Today in this class...

Chapter 4. Motion in Two and Three Dimensions

Last Thurs.
- Position and Displacement
- Average Velocity and Instantaneous Velocity
- Average Acceleration and Instantaneous Acceleration

Examples:
- 2D motion with a constant acceleration
- Projectile Motion
- Uniform circular motion

Recitation for Ch.4

2D Motion with a constant acceleration

Acceleration: $\vec{a} = a_x\mathbf{i} + a_y\mathbf{j}$, where a_x and a_y are constants.

Trick: separate x and y component motions

If the initial velocity $\vec{v}_0 = v_{0x}\mathbf{i} + v_{0y}\mathbf{j}$

and the initial position $\vec{r}_0 = x_0\mathbf{i} + y_0\mathbf{j}$ are given,

X-component motion

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$v_x^2 - v_{0x}^2 = 2a_x(x - x_0)$$

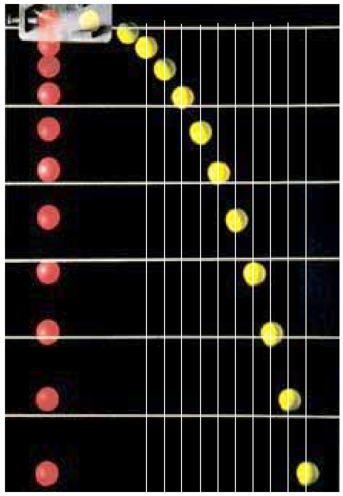
Y-component motion

$$v_y = v_{0y} + a_y t$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_y^2 - v_{0y}^2 = 2a_y(y - y_0)$$

Projectile Motion



Red ball: released from rest

Yellow ball: shot horizontally

Vertical motion:
Constant acceleration motion

a = -g, where g = 9.8 m/s ^{2}

Horizontal motion:

Constant velocity motion

Projectile Motion

If the initial velocity $\vec{v}_0 = v_{0x}\mathbf{i} + v_{0y}\mathbf{j}$

and the initial position $\vec{r}_0 = x_0\mathbf{i} + y_0\mathbf{j}$ are given,

X-component (horizontal) motion

$$v_x = v_{0x}$$

$$x = x_0 + v_{0x}t$$

Y-component (vertical) motion

$$v_y = v_{0y} + a_y t$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_y^2 - v_{0y}^2 = 2a_y(y - y_0)$$

$$\boxed{a_y = -g}$$

Projectile Motion (continued)

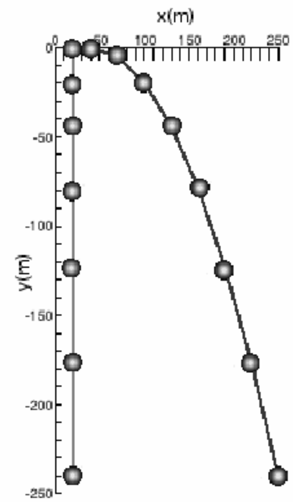
x and y motion happen independently so you can treat them separately

Connected by time:

$x = x_0 + v_{0x}t$

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

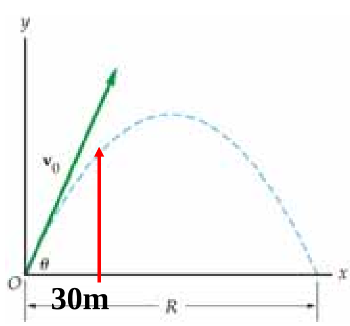
Since y(t) is a parabola and x is linear in time: y(x) is a parabola too



Example 2: Projectile Motion

Football place kicker is 30 m from goal post kicks ball at 20 m/s at 60^0 to ground. (a) Will it clear a goalpost at height 3 m? (b) What maximum height will it reach?

(c) What will be the range of the ball? (d) Hang time - ?



Given:

v_0=20 m/s	**(a) $y_{x=3m}$ - ?**
$\theta = 60^0$	**(b) y_{max}-?**
x_f=30 m	**(c) R - ?**
h = 3 m	**(d) $t_{in\ the\ air}$-?**
a=-9.8 m/s^2	

General Description of a Projectile Motion

$$\begin{cases} v_x = v_0 \cos(\theta) \\ v_y = v_0 \sin(\theta) + at \end{cases}$$

COMMENT: A trajectory of any projectile should be a parabola! Only if an object is launched vertically up its trajectory will be a vertical line.

$$\begin{cases} x: x = x_0 + v_{0x} t = x_0 + v_0 \cos(\theta) t \\ y: y = y_0 + v_0 \sin(\theta) t + \frac{1}{2} a t^2 \end{cases}$$

$$y = y_0 + v_0 \sin(\theta) \frac{x - x_0}{v_0 \cos(\theta)} + \frac{1}{2} a \left(\frac{x - x_0}{v_0 \cos(\theta)} \right)^2$$

Range (R) & max height (y_{max}) of a projectile above its launch site

$$\begin{cases} v_x = v_0 \cos(\theta) \\ v_y = v_0 \sin(\theta) - gt \end{cases}$$

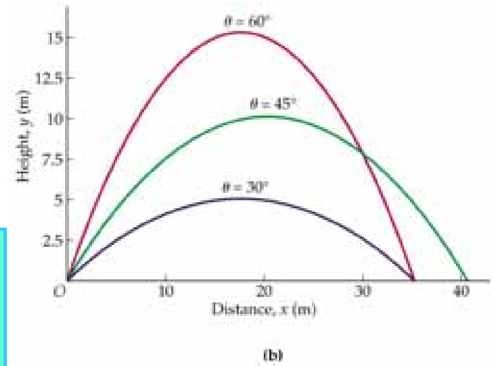
$$\left.\begin{matrix} \begin{cases} x: x = x_0 + v_{0x} t \\ y: y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \end{cases} \\ \begin{cases} v_x = v_0 \cos(\theta) = const \\ v_y = v_0 \sin(\theta) + at \end{cases} \end{matrix}\right\} \Rightarrow \begin{cases} R = \frac{v_0^2}{g} \sin(2\theta) \\ y_{max} = \frac{v_0^2 \sin^2 \theta}{2g} \end{cases}$$

Maximum range for given launch speed

$$\begin{cases} R = \dfrac{v_0^2}{g}\sin(2\theta) \\ R_{\max} \Rightarrow \theta = 45^0 \end{cases}$$

COMMENT:

According to our model, (neglecting air resistance) if the launch speed is kept const the max Range should be at a 45^0 angle. We can test it!



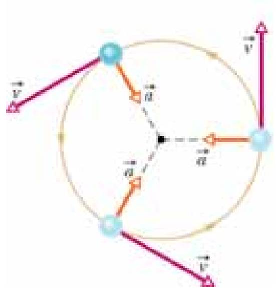
Uniform Circular Motion

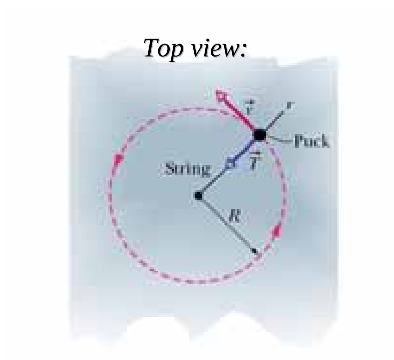
Centripetal acceleration

$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$





Note the direction of velocity and acceleration.

Proof: see Ch. 4, Sec. 7