Experimental Study on the Performance of Reduced Rank Timing Acquisition Scheme for Multiple Access Communications†

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***Abstract*— This work studies the performance of a data-driven reduced rank scheme for rapid timing acquisition in multiple access communications. Our results show that when only a limited amount of preamble bits are available, instead of the true second-order statistics (SOS), the reduced rank scheme provides reliable timing information. Exploiting the structure of the multiple access interference, the data driven reduced-rank timing acquisition scheme demonstrates almost near-far resistant performance in signal acquisition.**

I. INTRODUCTION

In code division multiple access (CDMA) communication systems, all the users share the same frequency band simultaneously, therefore, the multiple access interference (MAI) should be suppressed to guarantee the quality of service (QoS) for all the users. The near-far problem occurs when the interfering users have higher power level than the desired user at the receiver side. This drastically degrades the performance of the conventional matched filter based receiver for signal acquisition and detection. The multiuser detection (MUD) exploits the structure of MAI and provides a near-far resistant performance for signal detection in CDMA system. Synchronization, or timing acquisition is a critical prerequisite of multiple access communication systems. Consequently, to find a near-far resistant timing acquisition solution is very important in multiple access communications. From a signal processing perspective, timing acquisition is related to the traditional time delay estimation problem in the presence of MAI and ambient noise. By treating the combined effect of MAI and the ambient noise as a colored Gaussian noise, a large sample maximum likelihood (LSML) approach is proposed based on the assumption of knowing enough information bits of a desired user in [1]. In [2] [3], based on the assumption that a desired user transmits a fixed preamble along with its information bits, the maximum likelihood estimation (MLE) boils down to a peak search of an objective function, commonly named the compressed likelihood function (CLF) over the parameter space. The MLE of delay parameter depends on the knowledge on the data covariance matrix. In practice, this knowledge is commonly replaced by a preamble based sample covariance estimate. In communication applications, the preamble bits are scarce, therefore, a reliable low-complexity data-driven solution using only limited preamble bits is needed. In [4], we proposed a reduced-rank data-driven solution that avoids the matrix inversion for rapid timing acquisition in an asynchronous CDMA communication system, but the statistical performance of such a solution is not provided. In this work, we present our new results on the performance study of the solution with emphasis on the rank reduction and near-far resistant property when the number of preamble is less than the length of spreading codes.

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II. PROBLEM FORMULATION

In an asynchronous CDMA communication systems over the additive white Gaussian noise (AWGN) channel, the baseband data $r(t)$ can be modeled as,

$$r(t) = \sum_i \sum_{k=1}^{K} A_k b_k(i) s_k(t - \tau_k - iT) + n(t), \qquad (1)$$

where K is the number of active users; i is the symbol index; A_k, $b_k(i)$, τ_k, and $s_k(t)$ are the amplitudes, BPSK information bit, propagation delay, and signature waveform of the kth user, respectively; T is the symbol duration; $n(t)$ is a white Gaussian process with an average power σ^2. The signature waveform can be expressed as

$$s_k(t) = \sum_{l=1}^{L} s_k[l] p(t - lT_c), \qquad 0 \le t \le T$$

where $\{s_k[l], l = 1, 2, \ldots, L\}$ is a pseudo noise (PN) code sequence consisting of L chips that take values $\{-1, +1\}$; $p(t)$ is a rectangular pulse of chip duration T_c. Thus there are L chips per symbol and $T = LT_c$. Without loss of generality, we assume all the signal waveforms have unit energy, i.e., $\int_0^T s_k^2(t)dt = 1$.

