

HW: Collision (Due 11 pm **central time**, 12/9, Tuesday).

HW hints are posted on course web (<http://web.njit.edu/~kenahn>)

Minimum C grade required to fulfill pre-requisite condition

Final Exam: December 16th, Tuesday

Time : 8:30-11 am (arrive by 8:15am)

Room: KUPF 106 (different from common exams)

Bring scientific calculator & **photo ID**

Covers everything learned in this semester

Old exams are posted on web.

Last lecture: Dec. 9th, Tuesday

"Impulse" and "Momentum"

Last class ...

Collision in 2D

Center of mass

Today...

Motion of system of particles

(Motion of center of mass)

How could we analyze the motion of extended objects, or system of particles?



Reminder: Definition of center of mass (com)

$$x_{com} = \frac{m_1x_1 + m_2x_2 + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{com} = \frac{m_1y_1 + m_2y_2 + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

$$z_{com} = \frac{m_1z_1 + m_2z_2 + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

, where (x_i, y_i, z_i)

is the position of m_i

$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

Velocity of center of mass (com)

$$v_{x,com} = \frac{\Delta x_{com}}{\Delta t} = \frac{m_1 v_{x,1} + m_2 v_{x,2} + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i v_{x,i}}{\sum_i m_i}$$

$$(m_1 + m_2 + \dots)v_{x,com} = m_1 v_{x,1} + m_2 v_{x,2} + \dots = P_{net,x}$$

Similar for y and z components

$$\vec{v}_{com} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i \vec{v}_i}{\sum_i m_i}$$

$$\vec{P}_{net} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots = M \vec{v}_{com} \quad \text{where } M = m_1 + m_2 + \dots$$

Acceleration of center of mass (com)

$$a_{x,com} = \frac{\Delta v_{x,com}}{\Delta t} = \frac{m_1 a_{x,1} + m_2 a_{x,2} + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i a_{x,i}}{\sum_i m_i}$$

Similar for y and z components

$$\vec{a}_{com} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i \vec{a}_i}{\sum_i m_i}$$

$$M \vec{a}_{CM} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 + \dots + m_n \vec{a}_n$$

$$m_1 \vec{a}_1 = \vec{F}_{1,net} = \vec{F}_{1,ext} + \vec{F}_{1,int} \quad \& \quad m_2 \vec{a}_2 = \vec{F}_{2,net} = \vec{F}_{2,ext} + \vec{F}_{2,int}$$

Newton's 3rd law for internal forces: $\vec{F}_{\text{from 2 on 1}} + \vec{F}_{\text{from 1 on 2}} = 0$

$$\rightarrow \vec{F}_{1,int} + \vec{F}_{2,int} + \dots + \vec{F}_{n,int} = 0$$

$$M \vec{a}_{CM} = \vec{F}_{1,ext} + \vec{F}_{2,ext} + \vec{F}_{3,ext} + \dots + \vec{F}_{n,ext}$$

$$M \vec{a}_{com} = \vec{F}_{net,ext}$$

Newton's second law for center of mass

$$\vec{F}_{net,ext} = M \vec{a}_{com}$$

$\vec{F}_{net,ext}$: Sum of all *external* forces that act on the system

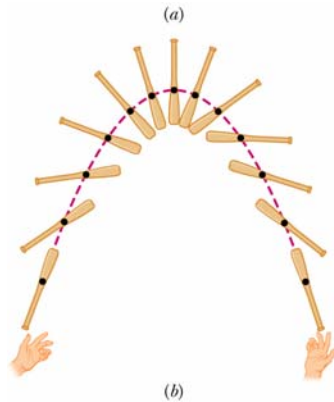
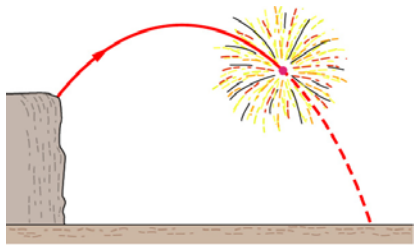
(*Internal* forces are *not* included)

$M = m_1 + m_2 + \dots$: Total mass of the system

\vec{a}_{com} : Acceleration of the center of mass

Internal forces do NOT change the motion of C.O.M.!!

Center of mass moves like a particle of mass M under the net external force.



Motion of COM is simple!

Example 1

A 2.0 kg particle has a velocity $(2.0 \mathbf{i} - 3.0 \mathbf{j})$ m/s, and a 3.0 kg particle has a velocity $(1.0 \mathbf{i} + 6.0 \mathbf{j})$ m/s. Find (a) velocity of the center of mass and (b) the total momentum of the system.

iClicker Quiz

Two objects with unknown mass and velocity collide and stick together moving at 3 m/s along x direction.

Assuming that net external force on the two objects is zero, what is the velocity of the center of mass before the collision?

- (a) 0
- (b) 3 m/s along x
- (c) -3 m/s along x
- (d) Not enough information

iClicker Quiz

An object is fired vertically upward. When it reaches the maximum height in 30 s, it explodes into two fragments. One fragment is twice heavier than the other. The lighter fragment moves downward at 20 m/s right after explosion. Assume no air resistance.

- (1) Find the velocity of the heavier fragment right after the explosion.
(a) 10 m/s upward, (b) 20m/s upward, (c) 40 m/s upward
- (2) From the explosion, it take _____ till the center of mass falls back to the ground.
(a) less than 30 s, (b) 30 s, (c) more than 30 s