On Disintegration of Near-Limit Cellular Flames

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Abstract

A strongly nonlinear geometrically-invariant model for the dynamics of near-limit cellular flame is proposed, where the flame evolution is governed by a system of equations for the flame interface and its temperature. The model generalizes its earlier weakly nonlinear version pertinent to a mildly perturbed planar flame. Numerical simulations of the new model show that at sufficiently high levels of heat losses the cellular flame resulting from the diffusive instability exhibits a tendency toward self-fragmentation, quite in line with direct numerical simulations of the associated reaction-diffusion system.

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1. Introduction

The laminar flames of low-Lewis-number premixtures are known to suffer a diffusive instability resulting in the formation of a non-planar cellular structure. The latter is most prominent in weak premixtures sensitive to radiative heat losses. In such systems, the cellular flame often breaks up into separate cap-like fragments which sometimes close upon themselves to form seemingly stationary spherical structures, the so-called flame balls [1,2]. Further increase of the heat losses leads to suppression of the fragments, terminating the combustion process altogether.

It has long been observed that near the stability threshold the flame structure becomes quasi-steady and quasi-planar. This allows for the asymptotic separation of the spatio-temporal coordinates and reduction of the effective dimensionality of the problem. In the case of the adiabatic or mildly non-adiabatic systems one thus ends up with a single evolution equation for the flame interface (front) advantageous both for physical analysis and numerical simulations. Yet, near the planar flame's quenching point the picture becomes more involved, and in this situation the flame evolution is governed by a coupled system of equations for the flame interface and its temperature [3]. In suitably chosen units the latter reads

$$\varphi_t + \frac{1}{2}\varphi_x^2 = \varphi_{xx} - \theta \tag{1}$$

$$\theta_t + \varphi_x\theta_x = \theta_{xx} + \frac{1}{3}\alpha\varphi_{xx} - \frac{1}{3}(\theta^2 + \nu) \ . \tag{2}$$

Here $\theta = \frac{1}{2}\beta(T - T_{ad})/(T_{ad} - T_u)$ is the reduced temperature of the flame interface, $F(x, y, t) = 0$; $\beta = T_a(T_{ad} - T_u)/T_{ad}^2$ is the Zeldovich number; T_a is the activation temperature; T_u, T_{ad} correspond to the temperature in the fresh premixture and the adiabatic temperature of the combustion products, respectively; $\alpha = \frac{1}{2}\beta(Le^{-1} - 1)$, Le being the Lewis number; $\nu = (h_c - h)/2h_c$, where $h = (U/U_{ad})^2 \ln(U_{ad}/U)$ is the scaled heat loss intensity; U is the planar flame speed; $U = U_{ad}$ at $h = 0$ (adiabatic limit) and $U = U_c = U_{ad}/\sqrt{e}$ at $h = h_c = 1/2e$ (quenching point). x, t are the scaled spatio-temporal coordinates referred to D_{th}/U_c and D_{th}/U_c^2, respectively. D_{th} is the thermal diffusivity of the mixture. $\varphi(x, t)$ is the perturbation of the planar flame referred to D_{th}/U_c; $F(x, y, t) = y + t - \varphi(x, t) = 0$, where $y = -t$ corresponds to the planar flame at the quenching point ($h = h_c, U = U_c$).

Negative ν pertains to a relatively low level of the heat losses allowing for planar configurations

$$\varphi = -\theta t, \quad \theta = -\sqrt{-\nu} \ . \tag{3}$$

For positive ν (high level of the heat losses) the planar configurations become unfeasible. Yet, for some range of positive ν the system (1)(2) allows for corrugated solutions [3-6]. The most interesting development, however, occurs at relatively high values of ν where the flame is expected to undergo self-fragmentation [7,8]. Precisely in this parameter range the model (1)(2) dealing with explicit functions of the spatial coordinates breaks down.