Denote the kth user's delay as $\tau_k = \nu_k T_c + \gamma_k$, where ν_k and γ_k are the integer and the fractional parts of τ_k with respect to the chip duration T_c. Within the i-th processing interval of length T, the chip-rate matched filtered and sampled data in (1) can be written in vector form as,

$$\mathbf{r}(i) = \underbrace{A_1\Big(b_1(i-1)\mathbf{u}_1^{(r)} + b_1(i)\mathbf{u}_1^{(l)}\Big)}_{\text{signal of interest}} + \underbrace{\sum_{k=2}^{K} A_k\Big(b_k(i-1)\mathbf{u}_k^{(r)} + b_k(i)\mathbf{u}_k^{(l)}\Big)}_{\text{MAI}} + \mathbf{n}(i), \quad (2)$$

where the i.i.d. white noise vectors $\mathbf{n}(i) \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I}_L)$. The $(L \times 1)$ vectors $\mathbf{u}_k^{(r)}$ and $\mathbf{u}_k^{(l)}$ are the effective signature vectors of the kth user, parameterized by the delay τ_k, i.e.,

$$\begin{aligned} \mathbf{u}_k^{(r)} &= (1-\tfrac{\gamma_k}{T_c})\mathbf{s}_k^{(r)}(\nu_k) + \tfrac{\gamma_k}{T_c}\mathbf{s}_k^{(r)}(\nu_k+1), \\ \mathbf{u}_k^{(l)} &= (1-\tfrac{\gamma_k}{T_c})\mathbf{s}_k^{(l)}(\nu_k) + \tfrac{\gamma_k}{T_c}\mathbf{s}_k^{(l)}(\nu_k+1), \end{aligned} \quad (3)$$

with $\mathbf{s}_k^{(r)}(\nu_k)$ and $\mathbf{s}_k^{(l)}(\nu_k)$ being the right and left portions of signature vectors $\mathbf{s}_k$ partitioned by the integer part of delay ν_k. That is,

$$\mathbf{s}_k^{(l)}(\nu_k) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ s_k[1] \\ s_k[2] \\ \vdots \\ s_k[L-\nu_k] \end{bmatrix}, \ \mathbf{s}_k^{(r)}(\nu_k) = \begin{bmatrix} s_k[L-\nu_k+1] \\ s_k[L-\nu_k+2] \\ \vdots \\ s_k[L] \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

In this work, we combined the MAI and the noise of (2) into one colored noise vector. Estimating delay of the desired user τ_1 then becomes jointly estimating signal parameters ν_1 and γ_1 in a colored noise.

III. Rapid Reduced-Rank Timing Acquisition

Normally a fixed preamble (M bits) associated with each user is used for timing acquisition. Then the signal of interest as well as the data model in (2) associated with the preamble bits can be simplified to

$$\mathbf{r}(i) = \beta_1\mathbf{u}_1(\tau_1) + \mathbf{e}(i), \qquad i = 1, 2, \ldots, M \quad (4)$$

where β_1 is an unknown scalar; the signal vector $\mathbf{u}_1(\tau_1) = \mathbf{u}_1^{(r)} + \mathbf{u}_1^{(l)}$ is parameterized by ν_1 and γ_1. The colored noise $\mathbf{e}(i)$ has zero mean and covariance matrix

$$\mathbf{Q} = \mathbf{U}\mathbf{A}_1^2\mathbf{U}^T + \sigma^2\mathbf{I}_L,$$

where $\mathbf{U} = \begin{bmatrix} \mathbf{u}_2^{(r)} & \mathbf{u}_2^{(l)} & \cdots & \mathbf{u}_K^{(r)} & \mathbf{u}_K^{(l)} \end{bmatrix}$, and $\mathbf{A}_1 = \mathbf{I}_2 \otimes \text{diag}\{A_2, A_3, \cdots, A_K\}$. Here "$\otimes$" denotes Kronecker product. When the number of preamble bits M and the number of users K are large, based on Gaussian approximation, the sample mean statistic of (4) can be modeled as

$$\hat{\mathbf{m}} = \frac{1}{M}\sum_{i=1}^{M}\mathbf{r}(i) \sim \mathcal{N}\Big(\beta_1\mathbf{u}_1(\tau_1), \frac{1}{M}\mathbf{Q}\Big).$$

The delay estimation is then obtained by searching the peak of the CLF [2] [3],

$$\hat{\tau}_1 = \arg\max_{\tau} \mathcal{J}(\tau) = \arg\max_{\tau} \frac{|\hat{\mathbf{m}}^T\hat{\mathbf{Q}}^{-1}\mathbf{u}_1(\tau)|^2}{\mathbf{u}_1^T(\tau)\hat{\mathbf{Q}}^{-1}\mathbf{u}_1(\tau)}, \quad (5)$$

where $\hat{\mathbf{Q}} = \frac{1}{M}\sum_{i=1}^{M}\mathbf{r}(i)\mathbf{r}^T(i) - \hat{\mathbf{m}}\hat{\mathbf{m}}^T$ is the sample estimate of the covariance matrix $\mathbf{Q}$.

