

Math 335-002
Homework #3

Due date: February 5 (problems 1-5) & February 7 (problems 6-8).

Please show all work in detail to receive full credit. Late homework is not accepted.

1. a) Find the unit normal to the surface $\ln(z + y^2) + \cos x = 1$ at point $(0, 1, 0)$.
a) Find the equation of the plane which is tangent to this surface at the same point.
2. Show that the vector field $\vec{V}(\vec{r}) = (\cos x \cos y, -\sin y \sin x + z^2, 2zy)$ is irrotational. Then, find the potential function for this field, $f(\vec{r})$, by integration, as we did in class.
3. Find the gradient of a scalar field $f(\vec{r}) = \exp(r)$, where r is the length of the position vector: $r = |\vec{r}|$. Show that the result can be written as $\vec{r} \exp(r) / r$ [note: there is nothing complicated in this problem: don't be scared of position vectors].
4. Consider the vector field $\mathbf{u} = (y^2, x^2, 0)$
 - a) Sketch this field in the x - y plane.
 - b) Find its divergence and its curl.
 - c) Find the set of points in space at which the value of the curl equals zero, and explain this answer in terms of your sketch. Explain the divergence result in terms of the sketch, as well.
5. Find the Laplacian of the scalar field $f(\vec{r}) = x^2 e^y + \sin(xz)$ by summing the 2nd order derivatives ($\nabla^2 f = \partial_x^2 f + \partial_y^2 f + \partial_z^2 f$), and show that you obtain the same result when you calculate it as $\text{div grad } f$ ($\nabla^2 f = \vec{\nabla} \cdot \vec{\nabla} f$)
6. Find the Laplacian of the vector field $\vec{V}(\vec{r}) = (x^2 + z^2, \cos x \cos y, x^2 e^y + \sin(xz))$
7. Re-write the right-hand side of the equation $\nabla^2 \vec{u} = \vec{\nabla}(\vec{\nabla} \cdot \vec{u}) - \vec{\nabla} \times (\vec{\nabla} \times \vec{u})$ using symbols *div*, *grad* and *curl*. Verify that this equation is satisfied for the vector field $\mathbf{u} = (y^2 + x^2, x^2 + z^2, x^2 + y^2)$.
8. Translate the following suffix notation equation into vector notation:

$$a_j b_j c_i + u_i = u_k b_i u_k$$