Math 335-002
Homework #6

Problems 1-5 are due March 5; Problems 6-8 are due March 7

Please show all work in detail to receive full credit. Late homework is not accepted.

In problems 1-5, calculate the surface integral $\iint_S \vec{\mathbf{u}} \cdot \vec{\mathbf{n}}\, dS$ of a vector field $\vec{\mathbf{u}}$ over the given surface:

1. $\vec{\mathbf{u}} = (e^x, y^2, x+y+z)$, and S is the part of the coordinate plane $z=0$ lying between the curves $y=x$ and $y=x^3$ in the positive quadrant ($x\geq 0$, $y\geq 0$). Assume $\vec{\mathbf{n}}$ points in the positive z direction.

2. $\vec{\mathbf{u}} = (x, 0, z)$, and S is the part of the surface $z = x + y^2$ with $z \leq 0$ and $x \geq -1$, which was sketched at the end of the lecture (consult problem 2.8 on page 43 if in doubt).

3. $\vec{\mathbf{u}} = (1, -y, -z)$, and S is the part of the flat surface $z = 1-x-2y$ lying in the octant $x\geq 0$, $y\geq 0$, $z\geq 0$; assume $\vec{\mathbf{n}}$ has a positive z component. Sketch this plane by indicating its intersections with the three coordinate surfaces ($x=0$, $y=0$ and $z=0$). Although this surface is not curved, it is still convenient to use the equation on page 35, with variables x and y as surface parameters: $\iint_S \mathbf{u}\cdot\mathbf{n}\, dS = \iint_S \mathbf{u}\cdot\left(\frac{\partial \mathbf{r}_s}{\partial x}\times\frac{\partial \mathbf{r}_s}{\partial y}\right) dx\, dy$

4. $\vec{\mathbf{u}} = (y, x, \ln(x+y))$, and S is the curved side of the cylinder $x^2+y^2 = 1$ lying between the planes $z=0$ and $z=1$ in the octant $x\geq 0$, $y\geq 0$, $z\geq 0$, with the normal pointing outward. Use variables y (or x) and z to parametrize this curved surface (Hint: the position vector will contain a square root, but everything simplifies in the end).

5. $\vec{\mathbf{u}} = (z, y, z)$, and S is the closed surface of a unit cube given by $0\leq x\leq 1$, $0\leq y\leq 1$, $0\leq z\leq 1$, with the unit vector pointing outward. This integral is calculated as a sum of six surface integrals corresponding to the six faces of the cube (see Problem 2.6 on p. 43)

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6. Calculate the volume enclosed between the surfaces $x + y^2 \leq z \leq 0$, $x \geq -1$ (the shape in problem 2). Hint: the most convenient order of integration is dz dx dy

7. Calculate the mass of a paraboloid-shaped object defined by $z + x^2 + y^2 \leq 1$, $z\geq 0$, given the mass density $\rho(x, y, z) = z^2 + 1$

8. Calculate the mass of an object defined by $0 \leq z \leq 1-x-2y$, $x\geq 0$, and $y\geq 0$ (as in problem 3), with a mass density of $\rho(x, y, z) = y^2$