Based on our previous work in [5] [6], a reduced-rank solution is proposed in [4] to facilitate the rapid timing acquisition due to the limited preamble bits in communication applications. The rank reduction technique alleviates the problem encountered in sample covariance matrix inversion especially when $M < L$, yet at the same time to obtain a data-driven and low-complexity timing acquisition. The rank-r solution of timing acquisition is obtained from [4]

$$\hat{\tau}_1 = \arg\max_{\tau} \mathcal{J}^{(r)}(\tau) = \arg\max_{\tau} \frac{|\hat{\mathbf{m}}^T\mathbf{w}_{\mathbf{u}_1}^{(r)}(\tau)|^2}{\mathbf{u}_1^T(\tau)\mathbf{w}_{\mathbf{u}_1}^{(r)}(\tau)}, \quad (6)$$

where vector $\mathbf{w}_{\mathbf{u}_1}^{(r)}(\tau)$ is a rank-r approximation to the Wiener filter vector defined as $\mathbf{w}_{\mathbf{u}_1}^{(r)}(\tau) = \hat{\mathbf{Q}}^{-1}\mathbf{u}_1(\tau)$. It can be iteratively calculated using the Krylov subspace method, or more explicitly, the conjugate gradient method [5] [7] [8]. Setting the initials as $\mathbf{d}_1 = \mathbf{u}_1(\tau)$, and $\mathbf{g}_1 = \mathbf{u}_1(\tau)$. Here $\mathbf{d}_i$ and $\mathbf{g}_i$ denote direction vector and residue vector, respectively. The combination coefficient and the vectors are updated as

$$\begin{gathered} \alpha_i = \|\mathbf{g}_i\|^2/\mathbf{d}_i^T\hat{\mathbf{Q}}\mathbf{d}_i, \\ \mathbf{w}_{\mathbf{u}_1}^{(i)}(\tau) = \mathbf{w}_{\mathbf{u}_1}^{(i-1)}(\tau) + \alpha_i\mathbf{d}_i, \\ \mathbf{g}_{i+1} = \mathbf{g}_i - \alpha_i\hat{\mathbf{Q}}\mathbf{d}_i, \\ \mathbf{d}_{i+1} = \mathbf{g}_{i+1} + (\|\mathbf{g}_{i+1}\|^2/\|\mathbf{g}_i\|^2)\mathbf{d}_i. \end{gathered}$$

At the r-th step of iteration, out of the rank-r Krylov subspace $\mathcal{K}_r(\hat{\mathbf{Q}}, \mathbf{u}_1(\tau))$, $\mathbf{w}_{\mathbf{u}_1}^{(r)}(\tau) = \sum_{i=1}^{r}\alpha_i\mathbf{d}_i$ is the optimal linear combination of the r conjugate direction vectors.

IV. Performance Study and Simulations

The application problem considered here is the propagation delay estimation of a desired user in a CDMA communication systems. The solution we are focusing on is the reduced-rank timing acquisition scheme in (6). In analyzing the performance of the scheme, we treat the propagation delay of the desired user, τ_1, as a deterministic but unknown parameter. The estimated delay $\hat{\tau}_1$ as a function of data, is a random variable. To study the performance of the reduced-rank timing acquisition scheme, we define a performance measure: the probability of correct timing acquisition

$$P_{acq} = P(|\hat{\tau}_1 - \tau_1| \leq \Delta\tau) = \int_{\tau_1-\Delta\tau}^{\tau_1+\Delta\tau} f(\hat{\tau}_1) d\hat{\tau}_1,$$

where $f(\hat{\tau}_1)$ is the probability density function (PDF) of $\hat{\tau}_1$. In this work, the performance analysis is carried out by designed computer simulations.

