

Lecture

Energy Work and Kinetic Energy Kinetic and Potential Energy

Physics 105 Spring 2009

Lecture 8

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ENERGY

Energy is a property of the state of an object: hard to define precisely

Energy is a scalar quantity. It does not have a direction associated with it

Energy is conserved. It can be transferred from one object to another or change in form, but not created or destroyed.

Units: joule = $\text{kg} \cdot \text{m}^2/\text{s}^2$

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Kinetic Energy

Kinetic Energy \equiv Energy of motion

$K = \frac{1}{2}mv^2$ for object moving with velocity v

$$K = \frac{1}{2}mv^2 \quad \left[\text{J} = \text{kg} \frac{\text{m}^2}{\text{s}^2} \right]$$

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Kinetic Energy: Orders of Magnitude

$$K = \frac{1}{2}mv^2 \text{ for object moving with velocity } v$$



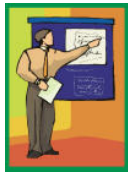
Earth orbiting sun: 2×10^{29} J

Car at 60 mph: 100,000 J

Nolan Ryan pitch: 300 J

Professor walking: 40 J

Angry bee: 0.005 J



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Why $K = \frac{1}{2}mv^2$?



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Why $K = \frac{1}{2}mv^2$?

Special case: Constant Acceleration

Remember result eliminating t :

$$v^2 - v_0^2 = 2a(x - x_0)$$

Multiply by $\frac{1}{2}m$:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = ma(x - x_0) = ma\Delta x$$

But $F=ma!$

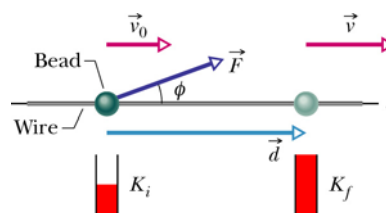
$$\Delta\left(\frac{1}{2}mv^2\right) = F\Delta x$$

Energy and Work

Kinetic energy

$$K = \frac{1}{2}mv^2 \quad \left[J = \text{kg} \frac{\text{m}^2}{\text{s}^2} \right]$$

Units of Work and Energy: Joule

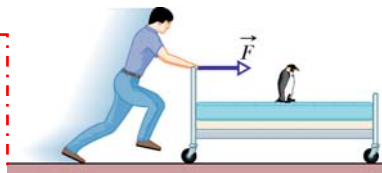


Work done by a constant force

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

Work-kinetic energy theorem

$$\Delta K = K_f - K_i = W$$



Work

Work \equiv Energy transferred by a force

Work done on an object is the energy transferred to/from it

$W > 0 \rightarrow$ energy added

$W < 0 \rightarrow$ energy taken away

$W = \vec{F} \cdot \vec{r} \equiv$ Work done on an object by a constant force \vec{F} while moving through a displacement \vec{r}

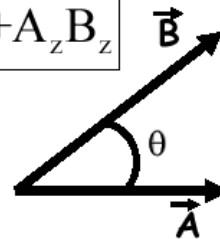
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Dot Product: Physical Meaning

$$\vec{A} \cdot \vec{B} = \underline{AB \cos \theta} = A_x B_x + A_y B_y + A_z B_z$$



$$\theta = 0 \rightarrow \vec{A} \cdot \vec{B} = \underline{AB}$$

$$\theta = 90^\circ \rightarrow \vec{A} \cdot \vec{B} = \underline{0}$$

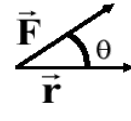
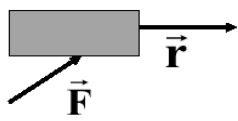
Dot product measures how much vectors are along each other

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What does $W = \vec{F} \cdot \vec{r}$ mean?



$$\begin{aligned}
 W &= \vec{F} \cdot \vec{r} \\
 &= F_x r_x + F_y r_y \\
 &= F r \cos\theta
 \end{aligned}$$

$W > 0$ if $\theta < 90^\circ$ \longrightarrow force is adding energy to object

$W < 0$ if $\theta > 90^\circ$ \longrightarrow force is reducing energy of object



$W = 0$ if $r = 0$ or $F = 0$ or $\vec{F} \perp \vec{r}$

Work Examples

Push on a wall

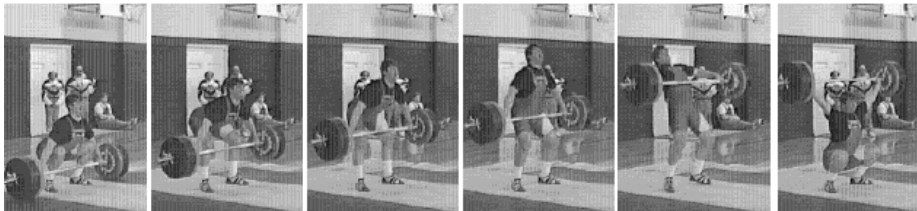
$W = 0$ since wall does not move ($\vec{r} = 0$)

