &= (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) \\ &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ &= \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k} \end{aligned}$$



Average and Instantaneous Velocity

$$\begin{aligned}\vec{v}_{\text{avg}} &= \frac{\Delta \vec{r}}{\Delta t} \\ &= \frac{\Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}}{\Delta t} \\ &= \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k}\end{aligned}$$

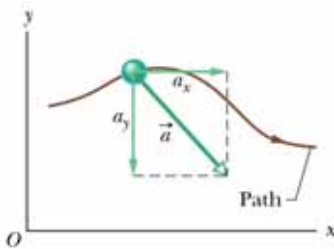


$$\begin{aligned}\vec{v} &= \frac{d\vec{r}}{dt} \\ &= \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) \\ &= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} \\ &= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\end{aligned}$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}$$

Average and Instantaneous Acceleration

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$



$$\begin{aligned}\vec{a} &= \frac{d\vec{v}}{dt} \\ &= \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) \\ &= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} \\ &= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\end{aligned}$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}$$

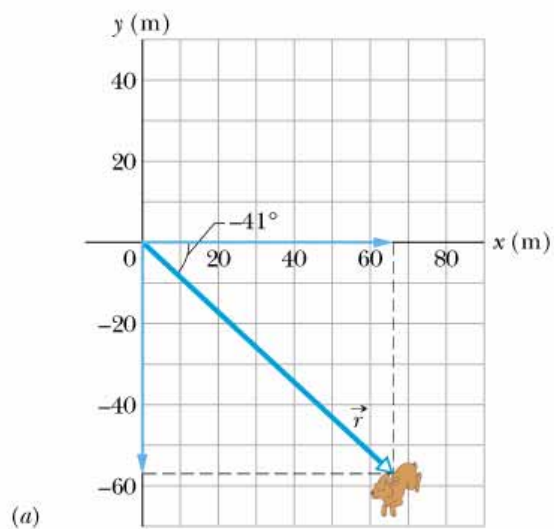
Sample Problem 4-2

For a running rabbit in a parking lot with a set of coordinate axes

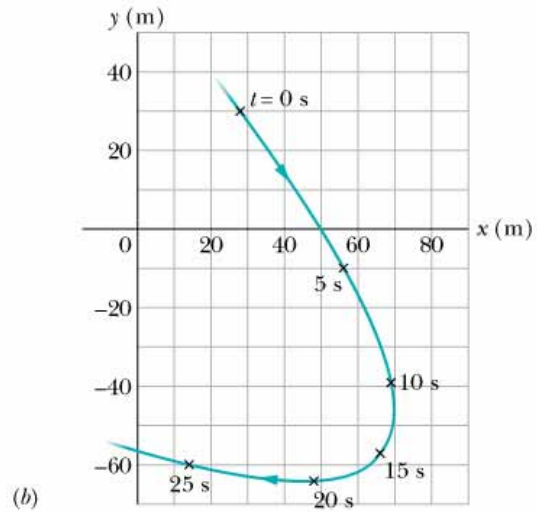
$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30$$

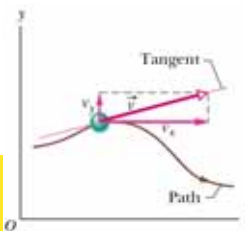
$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$



Plot of position from
t=0sec. to t=25 sec.



Instantaneous Velocity



$$\begin{aligned}\vec{v} &= \frac{d\vec{r}}{dt} \\ &= \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) \\ &= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} \\ &= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\end{aligned}$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}$$

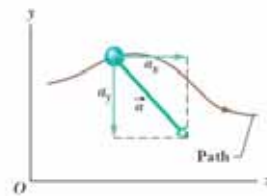
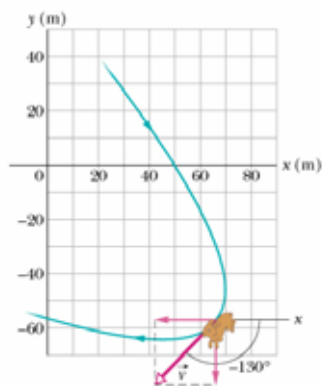
$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30$$

$$v_x = -0.62t + 7.2$$

$$v_y = 0.44t - 9.1$$

Instantaneous Acceleration



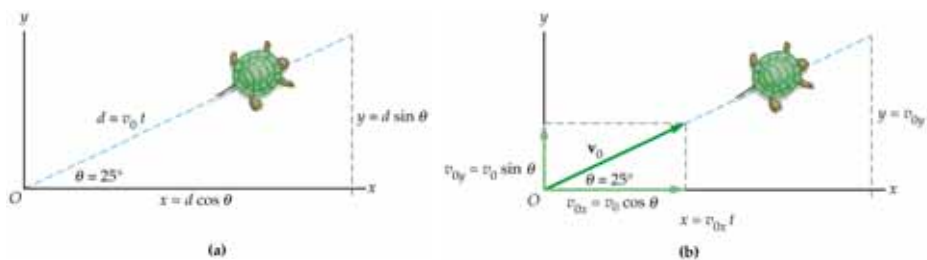
$$v_x = -0.62t + 7.2$$

$$v_y = 0.44t - 9.1$$

$$a_x = -0.62$$

$$a_y = 0.44$$

I. Observations: 2-D Motion



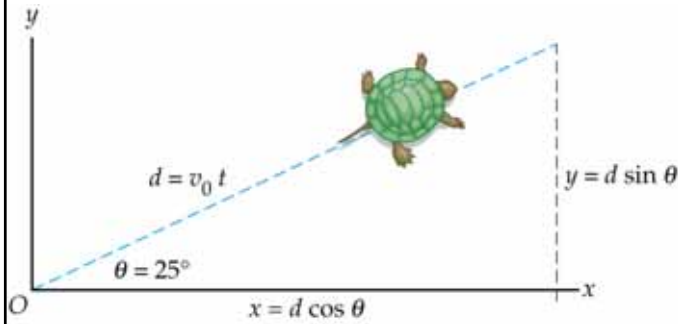
While describing any 2-D motion one can represent it as a superposition (a vector sum) of two 1-D motions!

Example: A turtle 2-D motion can be represented as its simultaneous motion in x direction and in y direction. Notice, the **velocity** is a vector and it has two components: \mathbf{v}_x and \mathbf{v}_y !

Example 1: Motion of a Turtle

A turtle starts at the origin and moves with the speed of $v_0=10$ cm/s in the direction of 25° to the horizontal.

- (a) Find the coordinates of a turtle 10 seconds later.
- (b) How far did the turtle walk in 10 seconds?



COMMENT:

Notice, you can solve the equations independently for the horizontal (x) and vertical (y) components of motion and then combine them!

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Sign on the attendance sheet, if you haven't today!