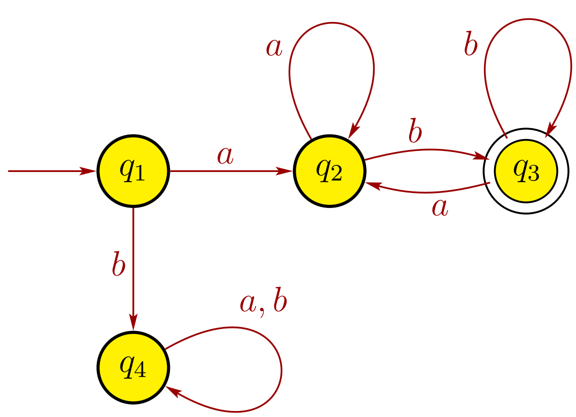
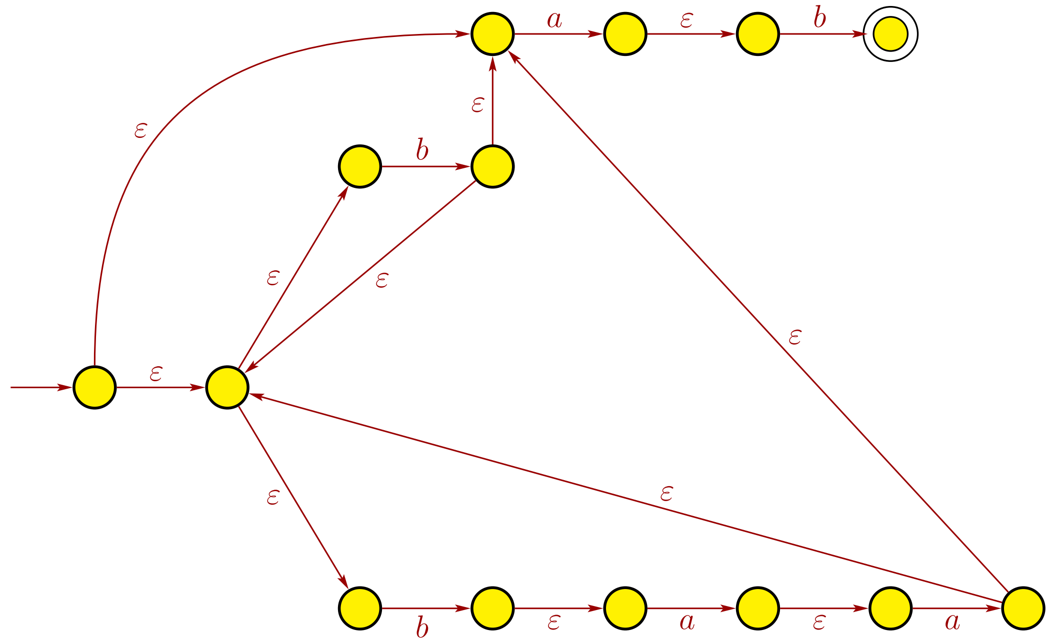
CS 341, Fall 2007
Solutions for Midterm, eLearning Section

1. (a) False. The regular expression $a^*b^*a^*$ generates the string $aabaaa \notin A$. Moreover, A is not a regular language, so it cannot have a regular expression.
   (b) False. The conclusion doesn't make sense at all since $A \times B$ consists of pairs of strings but $A \circ B$ just consists of strings.
   (c) True. Corollary 2.32.
   (d) False. The language $\{ 0^n1^n \mid n \geq 0\}$ is context-free (see slide 2-5), but not regular (see slide 1-90).
   (e) False. The language $\{ 0^n1^n \mid n \geq 0\}$ is context-free (see slide 2-5), but is infinite.
   (f) False. The language $A = \{abb\}$ is finite and has CFG with rule $S \rightarrow abb$.
   (g) True, by Theorem 1.47.
   (h) True, by Kleene's Theorem (Theorem 1.54).
   (i) True, by Theorem 2.20.
   (j) False. If A is recognized by an NFA, then A must be regular by Corollary 1.40.

