Math 340 * Review for Final Exam * Victor Matveev

Problem 1:

Assuming $|x|<<1$, for what values of x does the function evaluation below **completely fail** (i.e. relative error ≥ 100%) in double precision? What result would MATLAB produce for those values of x? Using Taylor polynomials, find **leading two terms** in the limiting behavior as $x\to 0$:

$$f(x) = \frac{x^5}{\sin(x^2) - \log(1+x^2)}$$

Problem 2:

Find values w_1, w_2 and x_2 so that the following integration rule has the highest degree of precision:

$$\int_0^1 \ln(x)\, f(x)\, dx = w_1 f(0) + w_2 f(x_2) \qquad \left(\text{Use the result } \int_0^1 x^m \ln(x) dx = -\frac{1}{(m+1)^2} \right)$$

Use the resulting integration rule to estimate $\int_0^1 \frac{\ln(x)}{1+x} dx$, comparing with the exact value $\int_0^1 \frac{\ln(x)}{1+x} dx = -\frac{\pi^2}{12} \doteq 0.8225$

Compare the accuracy of this estimate with the midpoint rule with $n=1$, noting that $\ln(2)\approx 0.69315$

Problem 3:

Find values of constants A, B and C so that the following finite difference approximates the 2^{nd} derivative of $f(x)$ at x_0. What is the error of this approximation? To check your answer, apply this formula to $f(x)=x^2$

$$Df(x_0) = A\, f(x_0 - 2h) + B\, f(x_0) + C\, f(x_0 + 3h)$$

Problem 4:

Find the value of constants γ_1 and γ_2 so that the following method of integrating a differential equation $dY/dx = f(x, Y)$ has second order of accuracy:

$$y_{n+1} = y_n + h\left[\gamma_1 f(x_n, y_n) + \gamma_2 f\left(x_n + \frac{2}{3}h,\; y_n + \frac{2}{3}h\, f(x_n, y_n) \right) \right]$$

Is this method absolutely stable?

Problem 5:

Find the value of constant k so that the following function is a cubic spline, and sketch this spline:

$$s(x) = \begin{cases} 1 + x + (x-1)^2 + (x-1)^3 & \text{if } 1 \le x \le 2 \\ 5 + 6(x-2) + k(x-2)^2 & \text{if } 2 \le x \le 3 \end{cases}$$

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Problem 6:

Use the least sum of squares method to fit a curve $y(x) = a\ \cos(x) + b$ to the data points $(0, 1)$, $(\pi/3,\ 0)$, $(\pi/2, -2)$