In the simulations, the numbers of asynchronous users are chosen as $K = 5$ and $K = 10$, respectively, the length of spreading codes is chosen as $L = 31$, and the gold codes with high correlations are used as the spreading codes. A normalized chip interval is chosen as $T_c = 1$. The propagation delays of all the multiple access users are randomly chosen within the symbol interval, and then fixed once chosen (as $\tau_1 = 10.3$, $\tau_2 = 16.8$, $\tau_3 = 25.4$, $\tau_4 = 11.2$, $\tau_5 = 18.6$, $\tau_6 = 3.0$, $\tau_7 = 27.5$, $\tau_8 = 7.1$, $\tau_9 = 21.9$, $\tau_{10} = 5.7$) for further study on the random behavior of the estimator from different independent trials. When $K = 5$ is chosen, only 5 users with propagation delays $\{\tau_k\}_{k=1}^5$ are considered. The SNR for the desired user is defined as $SNR(1)$ (in dB scales), and the SNRs for all the interfering users are chosen as $SNR(k) = SNR(1) + NFR(k = 2, 3, ..., K)$. Here NFR (in dB scales) is near-far ratio (NFR). Fig. 1 $\sim$ Fig. 6 are obtained when $K = 5$, and Fig. 7 $\sim$ Fig. 10 are obtained when $K = 10$. All the figures are obtained based on 1000 trials, $\Delta\tau = \{\frac{1}{4}T_c, \frac{2}{5}T_c\}$ are chosen, and the quantization interval is $\frac{1}{10}T_c$.

From (6), the rank 1 solution of the objective function can be obtained as

$$\mathcal{J}^{(1)}(\tau) = \frac{|\hat{\mathbf{m}}^T \mathbf{u}_1(\tau)|^2}{\mathbf{u}_1^T(\tau)\hat{\mathbf{Q}}\mathbf{u}_1(\tau)}.$$

It becomes the single user matched filter [9] when $\hat{\mathbf{Q}}$ is an identity matrix. Consequently, the rank 1 filter can not provide the reliable timing information in high NFR scenario. On the contrary, the reduced-rank scheme suppresses MAI by exploiting the structure of $\hat{\mathbf{Q}}$, providing near-far resistant property, as shown in Fig. 2, Fig. 3 and Fig. 8.

The timing acquisition probability P_{acq} increases with M, as shown in Fig. 1, Fig. 2 and Fig. 7, respectively. Fig. 4 and Fig. 8 show that the estimated propagation delay $\hat{\tau}_1$ to achieve the peak value of $f(\hat{\tau}_1)$ moves to the true delay $\tau_1 = 10.3$, and the shape of the curve becomes sharp when M increases. The curve is almost symmetric when $M = 100$, which indicates the estimate is asymptotically unbiased. It results from that $\hat{\mathbf{m}}$ is asymptotically unbiased estimate of $\beta_1\mathbf{u}_1(\tau_1)$. Fig. 1 and Fig. 7 also show that when K increases, more preamble bits are needed to improve the performance.

There is a rank to achieve the best P_{acq}, as shown in Fig. 1 and Fig. 7. In the scenario of $K = 5$ and $\Delta\tau = \frac{1}{4}T_c$, rank 6 scheme is the best solution when $NFR = 0dB$, while rank 10 scheme is the best solution when $NFR = 10dB$. In the scenario of $K = 10$ and $\Delta\tau = \frac{1}{4}T_c$, rank 11 scheme is the best solution when $NFR = 0dB$, while rank 16 scheme is the best solution when $NFR = 10dB$. It demonstrates the trend that in the same parameter setting, the rank to achieve the best P_{acq} increases from low NFR to high NFR scenario. It can be explained that in high NFR scenario, more rank is needed to de-correlate the desired user with the interfering users to suppress MAI. Fig. 6 shows if rank 7 scheme is used in high NFR scenario $(NFR = 20dB)$, the estimated propagation delay $\hat{\tau}_1$ to achieve the peak value of $f(\hat{\tau}_1)$ deviates from the true delay $\tau_1 = 10.3$, demonstrating the same trend.

