Final Exam: May 13th, Wednesday

Time : 8:30-11 am (arrive by 8:15 am)

Room: Cullimore Lecture Hall 3.

Lecture schedule

May 1st, Fri. : Sect. 006 & 008

May 5th, Tue., following Fri. schedule: Sect. 006 & 008
→ Last lecture (Review Session)

No class on May 4th, Mon, and May 6th, Wed.

<u>**"Impulse" and "Momentum"**</u>

Last class …

Example : Ballistic Pendulum

Collsion in 1D and 2D

Center of mass

Today…

Motion of center of mass

How could we analyze the motion of extended objects, or system of particles?

Reminder: Definition of center of mass (com)

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

$$z_{com} = \frac{m_1 z_1 + m_2 z_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

, where (x_i, y_i, z_i) is the position of m_i

$$\vec{r}_{com} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

Velocity of center of mass (com)

$$v_{x,com} = \frac{\Delta x_{com}}{\Delta t} = \frac{m_1 v_{x,1} + m_2 v_{x,2} + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i v_{x,i}}{\sum_i m_i}$$

$$(m_1 + m_2 + \ldots) v_{x,com} = m_1 v_{x,1} + m_2 v_{x,2} + \ldots = P_{net,x}$$

Similar for y and z components

$$\vec{v}_{com} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i \vec{v}_i}{\sum_i m_i}$$

$$\vec{P}_{net} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots = M \vec{v}_{com} \quad \text{where } M = m_1 + m_2 + \ldots$$

Acceleration of center of mass (com)

$$a_{x,com} = \frac{\Delta v_{x,com}}{\Delta t} = \frac{m_1 a_{x,1} + m_2 a_{x,2} + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i a_{x,i}}{\sum_i m_i}$$

Similar for y and z components

$$\vec{a}_{com} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i m_i \vec{a}_i}{\sum_i m_i}$$

$$M\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \dots + m_n\vec{a}_n$$

$m_1\vec{a}_1 = \vec{F}_{1,net} = \vec{F}_{1,ext} + \vec{F}_{1,\text{int}}$ & $m_2\vec{a}_2 = \vec{F}_{2,net} = \vec{F}_{2,ext} + \vec{F}_{2,\text{int}}$

Newton's 3rd law for internal forces: $\vec{F}_{\text{from 2 on 1}} + \vec{F}_{\text{from 1 on 2}} = 0$

→ $\vec{F}_{1,\text{int}} + \vec{F}_{2,\text{int}} + \dots + \vec{F}_{n,\text{int}} = 0$

$$M\vec{a}_{CM} = \vec{F}_{1,ext} + \vec{F}_{2,ext} + \vec{F}_{3,ext} + \dots + \vec{F}_{n,ext}$$

$$M\vec{a}_{com} = \vec{F}_{net,ext}$$

Newton's second law for center of mass

$$\vec{F}_{net,ext} = M\vec{a}_{com}$$

$\vec{F}_{net,ext}$: Sum of all *external* forces that act on the system

(*Internal* forces are *not* included)

$M = m_1 + m_2 + \dots$: Total mass of the system

$\vec{a}_{com}$: Acceleration of the center of mass

Internal forces do NOT change the motion of C.O.M.!!

Motion of center of mass under gravity force

Newton's second law for C.O.M.: $\vec{F}_{net,ext} = M\vec{a}_{com}$

Under gravity,

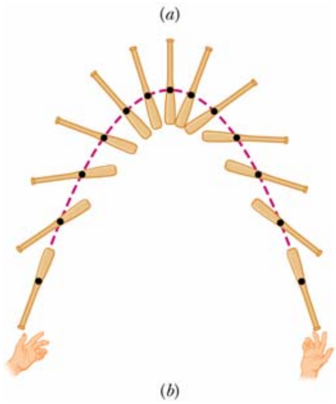
$$\vec{F}_{net,ext} = -m_1 g\mathbf{j} - m_2 g\mathbf{j} - \ldots$$
$$= -(m_1 + m_2 + \ldots)g\mathbf{j} = -Mg\mathbf{j}$$

$$\therefore -Mg\mathbf{j} = M\vec{a}_{com}$$

$$\therefore \vec{a}_{com} = -g\mathbf{j}$$ → Usual projectile motion

Center of mass moves like a particle of mass M under the net external force.

Motion of COM is simple!



Example 1

A 2.0 kg particle has a velocity (2.0 **i** -3.0 **j**) m/s, and a 3.0 kg particle has a velocity (1.0 **i** + 6.0 **j**) m/s. Find (a) velocity of the center of mass and (b) the total momentum of the system.

iClicker Quiz

Two objects with unknown mass and velocity collide and stick together moving at 3 m/s along x direction.

Assuming that net external force on the two objects is zero, what is the velocity of the center of mass before the collision?

(a) 0

(b) 3 m/s along x

(c) -3 m/s along x

(d) Not enough information