**CS 341, Fall 2009**
**Solutions for Midterm, eLearning Section**

1. (a) True. The language $\emptyset$ is finite, so slide 1-81 shows that it is regular. Corollary 2.32 then implies that $\emptyset$ is also context-free.

   (b) True. Suppose $A$ is finite. Then $A$ is regular by slide 1-81, so $A$ can't be nonregular.

   (c) True. Theorem 2.9.

   (d) False. Homework 5, problem 1(a), is a context-free language that is also regular since it has regular expression $0^*1^*0^*1^*0^*1^*(0 \cup 1)^*$.

   (e) True. If $B$ is regular, then so is $\overline{B}$ by Homework 2, problem 3. Then $A \cap \overline{B}$ is regular by slide 1-28.

   (f) False. The language $A = \{0, 1\}^*$, which is infinite, has regular expression $(0 \cup 1)^*$. Thus, Theorem 1.54 implies $A$ is regular.

   (g) False. The regular expression $1^*0^*1^*$ generates the string $001 \notin \{1^n0^n1^n \mid n \geq 0\}$, so it cannot be a correct regular expression for the language. In fact, the language is nonregular, so it cannot have a regular expression.

   (h) False. The derivation $S \Rightarrow 0$ generates the string 0, which is not in the language, so the CFG cannot be correct.

   (i) True. Slide 1-28.

   (j) False. The language $A = \{0^n1^n | n \geq 0\}$ is nonregular, and $B = \emptyset$ is a subset of $A$, but $B$ is regular since it is finite.

2. (a) Shorter strings appear before longer strings, and strings of the same length are in alphabetical order.

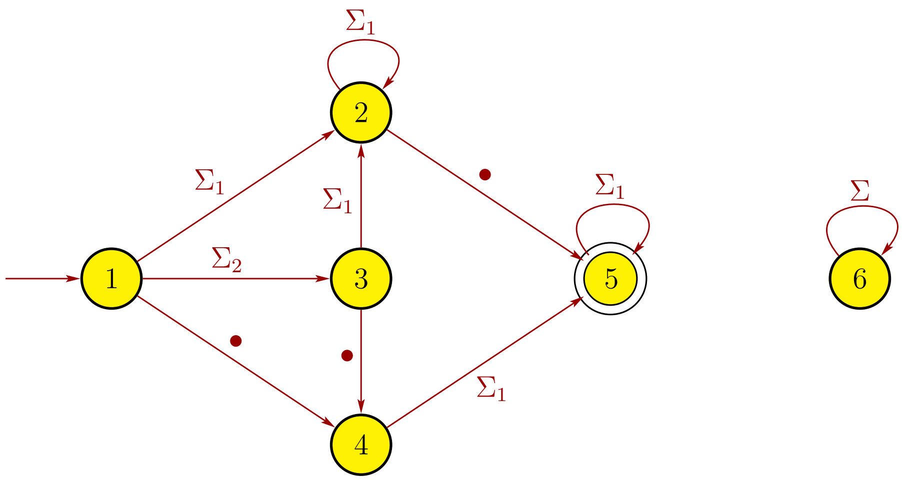
   (b)
   - $S \rightarrow Ya$ is not in Chomsky normal form since the CFG cannot have a right-hand side (RHS) that is a mix of terminals and variables.
   - $X \rightarrow YS$ is improper since $S$ cannot be on the RHS of a rule.
   - $Y \rightarrow \varepsilon$ is improper since $\varepsilon$ cannot be on the RHS of rule when the left side is not $S$.
   - $Y \rightarrow YXY$ is improper since the RHS cannot have more than two variables.

   (c) slide 1-53.

   (d) The set $D$ is closed under $f$ means that $x \in D$ implies $f(x) \in D$.

3. (a) Regular expression: $(+ \cup - \cup \varepsilon)(\Sigma_1\Sigma_1^*.\Sigma_1^* \;\cup\; .\Sigma_1\Sigma_1^*)$

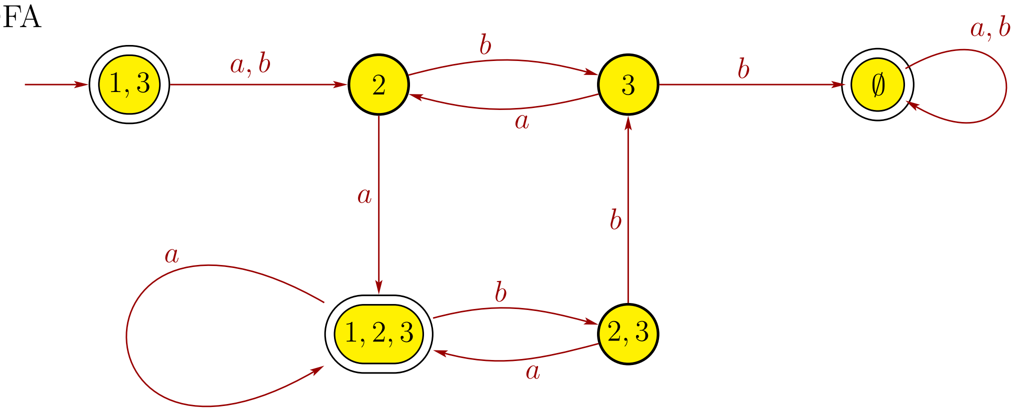
   (b) DFA:




All transitions not specified go to state 6.

4. Homework 3, problem 2.

5. DFA




6. $G = (V, \Sigma, R, S)$, with $V = \{S, X\}$, $\Sigma = \{a, b, c\}$, start variable $S$ and rules $S \to cSb \mid X$ and $X \to aXb \mid \varepsilon$.

7. Suppose that $A$ is a regular language. Let $p$ be the pumping length, and consider the string $s = a^p bba^p \in A$. Note that $|s| = 2p + 2 \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^i z \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that $x$ and $y$ have only $a$'s (together up to $p$ in total) and $z$ has the rest of the $a$'s at the beginning, followed by $bba^p$. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell bba^p$, where $j + k + \ell = p$ since $xyz = s = a^p bba^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell bba^p = a^{p+k} bba^p$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since it is not the same forwards and backwards because $k > 0$, which contradicts (i), so $A$ is not a regular language.