Final Exam: December 14th, Monday

Time : 11:30-14:00 (arrive by 11:15 am)

Room: Cullimore Lecture Hall 1

Final covers everything learned in this semester.

Check course website for sample problems, formula, old exams

Dec. 7th, Mon.: Last class (Quiz, Review)

Dec. 10th, Thur.: No class (Reading day)

<u>"Impulse" and "Momentum"</u>

Last class …

Collision in 2D

Today…

Elastic collision in 1D

Center of mass

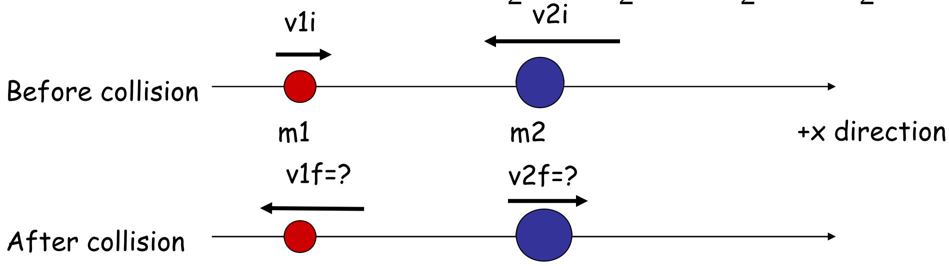
Motion of system of particles

(Motion of center of mass)

Elastic Collisions in One Dimension

Both total momentum & **total kinetic energy** are conserved.

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f} \quad \& \quad \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$



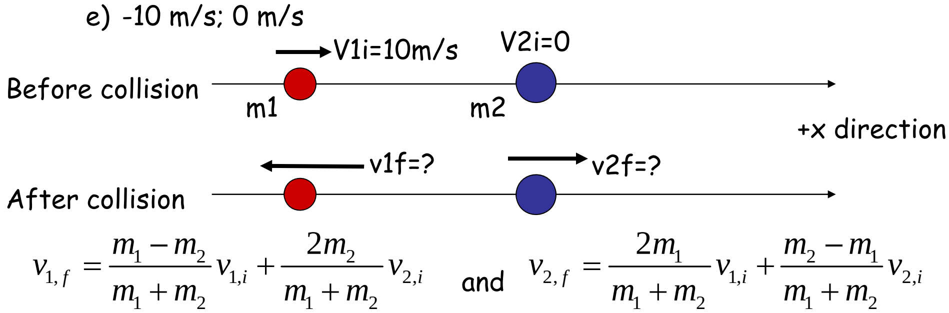
$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i} + \frac{2m_2}{m_1 + m_2} v_{2,i} \quad \text{and} \quad v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2,i}$$

(see text for proof)

iClicker Quiz

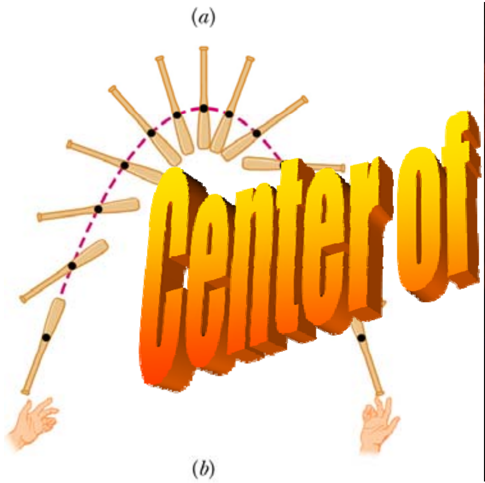
In 1D elastic collision, if m1=m2, vi1=+10 m/s, v2i=0, then, after collision v1f= ______ and v2f= ______.

