Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2008, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

_____
Signature and Date

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to be given on Saturday, October 18, 2008. The exam is to last 2.5 hours.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is recognized by an NFA, then A is a context-free language.

(b) TRUE FALSE — If A is a regular language and we add a finite number of strings to A, then the resulting language must be regular.

(c) TRUE FALSE — If A is a regular language and we remove a finite number of strings from A, then the resulting language must be regular.

(d) TRUE FALSE — A language L has a CFG if and only if L is recognized by a PDA.

(e) TRUE FALSE — If a language A is not regular, then it A must be infinite.

(f) TRUE FALSE — If a language A is regular, then it A must be finite.

(g) TRUE FALSE — A regular expression for the language $\{ 0^n 1^n \mid n \geq 0 \}$ is 0^*1^*.

(h) TRUE FALSE — $\emptyset$ is a context-free language.

(i) TRUE FALSE — The class of regular languages is closed under complementation.

(j) TRUE FALSE — If A and B are context-free languages, then so is $A \circ B$.

2. [**10 points**] Give short answers to each of the following parts. **Each answer should be at most three sentences. Be sure to define any notation that you use.**

   (a) Suppose f is a mapping defined on a domain D. What does it mean for D to be closed under f?

   (b) What does it mean for a context-free grammar $G = (V, \Sigma, R, S)$ to be in Chomsky normal form?

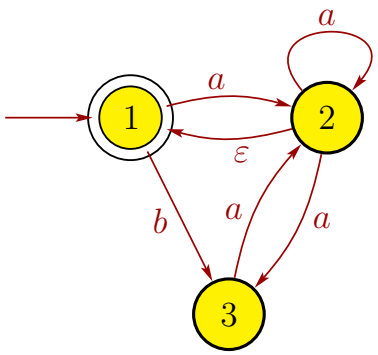
3. **[20 points]** Let Σ be the alphabet of all printable characters on a standard American computer keyboard. Let $\Sigma_1 = \{\texttt{0}, \texttt{1}, \ldots, \texttt{9}\}$ be the set of digits, and let $\Sigma_2 = \{\texttt{-}, \texttt{+}\}$ be the set of signs. Define L to be the set of all strings that represent floating-point numbers that are not in exponential notation. Specifically, L consists of strings that start with an optional sign, followed by zero or more digits, followed by a decimal point, and end with zero or more digits, where the string must have at least one digit and exactly one decimal point. Examples of strings in L are "`13.231`", "`-28.`" and "`.124`".

   (a) Give a regular expression for the language L.

   (b) Give a DFA for the language L with alphabet Σ. You only need to draw the graph; do not specify the DFA as a 5-tuple.

4. [**10 points**] Give an example of two context-free languages A and B whose intersection C is not context-free. Show that A and B are context-free by giving context-free grammars for them, but do not prove that C is not context-free.

5. **[10 points]** Convert the following NFA into an equivalent DFA.



Answer:

Scratch-work area

6. **[20 points]** Let $\Sigma = \{a, b\}$, and consider the language $A = \{ w \in \Sigma^* \mid w = w^{\mathcal{R}}, |w| \text{ is odd } \}$, where $w^{\mathcal{R}}$ denotes the reverse of w and $|w|$ denotes the length of w.

   (a) Give a context-free grammar G that generates A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

   (b) Give a pushdown automaton that recognizes A. You only need to draw the picture.

Scratch-work area

7. [**10 points**] Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then there are strings x, y, z such that $s = xyz$ and

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Prove that $A = \{\, c^{2n}a^nb^{2n} \mid n \geq 0 \,\}$ is not a regular language.