Math 335-002
Homework #11
Due date: April 23, 2007

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Problem 7.2 on page 121: If $\vec{\mathbf{u}}$ is a vector field, show that $\vec{\nabla}\cdot\vec{\mathbf{u}}$ is a scalar field (derive the transformation rule for $\vec{\nabla}\cdot\vec{\mathbf{u}}$ in terms of L_{ij})

2. Problem 7.9 on page 121: if Q_{ijkl} is a tensor of rank 4, show that Q_{ijjl} is a tensor of rank two (derive the transformation rule for Q_{ijjl} in terms of L_{ij})

3. Find the conductivity tensor of a material (defined by $j_i = \sigma_{ik} E_k$) given the following 3 measurements of current density at different values of electric field:

   $\boldsymbol{j}$ = (0.8, 0.2, 0) A/m^2 when $\boldsymbol{E}$ = (2, 0, 0) V/m
   $\boldsymbol{j}$ = (0.7, 1.3, 0) A/m^2 when $\boldsymbol{E}$ = (1, 3, 0) V/m
   $\boldsymbol{j}$ = (0.9, 0.6, 0.5) A/m^2 when $\boldsymbol{E}$ = (2, 1, 1) V/m

   Hint: determine the first column of σ_{ik} using the 1st measurement, then use these results along with the 2nd measurement to determine the 2nd column, and so on. Units of σ_{ik} are (A/m^2)/(V/m)=A/(V·m)=S/m.

4. Show that the conductivity tensor you found in problem 3 will be diagonalized by a rotation around the *z*-axis by π/4. To do this, use matrix multiplication to find $\sigma' = L\,\sigma\,L^T$ (the matrix form of the rule $\sigma'_{ij} = L_{ik} L_{jl} \sigma_{kl}$), where a rotation around the z-axis is given by

$$L_z(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   Sketch or describe how the layers of the material are oriented with respect to the old and the new coordinate systems. Finally, use matrix multiplication to verify that $LL^T = I$

5. Problem 7.11 on p. 130: B_{rs} is an anti-symmetric tensor, so $B_{rs} = -B_{sr}$. Show that the anti-symmetry persists in a rotated frame, i.e. $B'_{rs} = -B'_{sr}$.

6. Problems 7.15 on p. 130: Find an isotropic fourth-rank tensor that can be written in terms of ε_{ijk}