

Math 335-002 * Midterm examination
February 17, 2006

Please show all work to receive full credit. Notes and calculators are *not* allowed.

1. (8) Find the equation of line passing through points (1, 2, 1) and (0, 1, 2). Use any form you like.
2. (12) Consider the scalar field $f(x, y, z) = \ln(x + y + z) + \cos(xy)$. Calculate approximately $f(0.1, 0.1, 1.1)$ using the linear approximation for f around point $\vec{r}_0 = (0, 0, 1)$. Find the equation of plane tangent to the isosurface of f at point \vec{r}_0 .
3. (10) Sketch the isocurves of the scalar field $f(x, y) = \exp(y^2 - x)$; indicate the direction of the gradient.
4. (12) Sketch the vector field $\vec{u} = (x + y, x, 0)$. Calculate its divergence and curl.
5. (12) Simplify the following expression using either vector algebra or suffix notation: $(\vec{a} - \vec{b}) \cdot \vec{c} \times (\vec{b} + \vec{a})$
6. (A, 10) Simplify and convert into vector form:

$$a_l a_j b_m \delta_{pl} \delta_{pj} + \epsilon_{lkn} \epsilon_{mnq} \delta_{kp} a_q b_l a_p = \epsilon_{nqm} \delta_{nl} c_q b_l$$

(B, 8) Convert this equation into suffix notation (do not simplify):

$$\vec{a} \times (\vec{c} \times \vec{b}) \cdot \vec{c} = |\vec{b} + \vec{a}|^2$$
- 7 (10) Calculate the divergence of the vector field given by $\vec{u} = \vec{r}/r$, where \vec{r} denotes the position vector, and $r = |\vec{r}|$.
- 8 (12) Consider vector field $\vec{F} = (y^3, z \exp(zy) + 3y^2x, y \exp(zy))$. Check whether \vec{F} is irrotational (conservative). Find the potential of the field \vec{F} , $\phi(x, y, z)$, and calculate its Laplacian.
- 9 (8pts) The following sketch (over) shows the values of two vector fields, \vec{v} and \vec{w} , at a select set of points forming a circle. It illustrates one of the results listed below. Show the relationship between vector fields \vec{v} and \vec{w} and the result you have chosen from the list. Draw several more vectors to complete the sketch.

(a) $\nabla \cdot \nabla \times \vec{u} = 0$ (a) $\nabla \times \nabla f = 0 = 0$ (c) $\vec{\nabla} \times (\vec{\nabla} \times \vec{u}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{u}) - \nabla^2 \vec{u}$