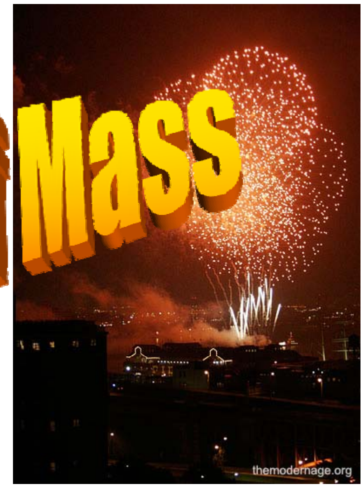
a) 5 m/s; 5 m/s
b) -5 m/s; 5 m/s
c) 0 m/s; 10 m/s
d) 0 m/s; -10 m/s
e) -10 m/s; 0 m/s



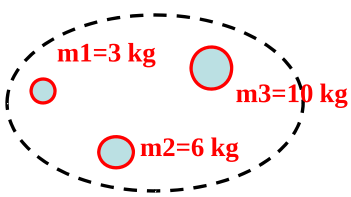
$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i} + \frac{2m_2}{m_1 + m_2} v_{2,i} \quad \text{and} \quad v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2,i}$$

How could we analyze the motion of extended objects, or system of particles?





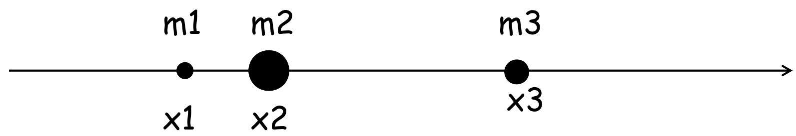
Concept of Center of Mass



For a system of particles or an extended object,

"Center of mass" is an "average" position for mass distribution.

Definition of center of mass (com) in 1D



In 1D,

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

, where x_i is position of mass m_i

Definition of center of mass (com) in 2D, 3D

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

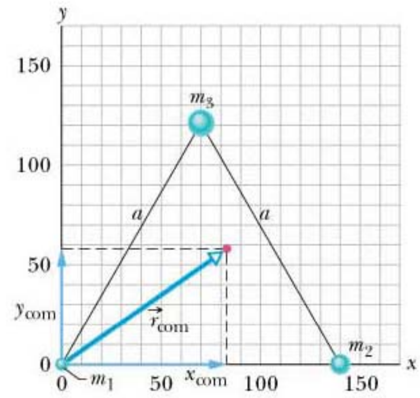
$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

$$z_{com} = \frac{m_1 z_1 + m_2 z_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

$$\vec{r}_{com} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

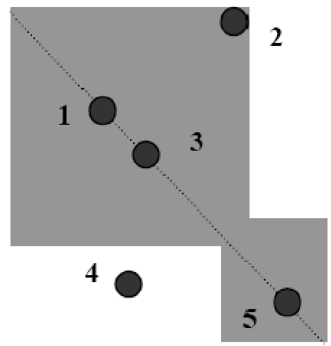
, where (x_i, y_i, z_i) is the position of m_i

Example 3
Three particles of masses *m*1 = 1.1 kg, *m*2 = 2.5 kg, and *m*3 = 3.4 kg are located as shown in the figure: m1 is at (0,0), m2 is at (140 m,0), and m3 is at (70 m, 120 m). Find the coordinate of the center of mass.



iClicker Quiz
A two-section piece, represented by the gray area on the figure, is cut from a metal plate of uniform thickness. The point that corresponds to the center of mass of this piece is closest to

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5



Velocity of center of mass (com)

$$v_{x,com} = \frac{\Delta x_{com}}{\Delta t} = \frac{m_1 v_{x,1} + m_2 v_{x,2} + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i v_{x,i}}{\sum_i m_i}$$

$$(m_1 + m_2 + \ldots) v_{x,com} = m_1 v_{x,1} + m_2 v_{x,2} + \ldots = P_{net,x}$$

Similar for y and z components

$$\vec{v}_{com} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i \vec{v}_i}{\sum_i m_i}$$

$$\vec{P}_{net} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots = M \vec{v}_{com}$$ where $M = m_1 + m_2 + \ldots$