Fig. 5 and Fig. 10 show that the estimated propagation delay $\hat{\tau}_1$ to achieve the peak value of $f(\hat{\tau}_1)$ moves to the true delay $\tau_1 = 10.3$ when r increases. $f(\hat{\tau}_1 = 10.3)$ increases to the maximum at $r = 10$ when $K = 5$, and at $r = 20$ when $K = 10$, respectively, then decreases with the increase of the rank. It shows that by exploiting the structure of MAI which is estimated based on limited amount of preamble bits, a scheme of rank $r > 2K$ (the number of the virtue users in the system) will degrade the performance. If the preamble bits are large, the scheme of rank $r > 2K$ will keep a steady performance, as shown in Fig. 7 when $M = 50$.

V. Conclusions

This work investigates the applicability of a data-driven reduced rank scheme for rapid timing acquisition in multiple access communications. The performance of the scheme is studied with special emphasis in the aspect of near-far resistant property and rank choice. The results show that scheme can provide reliable timing information at low complexity with a limited amount of preambles.

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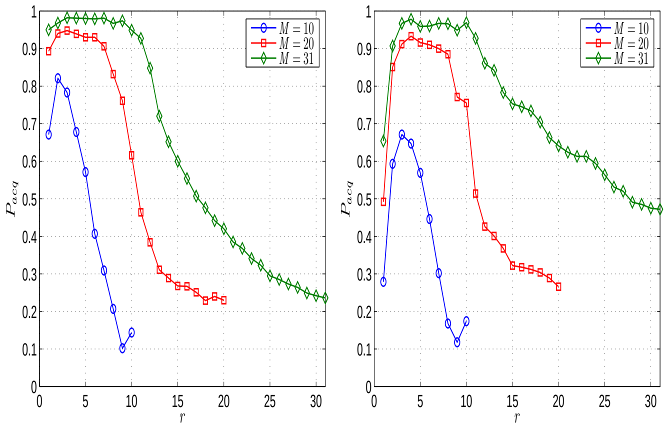
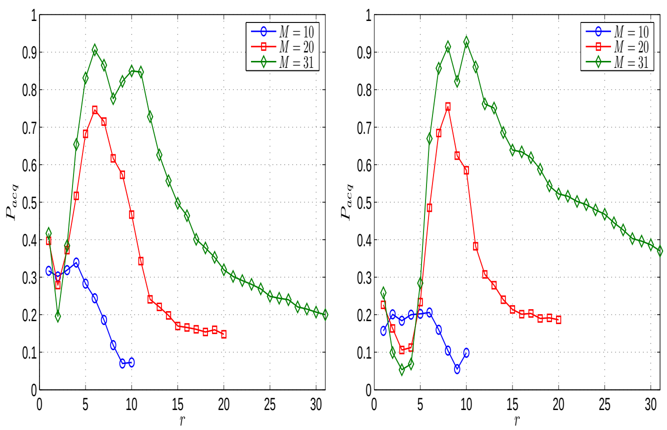
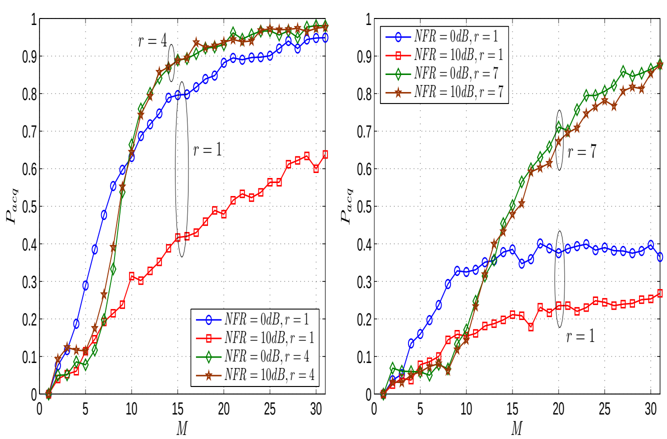


(a) $\Delta\tau = \frac{2}{5}T_c$, $NFR = 0dB$. (b) $\Delta\tau = \frac{2}{5}T_c$, $NFR = 10dB$.



