

Math 340 * Quiz #6

April 26, 2012

Your Name: _____

Consider the following initial value problem:
$$\begin{cases} \frac{dY}{dx} = -Y^2 \\ Y(0) = 1 \end{cases}$$

- 1) Without solving this equation, make a rough sketch of the solution $Y(x)$ by examining the rate of change of Y as a function of Y .
- 2) Obtain the exact solution $Y(x)$ using separation of variables, and evaluate $Y(\mathbf{0.5})$ (check that your solution agrees with the sketch in part 1).
- 3) Find the approximate solution at $x = \mathbf{0.5}$, using the following methods:
 - a) Euler method with $h = 0.5$
 - b) Euler method with $h = 0.25$
 - c) Richardson's extrapolation of $Y(0.5)$ using above two approximations
 - d) One step of the Heun's method with $h = 0.5$:

$$y_1 = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_0 + hf(x_0, y_0))]$$