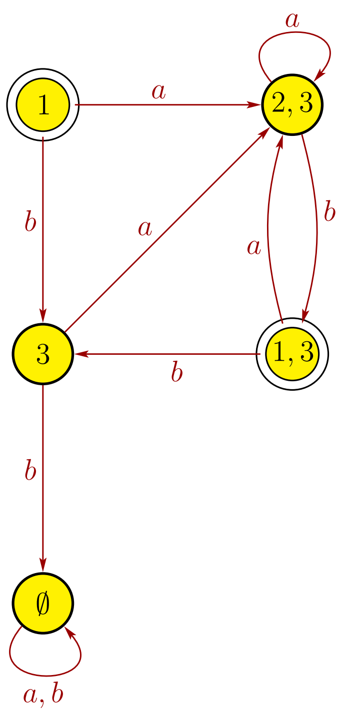
2. (a) Lexicographic order means that shorter strings appear before longer strings, and strings of the same length are in alphabetical order.
   (b) For a CFG $G = (V, \Sigma, R, S)$ to be in Chomsky normal form, each of its rules must have one of three forms: $A \rightarrow BC$, $A \rightarrow x$, or $S \rightarrow \varepsilon$, where A, B, C are variables, B and C are not the start variable, x is a terminal, and S is the start variable.
   (c) $a(a \cup b)^*b$
   (d) DFA



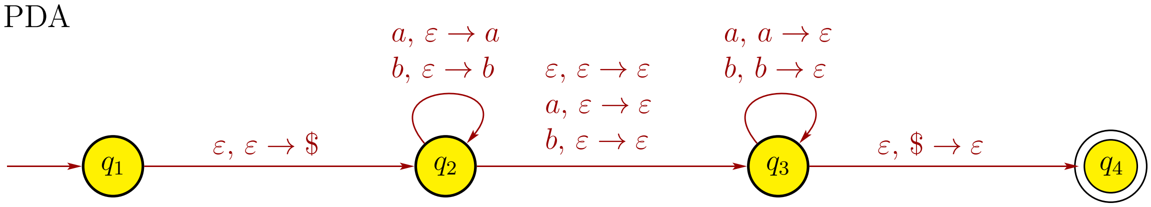
3. NFA



4. DFA:



5. (a) $G = (V, \Sigma, R, S)$, where $V = \{S\}$, $\Sigma = \{a, b\}$, and the rules are $S \to aSa \mid bSb \mid a \mid b \mid \varepsilon$.
   (b) PDA



6. No, the class of context-free languages is not closed under intersection. Consider the languages $A = \{a^n b^n c^k \mid n, k \geq 0\}$ and $B = \{a^n b^k c^n \mid n, k \geq 0\}$. A CFG for A is $G_1 = (V_1, \Sigma, R_1, S_1)$, with $V_1 = \{S_1, X, Y\}$, $\Sigma = \{a, b, c\}$, and rules $S_1 \to XY$, $X \to aXb \mid \varepsilon$, $Y \to cY \mid \varepsilon$. A CFG for B is $G_2 = (V_2, \Sigma, R_2, S_2)$, with $V_2 = \{S_2, Z\}$, $\Sigma = \{a, b, c\}$, and rules $S_2 \to aS_2c \mid Z$, $Z \to bZ \mid \varepsilon$. Then $A \cap B = \{a^n b^n c^n \mid n \geq 0\}$, which is not context-free (see page 2-105 of the notes).

7. Suppose that $A = \{w \in \Sigma^* \mid w = w^{\mathcal{R}}\}$ is a regular language. Let p be the pumping length, and consider the string $s = a^p b a^p \in A$. Note that $|s| = 2p + 1 \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^i z \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the first set of a's, followed by ba^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell b a^p$, where $j + k + \ell = p$ since $xyz = s = a^p b a^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell b a^p = a^{p+k} b a^p$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since $w \neq w^{\mathcal{R}}$, which is a contradiction, so A is not a regular language.