## Math 630 - Linear Algebra and Its Applications

Instructor: Prof. X. Sheldon Wang

## Quiz 3

# (Closed book)

Assigned: 8:00pm, Mar. 3rd, 2005 Due: 9:00pm, Mar. 3rd, 2005

## Problem 1 (25 points)

Find the length of  $\mathbf{a} = (2, -2, 1)$  and write down two independent vectors that are perpendicular to  $\mathbf{a}$ .

# Problem 2 (25 points)

Factor

$$\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & 0 \end{bmatrix}$$

into QR, recognizing that the first column is already a unit vector.

#### Problem 3 (25 points)

Use Gram-Schmidt to construct an orthonormal pair  $\mathbf{q}_1$  and  $\mathbf{q}_2$  from  $\mathbf{a}_1 = (4, 5, 2, 2)$  and  $\mathbf{a}_2 = (1, 2, 0, 0)$ . Express  $\mathbf{a}_1$  and  $\mathbf{a}_2$  as combinations of  $\mathbf{q}_1$  and  $\mathbf{q}_2$  and write down the triangular  $\mathbf{R}$  in  $\mathbf{A} = \mathbf{QR}$ .

#### Problem 4 (25 points)

(a) Find an orthonormal basis for the column space of

$$\mathbf{A} = \begin{bmatrix} 1 & -6 \\ 3 & 6 \\ 4 & 8 \\ 5 & 0 \\ 7 & 8 \end{bmatrix}.$$

(b) Find the least square solution to  $\mathbf{A}\mathbf{x} = \mathbf{b}$ , if  $\mathbf{b} = (-3, 7, 1, 0, 4)^T$ .