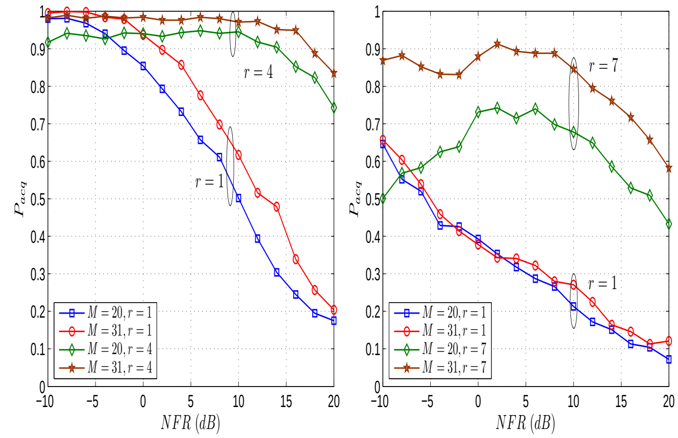
(c) $\Delta\tau = \frac{1}{4}T_c$, $NFR = 0dB$. (d) $\Delta\tau = \frac{1}{4}T_c$, $NFR = 10dB$.

Fig. 1. Acquisition probability of the reduced-rank scheme as a function of rank, with varying preamble length M, for a desired user with $SNR(1) = 10dB$. Total user number $K = 5$, and the length of spreading codes $L = 31$.



(a) $\Delta\tau = \frac{2}{5}T_c$. (b) $\Delta\tau = \frac{1}{4}T_c$.

Fig. 2. Acquisition probability of the reduced-rank scheme as a function of preamble length M, with varying NFR, for a desired user with $SNR(1) = 10dB$. Total user number $K = 5$, the length of spreading codes $L = 31$, and rank $r = \{1{,}4{,}7\}$. Note: rank 4 and rank 7 are near-far resistant, respectively.



(a) $\Delta\tau = \frac{2}{5}T_c$. (b) $\Delta\tau = \frac{1}{4}T_c$.

Fig. 3. Acquisition probability of the reduced-rank scheme as a function of NFR, with varying M, for a desired user with $SNR(1) = 10dB$. Total user number $K = 5$, the length of spreading codes $L = 31$, and rank $r = \{1{,}4{,}7\}$. Note: rank $r = \{4{,}7\}$ are almost near-far resistant.

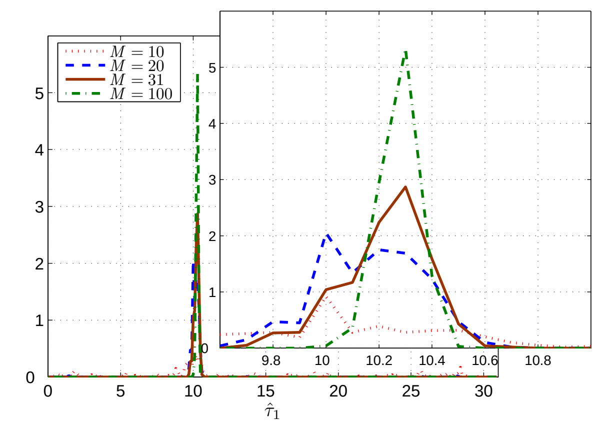


Fig. 4. Simulated PDF of the estimated delay $\hat{\tau}_1$, with $r = 7$, $NFR = 10dB$ and varying preamble length M, for a desired user with $SNR(1) = 10dB$ and the true delay $\tau_1 = 10.3$. Total user number $K = 5$, and the length of spreading codes $L = 31$.

