Math 630 Linear Algebra

Quiz # 6

Problem 1

$$\underline{A} = \begin{bmatrix} 3 & 6 \\ 6 & 16 \end{bmatrix}, \quad \underline{L} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad \underline{D} = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

d_{ii} matches with the coefficients of the squares.
l_{ji} $i \le j \le 2$, $1 \le i \le 2$ matches with the combinations in the squares.

Problem 2

$$\underline{A} = \underline{Q}\underline{\Lambda}\underline{Q}^T, \quad \underline{A}^2 = \underline{Q}\underline{\Lambda}^2\underline{Q}^T, \quad \underline{A}^{-1} = \underline{Q}\underline{\Lambda}^{-1}\underline{Q}^T$$

$\lambda_i > 0 \Rightarrow \lambda_i^2, \ 1/\lambda_i > 0$.

Problem 3

$$\underline{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} \underline{1} & 2 & 3 \\ 0 & \underline{1} & -2 \\ 0 & 0 & \underline{\underline{-4}} \end{bmatrix}$$

$\underline{A}$: indefinite, so is $\underline{A}^{-1}$

$$\underline{B} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 6 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \underline{\underline{1}} & 2 & 0 & 0 \\ 0 & \underline{\underline{2}} & -2 & 0 \\ 0 & 0 & \underline{\underline{3}} & -2 \\ 0 & 0 & 0 & \underline{\underline{\frac{5}{3}}} \end{bmatrix}$$

all pivots are positive $\Rightarrow$ $\underline{B}$ positive definite, so $\underline{C} = -\underline{B}$ is negative definite.

There $\underline{\underline{\text{is}}}$ a real solution for $-\underline{x}^T \underline{A} \underline{x} = 1$.

Problem 4

$$\underline{A} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & -\frac{3}{2} & \frac{3}{2} \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{L} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & -1 & 1 \end{bmatrix}$$

$$\underline{x}^T \underline{A}\, \underline{x} = 2\left(x_1 - \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3\right)^2 + \tfrac{3}{2}\left(x_2 - x_3\right)^2$$

$$\underline{B} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{L} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\underline{x}^T \underline{B}\, \underline{x} = \left(x_1 + x_2 + x_3\right)^2$$