Math 335-002
Homework #2

Due date: January 29 (problems 1-4) & January 31 (problems 5-8).

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Find the equation of a straight line which passes through the points (1, 2, -2) and (3,-1,1), in the cross product form ($\mathbf{r} \times \mathbf{u} = \mathbf{b}$).

2. Find the equation of the plane that is perpendicular to the vector (-1,2,2) and passes through the point (1, 0, 3). Use the dot product vector form ($\mathbf{r} \cdot \mathbf{a} = c$)

3. Find the equation of the plane that contains the points (1,0,0), (0,2,0), and (0,0,3). Use the same dot product form, $\mathbf{r} \cdot \mathbf{a} = c$. (Hint: you will need a simple intermediate step involving vector algebra).

4. Simplify the following expressions (hint: use the properties of the triple vector products). Bold letters indicate vectors.

   a) $((\mathbf{a} \times \mathbf{b}) \times \mathbf{b}) \times \mathbf{a}$ b) $(\mathbf{v}-\mathbf{u}) \times (\mathbf{v}-\mathbf{w}) \cdot (\mathbf{u}-\mathbf{w})$ c) $(\mathbf{u} \times \mathbf{v}) \cdot (\mathbf{w} \times \mathbf{u}) \times \mathbf{u}$

5. Sketch the following 2D fields (one of them is scalar, the other is a vector field):

   a) $F(x, y) = \ln(x\,y)$ (don't be intimidated – the isocurves are simple)
   b) $\mathbf{V}(x, y) = (x, x+y)$

6. Find the gradient of a 2D scalar field $f(\mathbf{r}) = \mathbf{r} \cdot \mathbf{a}$, where $\mathbf{r}$ is a constant vector. For $\mathbf{a} = (1, 1)$, sketch separately the scalar field (by showing its isocurves) and its gradient, which is a vector field. Comment on the geometry of the two sketches.

7. Consider a 2D scalar field $f(\mathbf{r}) = f(x, y) = y\,e^x + x \ln y$. Calculate its gradient , and use it to find the normal vector to one of the isocurves of this field, $y\,e^x + x \ln y = 1$, at point (0, 1). Don't try to draw the curve – it's not easy.

8. Find the gradient for a 3D scalar field $f(\mathbf{r}) = z\,e^x(1 + \ln y)$ (it will have 3 components, since there are now 3 coordinates). Calculate *approximately* the value of the field $f(\mathbf{r})$ at point $\mathbf{r} = (0.05, 1.1, 1.2)$, using the linear approximation for the field around point $\mathbf{r}$ $\mathbf{r}_o = (0, 1, 1)$:

$$f(\mathbf{r}) - f(\mathbf{r_o}) \approx (\text{grad}\, f)(\mathbf{r_o}) \cdot (\mathbf{r} - \mathbf{r_o})$$

Naturally, the estimate you obtained for $f(\mathbf{r})$ should roughly agree with its exact value (compare the two).