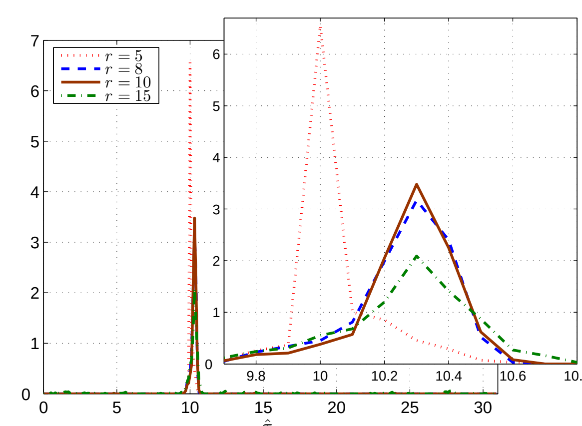


Fig. 5. Simulated PDF of the estimated delay $\hat{\tau}_1$, with $M = 31$, $NFR = 10dB$ and varying rank r, for a desired user with $SNR(1) = 10dB$ and the true delay $\tau_1 = 10.3$. Total user number $K = 5$, and the length of spreading codes $L = 31$.

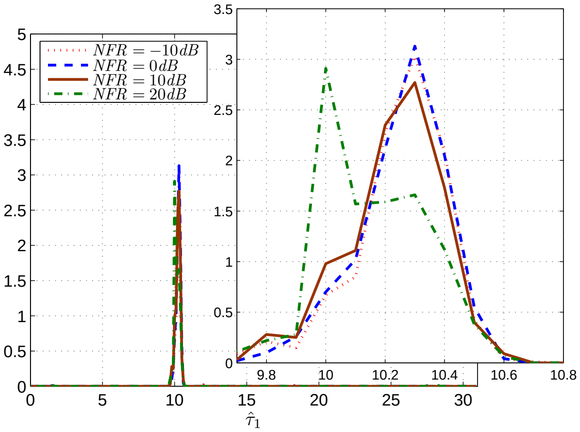
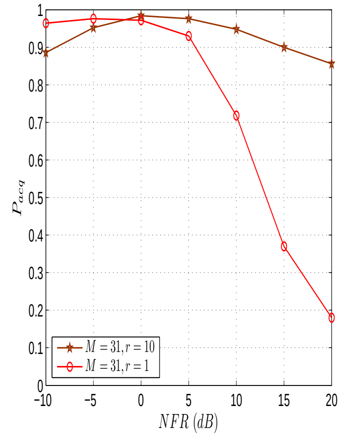
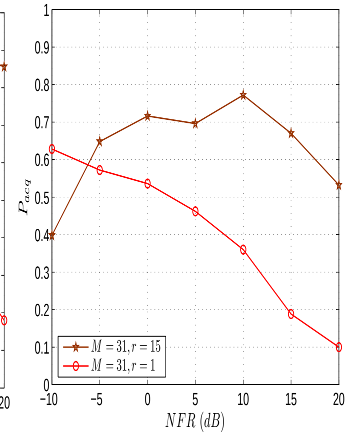


Fig. 6. Simulated PDF of the estimated delay $\hat{\tau}_1$, with $M = 31$, $r = 7$ and varying NFR, for a desired user with $SNR(1) = 10dB$ and the true delay $\tau_1 = 10.3$. Total user number $K = 5$, and the length of spreading codes $L = 31$.

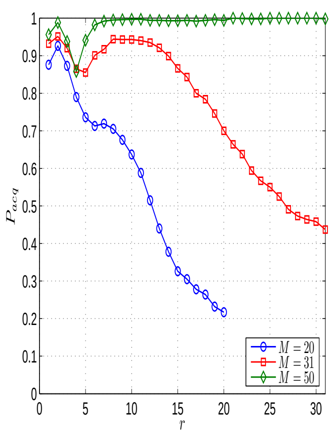


(a) $\Delta\tau = \frac{2}{5}T_c$.

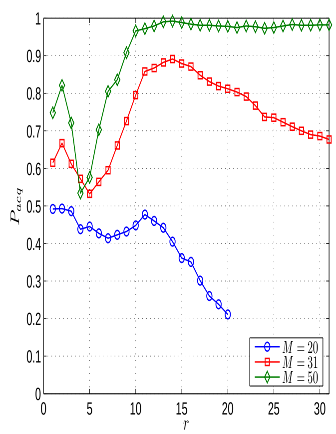


(b) $\Delta\tau = \frac{1}{4}T_c$.