Acceleration of center of mass (com)

$$a_{x,com} = \frac{\Delta v_{x,com}}{\Delta t} = \frac{m_1 a_{x,1} + m_2 a_{x,2} + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i a_{x,i}}{\sum_i m_i}$$

Similar for y and z components

$$\vec{a}_{com} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i \vec{a}_i}{\sum_i m_i}$$

$$M\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n$$

$m_1\vec{a}_1 = \vec{F}_{1,net} = \vec{F}_{1,ext} + \vec{F}_{1,\text{int}}$ & $m_2\vec{a}_2 = \vec{F}_{2,net} = \vec{F}_{2,ext} + \vec{F}_{2,\text{int}}$

Newton's 3[rd] law for internal forces: $\vec{F}_{\text{from 2 on 1}} + \vec{F}_{\text{from 1 on 2}} = 0$

→ $\vec{F}_{1,\text{int}} + \vec{F}_{2,\text{int}} + \ldots + \vec{F}_{n,\text{int}} = 0$

$$M\vec{a}_{CM} = \vec{F}_{1,ext} + \vec{F}_{2,ext} + \vec{F}_{3,ext} + \cdots + \vec{F}_{n,ext}$$

$$M\vec{a}_{com} = \vec{F}_{net,ext}$$

Newton's second law for center of mass

$$\vec{F}_{net,ext} = M\vec{a}_{com}$$

$\vec{F}_{net,ext}$: Sum of all *external* forces that act on the system

(*Internal* forces are *not* included)

$M = m_1 + m_2 + \ldots$: Total mass of the system

$\vec{a}_{com}$: Acceleration of the center of mass

Internal forces do NOT change the motion of C.O.M.!!

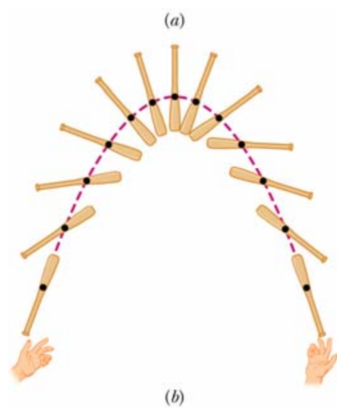
Motion of center of mass under gravity force

Newton's second law for C.O.M.: $\vec{F}_{net,ext} = M\vec{a}_{com}$

Under gravity,

$$\vec{F}_{net,ext} = -m_1 g\mathbf{j} - m_2 g\mathbf{j} - \ldots$$
$$= -(m_1 + m_2 + \ldots) g\mathbf{j} = -Mg\mathbf{j}$$
$$\therefore -Mg\mathbf{j} = M\vec{a}_{com}$$
$$\therefore \vec{a}_{com} = -g\mathbf{j}$$ → Usual projectile motion

Center of mass moves like a particle of mass M under the net external force.



Motion of COM is simple!

Example 1

A 2.0 kg particle has a velocity (2.0 **i** -3.0 **j**) m/s, and a 3.0 kg particle has a velocity (1.0 **i** + 6.0 **j**) m/s. Find (a) velocity of the center of mass and (b) the total momentum of the system.

iClicker Quiz

Two objects with unknown mass and velocity collide and stick together moving at 3 m/s along x direction.

Assuming that net external force on the two objects is zero, what is the velocity of the center of mass before the collision?

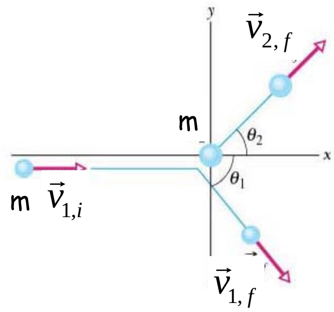
(a) 0

(b) 3 m/s along x

(c) -3 m/s along x

(d) Not enough information

Example 1: 90 degree deflection rule in a game of pool



Assume that the collision is elastic, and the two balls have the same mass.

Show that the angle between the outgoing balls is 90 degree.

(No forward, back or side spin is in effect.)