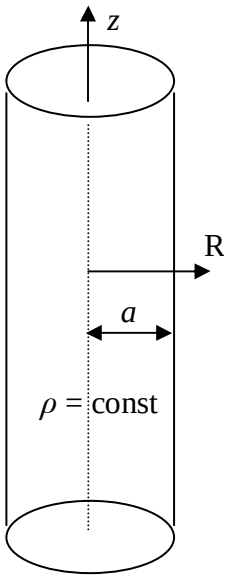
Math 335-002
Homework #10
Problems 1-4 due April 16; problem 5 due April 18, 2007

Please show all work in detail to receive full credit. Late homework is not accepted.



1. Consider a very long cable (cylinder) of radius *a* with a constant charge density inside the cable, ρ, and zero charge density outside the cable. Use **cylindrical** coordinates to find the electric potential Φ and the radial component of electric field E_R both inside and outside of the cable, by solving (integrating) $\nabla^2\Phi = -\rho/\varepsilon_0$ inside the cable, and $\nabla^2\Phi = 0$ outside of the cable, as we did in class for a sphere in spherical coordinates. Assume that Φ depends only on R, the distance from the z-axis, which is the axis of the cable: $\Phi=\Phi(R)$. Use Eq. 6.16 for the Laplacian. Assume that E_R is continuous across the surface of the cable; this condition will fix one of the integration constant. See example 8.3 on page 136 if in doubt. Hint: outside of the cable Φ will depend on the logarithm of R.

2. Consider two point charges located at Cartesian points (0,1,0) and (1,0,0), with electric charges equal to Q and 2Q, respectively. Find the potential and the electric field at point (1,1,0) using the superposition principle ($\Phi=\Phi_1+\Phi_2$, $\boldsymbol{E}=\boldsymbol{E}_1+\boldsymbol{E}_2$).

3. Consider an electromagnetic wave propagating in the *z*-direction, with the electric field polarized in the *y*-direction: $\vec{\mathbf{E}} = \vec{\mathbf{E}}(y) = \{0, A\cos(k z - \omega t), 0\}$, where *A* is a constant wave amplitude, *k* is the wave number, and $\omega = k c$ is the angular frequency. Show that $\vec{\mathbf{E}}$ satisfies the wave equation. Calculate the corresponding magnetic field $\vec{\mathbf{B}}$ by calculating $\vec{\nabla}\times\vec{\mathbf{E}}$ and then integrating with respect to time, as we did in class: $\vec{\mathbf{B}} = \int \vec{\nabla}\times\vec{\mathbf{E}}\, dt$

4. Problem 8.5 on page 139. Note that in this problem $\vec{\mathbf{E}}_0$ is a constant vector, *f* is a scalar field with an argument $(\vec{\mathbf{k}}\cdot\vec{\mathbf{x}} - \omega t)$, $\vec{\mathbf{k}}$ is a constant "wave vector" pointing in the direction of the wave, and $\vec{\mathbf{x}} = \vec{\mathbf{r}}$ is the position vector. Fill in the missing solution steps, as usual.

5. The New York public radio broadcasts on the frequency *f* =94 MHz. What is the wavelength of the radiowaves produced by the radio station?