In the current study a geometrically-invariant (GI) extension of the model (1)(2) is proposed which is free of the above restrictions. The GI model may be either 'derived' using the appropriate gradient expansion in the intrinsic coordinates, or may be built as a GI extrapolation based on general geometrical considerations. In this paper we adopt the latter strategy as less formalistic and perhaps more appealing physically. However, we regard both approaches as legitimate and, in view of the lack of a more rigorous alternative, supplementing and substantiating each other. The perturbative derivation of the GI model for the new-limit flames will be presented elsewhere.

2. Formulation

The adiabatic counterpart for the weakly nonlinear model (1)(2) is a single equation for the flame interface [9],

$$\varphi_t + \frac{1}{2}\varphi_x^2 + (\alpha - 1)\varphi_{xx} + 4\varphi_{xxxx} = 0 \tag{4}$$

where D_{th}/U_{ad} and D_{th}/U_{ad}^2 are used as spatio-temporal scales.

As has been shown previously [10], the geometrically invariant extension of Eq. (4) reads

$$V_n = 1 + (\alpha - 1)\kappa + 4\kappa_{ss} \tag{5}$$

where V_n is the normal velocity of the flame interface, κ is its curvature and s is the arc-length along the curved interface.

If the interface may be described by an explicit function of the spatial coordinates, i.e. $F(x,y,t) = y - f(x,t) = 0$, then

$$V_n = -\frac{f_t}{\sqrt{1+f_x^2}}, \quad \kappa = -\frac{f_{xx}}{(1+f_x^2)^{3/2}}, \quad \text{and} \quad ds = \sqrt{1+f_x^2}\,dx\ . \tag{6}$$

For a mildly disturbed planar flame,

$$f(x,t) = -t + \varphi(x,t), \quad V_n \simeq 1 - \varphi_t - \frac{1}{2}\varphi_x^2, \quad \kappa \simeq -\varphi_{xx}, \quad ds \simeq dx \tag{7}$$

and Eq. (5) converts into Eq. (4).

The above observations suggest the following GI extension of Eqs. (1)(2),

$$V_n = 1 + \kappa + \theta \tag{8}$$

$$\mathcal{D}_t\theta = \theta_{ss} - \frac{1}{3}\alpha\kappa - \frac{1}{3}(\theta^2 + \nu) \tag{9}$$

where the term $\mathcal{D}_t\theta$ is supposed to be a GI counterpart for the combination $\theta_t + \varphi_x\theta_x$. This term does not have an analog in Eq. (4). Its geometrical meaning, however, is readily revealed through the following reasoning. Consider the flame interfaces

$$F(x,y,t) = 0 \quad \text{and} \quad F(x,y,t+\Delta t) = 0$$

at two successive instants of time, t and $t + \Delta t$.

Let A and B be two points on the interfaces such that the vector $\overrightarrow{AB}$ is orthogonal to $F(x,y,t) = 0$ (Fig. 1). Introduce the Lagrangian-type derivative

$$\mathcal{L}_t\theta = \lim_{\Delta t \to 0} \frac{\theta_B - \theta_A}{\Delta t} \tag{10}$$

where θ_A and θ_B are the temperatures at the points A and B.

If the flame interface is defined as $F(x,y,t) = y - f(x,t) = 0$, one may easily show that

$$\mathcal{L}_t\theta = \theta_t - f_t f_x \theta_x/(1+f_x^2) \ , \tag{11}$$

or in the intrinsic coordinates,

$$\mathcal{L}_t\theta = \theta_t + \theta_s \int_0^s \kappa(s',t)ds'. \tag{12}$$

For a weakly perturbed planar flame $f(x,t) = -t + \varphi(x,t)$, Eq. (11) yields

$$\mathcal{L}_t\theta \simeq \theta_t + \varphi_x\theta_x \tag{13}$$

thus recovering the left side of Eq. (2). In light of this outcome one may naturally set

$$\mathcal{D}_t\theta = \mathcal{L}_t\theta \tag{14}$$

thereby completing formulation of the GI model (8)(9).