Work due to Gravity

A weightlifter does work when lifting a weight

$$W = mgh$$

(h is the vertical drop)

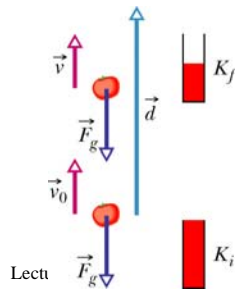


Work Done by a Gravitational Force

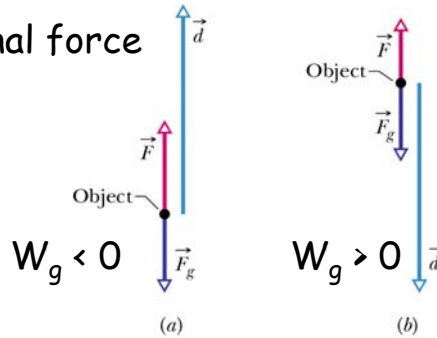
Work done by gravitational force

$$W_g = mgd \cos \theta$$

Tomato thrown upward



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Lifting/lowering an object

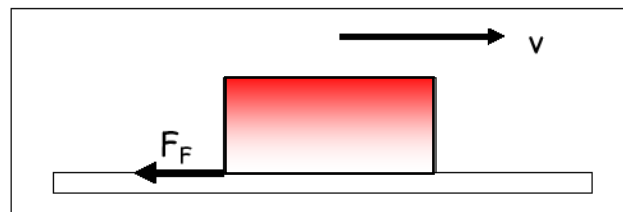
Change in kinetic energy:

$$\Delta K = K_f - K_i = W_a + W_g$$

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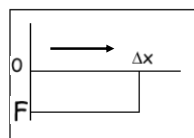
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Work due to Friction



The frictional force always opposes the motion

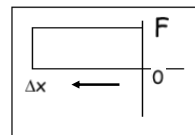
Moving to the right



$$W = -|F| \Delta x$$

$$\Delta x > 0$$

Moving to the left



$$W = +|F| \Delta x$$

$$\Delta x < 0$$

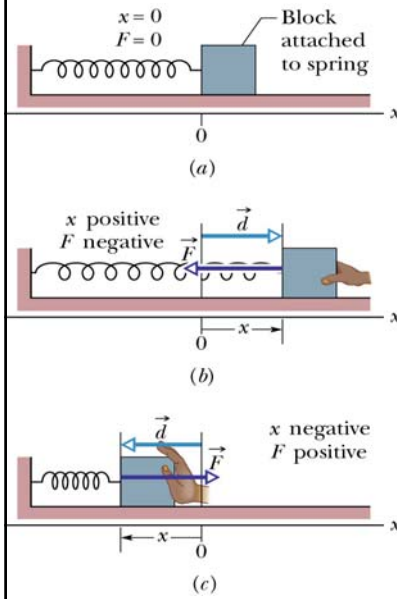
W negative in both cases

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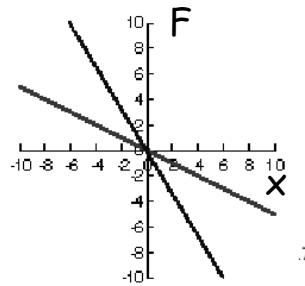
Restoring Force



Equilibrium - no force
 Stretched - force towards equilibrium point

$$F = -kx$$

Hooke's Law



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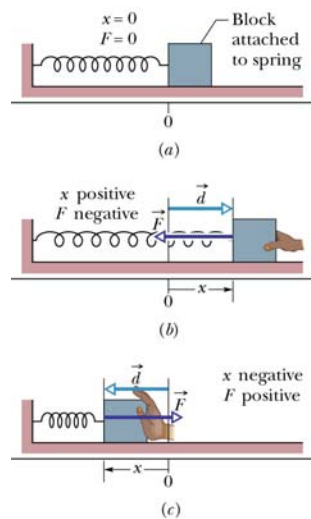
Work Done by a Spring Force

Hooke's law: $\vec{F} = -k\vec{d}$

Work done by a spring force:

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

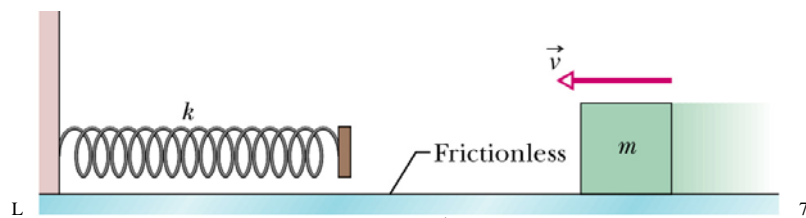
$$\begin{aligned} W_s &= \sum F_j \Delta x \\ W_s &= \int_{x_i}^{x_f} F dx \\ &= \int_{x_i}^{x_f} -kx dx = -k \int_{x_i}^{x_f} x dx \\ &= -\frac{1}{2}k [x^2]_{x_i}^{x_f} \\ &= -\frac{1}{2}k(x_f^2 - x_i^2) = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \end{aligned}$$



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Sample Problem 7-8

A block of mass $m = 0.40 \text{ kg}$ slides across a horizontal frictionless counter with a speed of $v = 0.50 \text{ m/s}$. It runs into and compresses a spring of spring constant $k = 750 \text{ N/m}$. When the block is momentarily stopped by the spring, by what distance d is the spring compressed?



Work Done by a General Variable Force

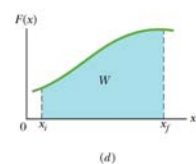
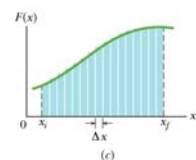
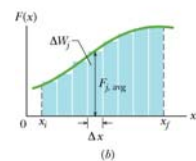
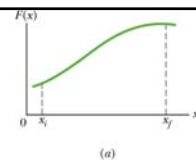
Work: variable force

$$W = \int_{x_i}^{x_f} F(x) dx$$

- Calculus
- Divide area under curve
- Add increments of W (numerically)
- Analytical form?
- Integration!!!

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Power

Work doesn't depend on the time interval

Work to climb a flight of stairs ~3000 J

10 s
1 min
1 hour

Power is work done per unit time

Average Power

$$P_{avg} = \frac{W}{\Delta t}$$

Instantaneous Power $P = dW/dt = F dx/dt = Fv$

Units $\frac{\text{Work}}{\text{time}}$

$$\frac{1 \text{ J}}{1 \text{ s}} = 1 \text{ Watt}$$

$$1 \text{ hp} = 746 \text{ W}$$

$$P = \frac{1}{2} * 60 \text{ kg} * (5 \text{ m/s})^2$$

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Power

Average Power

$$P_{avg} = \frac{W}{\Delta t}$$

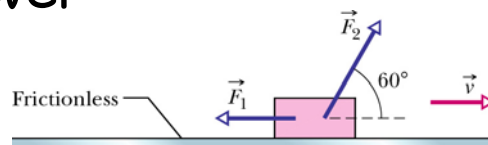
Units: Watts

Instantaneous Power

$$P = \frac{dW}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P = \frac{dE}{dt}$$



Sample Problem 7-10: Two constant forces F_1 and F_2 acting on a box as the box slides rightward across a frictionless floor. Force F_1 is horizontal, with magnitude 2.0 N , force F_2 is angled upward by 60° to the floor and has a magnitude of 4.0 N . The speed v of the box at a certain instant is 3.0 m/s .

- What is the power due to each force acting on the box? Is the net power changing at that instant?
- If the magnitude F_2 is, instead, 6.0 N , what is now the net power, and is it changing?

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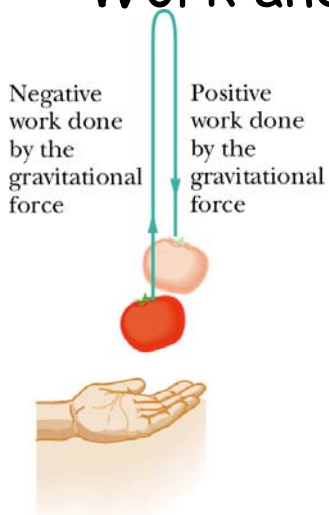
- **Potential Energy and Conservation of Energy**
- **Conservative Forces**
- **Gravitational and Elastic Potential Energy**
- **Conservation of (Mechanical) Energy**
- **Potential Energy Curve**
- **External and Internal Forces**

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Work and Potential Energy



Potential Energy

$$\Delta U = -W$$

General Form:

$$W = \int_{x_i}^{x_f} F(x) dx$$

$$\Delta U = - \int_{x_i}^{x_f} F(x) dx$$

Gravitational Potential Energy

$$U = mgy$$

Elastic Potential Energy

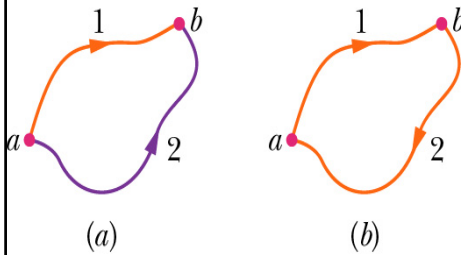
$$U = \frac{1}{2}kx^2$$

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Path Independence of Conservative Forces



- The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.
- The net work done by a conservative force on a particle moving around every closed path is zero.

