Math 335-002
Homework #7
Due date: March 19, 2006

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Past material problem: use suffix notation to differentiate the expression $\vec{\nabla}\cdot\left(\vec{\mathbf{r}}\,\ln r\right)$, where $r \equiv |\vec{\mathbf{r}}| \equiv \sqrt{x^2+y^2+z^2} \equiv \sqrt{x_1^2+x_2^2+x_3^2} \equiv \sqrt{r_1^2+r_2^2+r_3^2}$ (these different notations are identical). Use the chain rule $\partial_i f(r) = f'(r)\partial_i r$, and recall that $\partial_i r = x_i / r$. Look at the posted solutions to homework #5 if in doubt.

2. Use the divergence theorem to evaluate the integral $\oiint_S \vec{\mathbf{u}}\cdot\vec{\mathbf{n}}\,dS$, where $\vec{\mathbf{u}}=(z+y,\ x^3+z^3,\ x^5+z^5)$, and S is the spherical surface $z^2 + x^2 + y^2 = 1$ (Hint: I did this in class for a different $\vec{\mathbf{u}}$: use the divergence theorem to replace this integral with a simpler volume integral; use z as the outer variable in the volume integral).

3. Problems 5.3 and 5.4 on page 90.

4. Verify the divergence theorem $\left(\iiint_V \vec{\nabla}\cdot\vec{\mathbf{u}}\,dV = \oiint_S \vec{\mathbf{u}}\cdot\vec{\mathbf{n}}\,dS\right)$ by calculating both the volume integral and the surface integral, for the vector field given by $\vec{\mathbf{u}}=(0,e^y,0)$, where volume V is the unit cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$. Consult problem 5.2 if in doubt (we did it in class).

5. Verify the divergence theorem $\left(\iiint_V \vec{\nabla}\cdot\vec{\mathbf{u}}\,dV = \oiint_S \vec{\mathbf{u}}\cdot\vec{\mathbf{n}}\,dS\right)$ by calculating both the volume integral and the surface integral, for the vector field given by $\vec{\mathbf{u}}=(0,\ 0,\ 1-z)$, where volume V is a tetrahedron $z + x + y \le 1$, $x \ge 0$, $y \ge 0$, $z \ge 0$. When calculating the integral over the closed surface, remember that the normal should point *outside* the volume. (Hint: the surface is composed of four separate pieces; two of those surfaces give a zero contribution to the integral).