Note that in the adiabatic limit ($h = 0$) the flame dynamics may also be described by a system analogous to (8)(9). In this case the linear-stability analysis suggests the following GI model,

$$V_n = 1 + \kappa + \theta \tag{15}$$

$$\mathcal{D}_t\theta = \theta_{ss} - \alpha\kappa - \theta \ . \tag{16}$$

For a weakly-perturbed planar flame when α is close to unity Eqs. (14)(15) reduce to Eq. (4).

3. Numerical simulations

For negative ν ($h < h_c$) or sufficiently small positive ν the developing cellular flame seems to endure for any time interval however long [3-7]. Yet, for relatively high ν the cellular flame is expected to display a tendency toward self-fragmentation. The effect should manifest itself as the solution blow-up within a finite time interval. Indeed, direct numerical simulation of the GI system (8)(9) for positive ν confirms such a tendency. Figure 2 shows several snapshots of a closed flame front prior to the solution blow-up. As the evolution approaches the critical time one observes the development of bubbles of the unburned matter intruding, as it were, into the internal domain behind the front, while the temperature of the bubbles drops rather drastically.

The computation started from a slightly perturbed circular front of radius 5 and near-constant (zero) temperature. The time increments between the snapshots corresponding to the increasing size of the bubble are as follows: $\Delta t = 0.1$, 0.06, and 0.03. A straightforward finite-difference explicit algorithm with grid homogenization similar to that described in [11] was employed. The numerical solution of the GI system (8)(9) in greater volume and detail will be presented elsewhere.

The near-circular nature of the bubbles emerging prior to the solution blow-up suggests that their dynamics may be captured by a radially-symmetric solution of Eqs. (8)(9). In this case $V_n = -R_t$, $\kappa = R^{-1}$, $\theta_{ss} = 0$ and Eqs. (8)(9) become

$$R_t = -1 - R^{-1} - \theta \tag{17}$$

$$\theta_t = -\frac{1}{3}\alpha R^{-1} - \frac{1}{3}(\theta^2 + \nu)\ . \tag{18}$$

At $R \to +\infty$, $\theta \to -\infty$, Eqs. (16)(17) yield

$$\theta \simeq \frac{3}{t - t_0}, \quad R \simeq 3\ln\left(\frac{1}{t_0 - t}\right) \tag{19}$$

where t_0 pertains to the point of blow-up. Thus indeed the flame disintegration occurs within a finite time interval, and is accompanied by a dramatic drop of temperature at the cells' cusps. This picture is quite in line with the numerical solution of the associated reaction-diffusion system [7,8].

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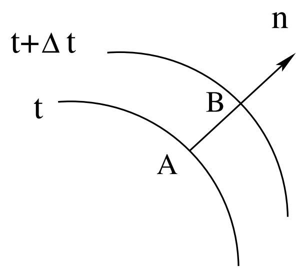

Figure 1

Figure 1. Diagram illustrating the geometrical meaning of the derivative $\mathcal{L}_t\theta$.

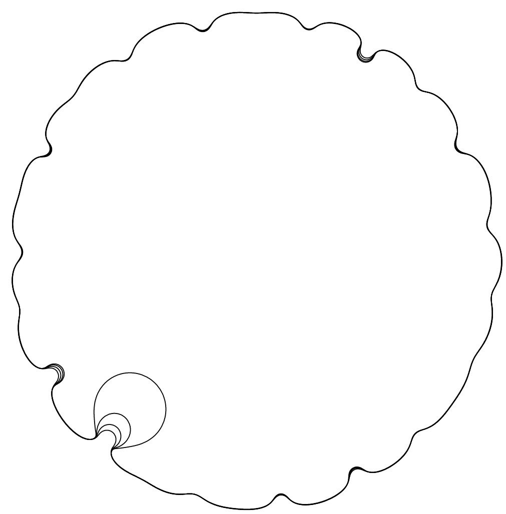

Figure 2

Figure 2. Numerical simulation of Eqs. (8)(9) in the parameter range pertinent to the flame disintegration ($\alpha = 1.5$, $\nu = 0.05$). Flame front at several consecutive instants of time starting from $t = 97.2$ prior to the solution blow-up. The flame front radius is about 108 units.