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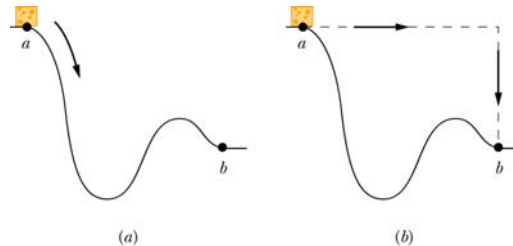
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Path Independence of Conservative Forces

Sample Problem 8-1: A 2.0 kg block slides along a frictionless track from a to point b . The block travels through a total distance of 2.0 m, and a net vertical distance of 0.8 m. How much work is done on the block by the gravitational force?

$$U = mgy$$



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Conservation of Mechanical Energy

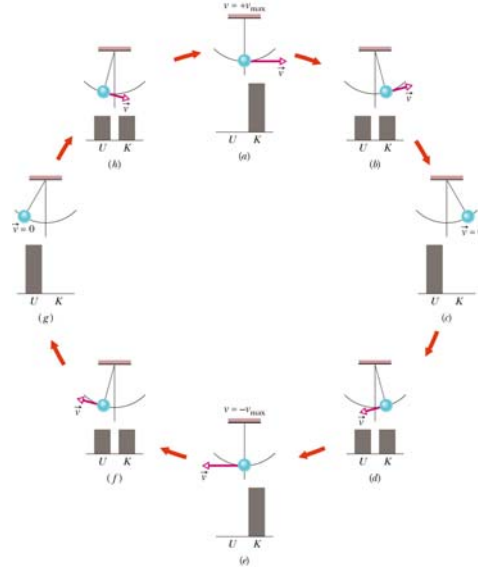
Mechanical Energy

$$E_{\text{mec}} = K + U$$

Conservation of Mechanical Energy

$$K_2 + U_2 = K_1 + U_1$$

In an isolated system where only conservative forces cause energy changes, the kinetic and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.



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Kinetic Energy:

$$K = \frac{1}{2}mv^2$$

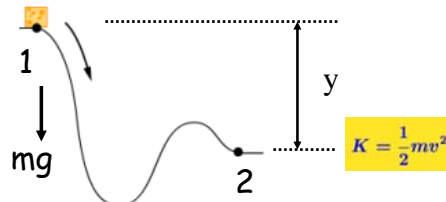
Potential Energy:

$$\Delta U = -W$$

• Gravitation: $U = mgy$

• Elastic (due to spring force): $U = \frac{1}{2}kx^2$

$$U = mgy$$



$U \rightarrow K$

Conservation of Mechanical Energy

$$E_{\text{mec}} = K + U$$

$$K_2 + U_2 = K_1 + U_1$$

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Kinetic Energy:

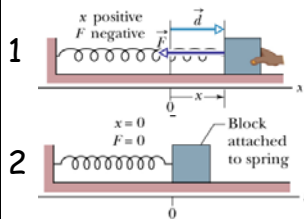
$$K = \frac{1}{2}mv^2$$

Potential Energy:

$$\Delta U = -W$$

• Gravitation: $U = mgy$

• Elastic (due to spring force): $U = \frac{1}{2}kx^2$



$$U = \frac{1}{2}kx^2 \quad K = 0$$

$U \leftrightarrow K$

$$U = 0 \quad K = \frac{1}{2}mv^2$$

Conservation of Mechanical Energy

$$E_{\text{mec}} = K + U$$

$$K_2 + U_2 = K_1 + U_1$$

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QZ#8

Name, ID#, Section #

SF

4 days



NYC

NYC - SF by train. On a regular schedule

It takes 4 days for 1 way trip

One train per day; starts at 1 pm in NYC

and arrives in 4 days at 1 pm to SF.

Quickly unload - load and go back.

- Calculate the work of the engine when the train of the mass 10,000 kg accelerates to $v = 72 \text{ km/h}$ from zero at the departure from NYC.
- Calculate the work done by the breaks (friction force) when the train slows down from $v = 72 \text{ km/h}$ to $v = 0$ arriving to SF.
- How many other trains will our train meet during one way trip?

