Math 335-002
Homework #5

Problems 1-6 due Feb 26; Problems 7-9 due Feb 28

Group work on the homework is **not** allowed. Please show all work in detail to receive full credit. Late homework is not accepted.

1. Use suffix notation to simplify (bold symbols are vectors):

   a) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ b) $\mathbf{a} \times (\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b})$

2. Use suffix notation to find the product rule for $\vec{\nabla} \times (f\,\vec{\mathbf{u}})$

3. Use suffix notation to simplify (here $r = |\vec{\mathbf{r}}|$):

   a) $\vec{\nabla} \cdot (\vec{\mathbf{r}} \times \vec{\mathbf{u}})$ b) $\vec{\nabla} \cdot (r^2\,\vec{\mathbf{r}})$ c) $\vec{\nabla} \cdot (\vec{\mathbf{r}}\,/\,r^2)$ d) $\vec{\nabla} \times (r^2\,\vec{\mathbf{r}})$

4. (4.12 on p.82) Use suffix notation or product rules to show the following (here f is a scalar field):

   a) $\vec{\nabla} \times (f\,\vec{\nabla} f) = 0$
   b) $\vec{\nabla} \cdot (f\,\vec{\nabla} f) = |\vec{\nabla} f|^2 + f\,\nabla^2 f$

5. (4.15 on p. 82): Show that $\vec{\nabla} \cdot (\nabla^2\,\vec{\mathbf{u}}) = \nabla^2 (\vec{\nabla} \cdot \vec{\mathbf{u}})$ using two different methods: (a) suffix notation, and (b) using the equation $\nabla^2\,\vec{\mathbf{u}} = \vec{\nabla}(\vec{\nabla} \cdot \vec{\mathbf{u}}) - \vec{\nabla} \times (\vec{\nabla} \times \vec{\mathbf{u}})$

6. Problem 4.16 on p. 82 (Laplace's equation is $\nabla^2 \varphi = 0$)

7. Find the line integral of the vector field $\vec{\mathbf{u}} = (x^2, y^{1/3}, z)$ along the curve given by $x=t^2$, $y=e^{3t}$, $z=e^{2t}$, for t varying from 0 to 1 (if in doubt, see problem 2.3 on p. 31).

8. Consider a conservative force $\boldsymbol{F} = -\nabla \varphi$ with a potential energy φ given by $\varphi = r^2$. Use line integration to calculate the work done by this force along the parabola $y=x^2$, for x varying from 0 to 1 (assume $z=0$). Compare this value with the difference in potential energy between the endpoints of the curve, $\varphi(\mathrm{B})-\varphi(\mathrm{A})$.

9. Calculate the line integral of a vector field $\vec{\mathbf{u}} = (y^2, -x, 0)$ over the following curves connecting points A=(1,0,0) and B=(0,1,0):

   a. A horizontal line connecting point A and the origin (0,0,0) plus a vertical line connecting the origin and point B.
   b. A circular arc connecting points A and B (recall that trigonometric functions parametrize this circle)
   c. A straight line connecting points A and B

Compare the three results. Is $\vec{\mathbf{u}}$ a conservative vector field? Calculate the curl of $\vec{\mathbf{u}}$ to check your conclusion.