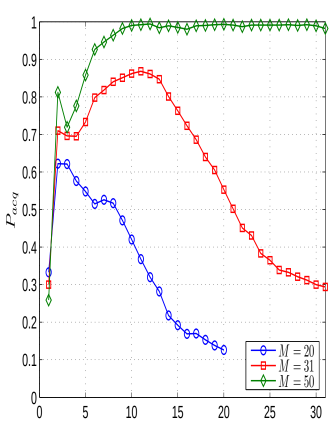
Fig. 8. Acquisition probability of the reduced-rank scheme as a function of NFR, for a desired user with $SNR(1) = 10dB$. Total user number $K = 10$, the length of spreading codes $L = 31$, and rank $r = \{1,10,15\}$. Note: rank $r = \{10,15\}$ are almost near-far resistant.



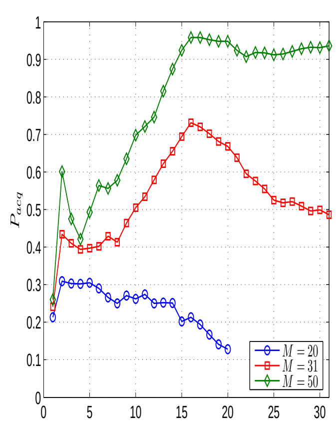
(a) $\Delta\tau = \frac{2}{5}T_c$, $NFR = 0dB$.



(b) $\Delta\tau = \frac{2}{5}T_c$, $NFR = 10dB$.



(c) $\Delta\tau = \frac{1}{4}T_c$, $NFR = 0dB$.



(d) $\Delta\tau = \frac{1}{4}T_c$, $NFR = 10dB$.

Fig. 7. Acquisition probability of the reduced-rank scheme as a function of rank, with varying preamble length M, for a desired user with $SNR(1) = 10dB$. Total user number $K = 10$, and the length of spreading codes $L = 31$.

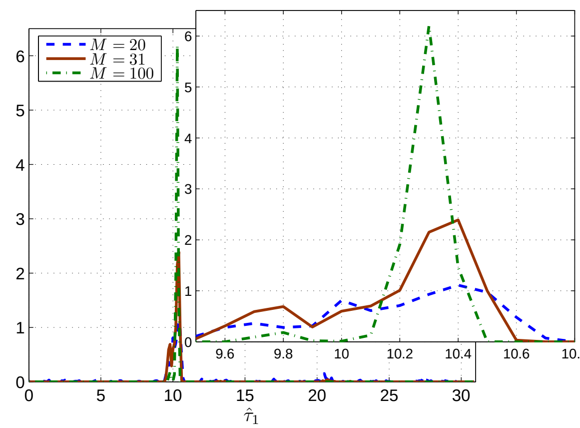


Fig. 9. Simulated PDF of the estimated delay $\hat{\tau}_1$, with $r = 16$, $NFR = 10dB$ and varying preamble length M, for a desired user with $SNR(1) = 10dB$ and the true delay $\tau_1 = 10.3$. Total user number $K = 10$, and the length of spreading codes $L = 31$.

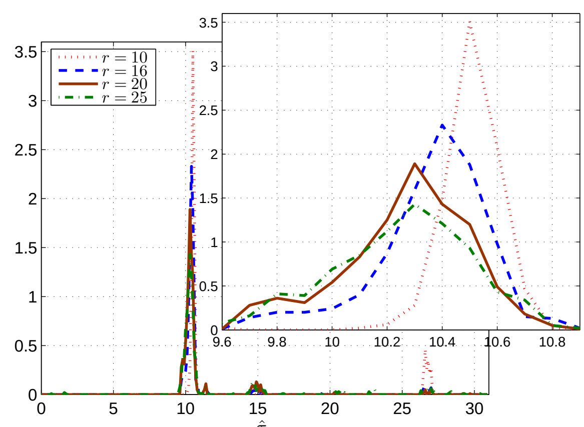


Fig. 10. Simulated PDF of the estimated delay $\hat{\tau}_1$, with $M = 31$, $NFR = 10dB$ and varying rank r, for a desired user with $SNR(1) = 10dB$ and the true delay $\tau_1 = 10.3$. Total user number $K = 10$, and the length of spreading codes $L = 31$.