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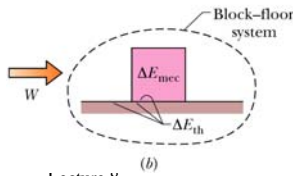
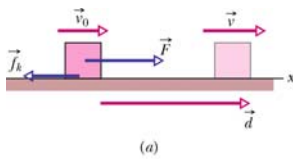
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Conservation of Energy

Thermal Energy/Friction

$$\Delta E_{th} = f_k d$$

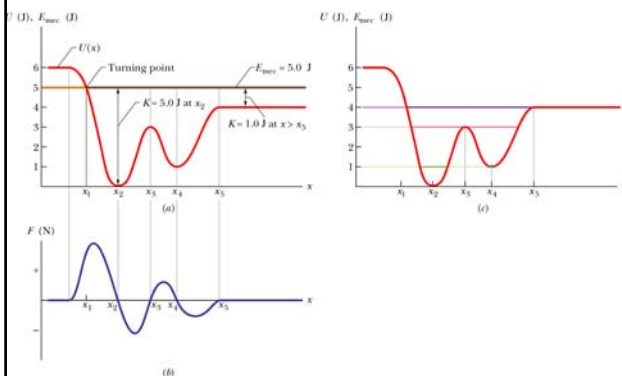


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- The total energy of a system can change only by amounts of energy that are transferred to or from the system.
- The total energy E of an isolated system cannot change.

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

Potential Energy Curve



1D Motion

$$F(x) = -\frac{dU(x)}{dx}$$

Turning Points

Equilibrium Points

- Neutral Equilibrium
- Unstable Equilibrium
- Stable Equilibrium

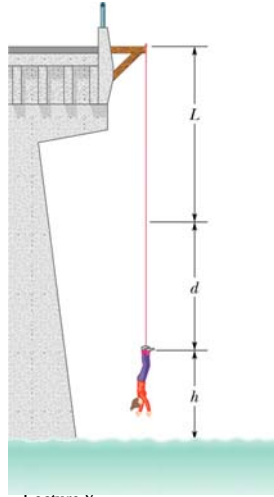
A plot of $U(x)$, the potential energy function of a system containing a particle confined to move along the x axis. There is no friction, so mechanical energy is conserved.

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Sample Problem 8-4



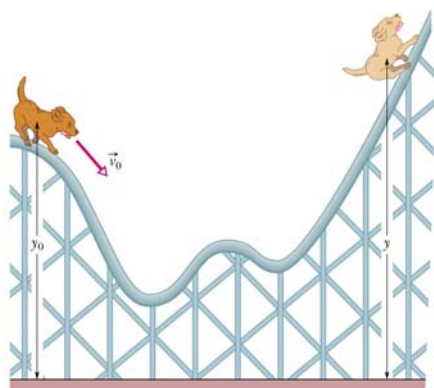
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A 61.0 kg bungee-cord jumper is on a 45.0 m bridge above a river. The elastic bungee cord has a relaxed length of $L = 25.0$ m. Assume that the cord obeys Hooke's law, with a spring constant of 160 N/m. If the jumper stops before reaching the water, what is the height h of her feet above the water at her lowest point?

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Sample Problem 8-8



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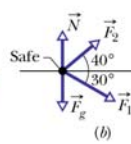
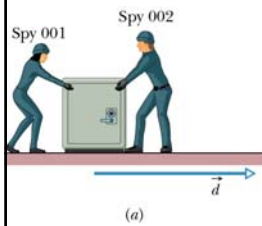
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A circus beagle of mass $m = 6.0$ kg runs onto the left end of a curved ramp with speed $v_0 = 7.8$ m/s at height $y_0 = 8.5$ m above the floor. It then slides to the right and comes to a momentary stop when it reaches a height $y = 11.1$ m from the floor. The ramp is not frictionless. What is the increase ΔE_{th} in the thermal energy of the beagle and the ramp because of the sliding?

Sample Problem 7-2

Two industrial spies sliding an initially stationary 225 kg floor safe a displacement d of magnitude 8.50 m , straight toward their truck. The push F_1 of spy 001 is 12.0 N , directed at an angle of 30° downward from the horizontal; the pull F_2 of spy 002 is 10.0 N , directed at 40° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.



- What is the net work done on the safe by the forces F_1 and F_2 during the displacement d ?
- During the displacement, what is the work W_g done on the safe by the gravitational force F_g and what is the work done on the safe by the normal force N from the floor?
- The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?