Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2009, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

_____
Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to last 2.5 hours and is to be given on Saturday, October 10, 2009.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
    1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
    2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
    3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — $\emptyset$ is a context-free language.

(b) TRUE FALSE — If A is a nonregular language, then A must be infinite.

(c) TRUE FALSE — Every context-free language has a context-free grammar in Chomsky normal form.

(d) TRUE FALSE — If A is a context-free language, then A must be nonregular.

(e) TRUE FALSE — If A and B are regular languages, then $A \cap \overline{B}$ must be regular.

(f) TRUE FALSE — If a language A is regular, then it A must be finite.

(g) TRUE FALSE — The language $\{ 1^n 0^n 1^n \mid n \geq 0 \}$ has regular expression $1^*0^*1^*$.

(h) TRUE FALSE — The language $\{ 1^n 0^n \mid n \geq 0 \}$ has context-free grammar $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{0, 1\}$, start variable S, and rules $S \rightarrow 1S0 \mid 0$.

(i) TRUE FALSE — The class of regular languages is closed under intersection.

(j) TRUE FALSE — If A is a nonregular language and $B \subseteq A$, then B must be nonregular.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

   (a) What does it mean for a sequence of strings to be in *lexicographic order*?

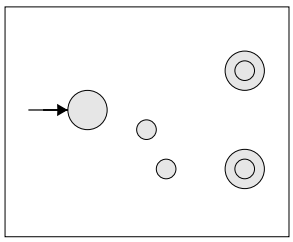
   (b) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

$$\begin{aligned} S &\rightarrow Ya \mid XX \mid \varepsilon \\ X &\rightarrow YS \mid b \\ Y &\rightarrow \varepsilon \mid YXY \end{aligned}$$

   Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

(c) Suppose that language A_1 is recognized by NFA N_1 below. Note that the transitions are not drawn in N_1. Draw a picture of an NFA for A_1^*.

N_1



(d) Suppose f is a mapping defined on a domain D. What does it mean for D to be closed under f?

3. **[20 points]** Let Σ be the alphabet of all printable characters on a standard American computer keyboard. Let $\Sigma_1 = \{\texttt{0}, \texttt{1}, \ldots, \texttt{9}\}$ be the set of digits, and let $\Sigma_2 = \{\texttt{-}, \texttt{+}\}$ be the set of signs. Define L to be the set of all strings that represent floating-point numbers that are not in exponential notation. Specifically, L consists of strings that start with an optional sign, followed by zero or more digits, followed by a decimal point, and end with zero or more digits, where the string must have at least one digit and exactly one decimal point. Examples of strings in L are "`13.231`", "`-28.`" and "`.124`".

   (a) Give a regular expression for the language L.

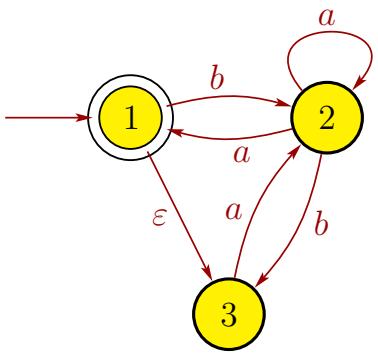
   (b) Give a DFA for the language L with alphabet Σ. You only need to draw the graph; do not specify the DFA as a 5-tuple.

4. [**10 points**]

(a) Show by giving an example that, if M is an NFA that recognizes language C, swapping the accept and non-accept states in M does not necessarily yield a new NFA that recognizes C. Explain your answer.

(b) Is the class of languages recognized by NFAs closed under complements? Explain your answer.

5. **[10 points]** Convert the following NFA into an equivalent DFA.



Answer:

Scratch-work area

6. **[10 points]** Give a context-free grammar G that generates the language

$$A = \{\, c^i a^j b^k \mid i, j, k \geq 0 \text{ and } i + j = k \,\}.$$

Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

Scratch-work area

7. [**10 points**] Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then there are strings x, y, z such that $s = xyz$ and

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Let $\Sigma = \{a, b\}$, and consider the language $A = \{\, w \in \Sigma^* \mid w = w^{\mathcal{R}}, |w| \text{ is even} \,\}$, where $w^{\mathcal{R}}$ denotes the reverse of w and $|w|$ denotes the length of w. Prove that A is not a regular language.