Comparing the effect of Carrier Frequency Offset on OFDM and Single-Carrier Block Transmission in AWGN Channels

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***Abstract*— Both OFDM and single-carrier block transmission (SCBT) are promising candidates for the future high date rate communication, which well handle the frequency selective channels with a low implementation complexity. However, the sensitivity to carrier frequency offset, caused by Doppler effect or the carrier frequency difference between transmitter and receiver, is always a major concern of these block transmission systems.**

Actually, a controversy in comparing the carrier frequency offset effect on OFDM and SCBT systems has already been witnessed in the literature for a long time. In this paper, we first analyze the effects of carrier frequency offset on these two systems, then by carefully investigating existing works, we classify the problem into two categories: carrier frequency offset analysis with compensation and without compensation. Finally, we draw conclusions based on theoretical and numerical results that the BER performance of OFDM systems is more sensitive to carrier frequency offset than SCBT systems for the BPSK modulation in AWGN channels, although the difference of the BER performance between two systems is much smaller than theoretically predicted in previous works, if a fair comparison environment is used. Moreover, this small performance gap vanishes in higher level constellation like QPSK.

***Index Terms*— Carrier frequency offset, OFDM, SC, block transmission**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an attractive modulation scheme used in broadband communication systems which encounter large delay spread. It has been adopted in various applications, e.g., digital subscriber line (DSL), digital video/audio broadcasting (DVB/DAB), IEEE 802.11a wireless local area network (or HIPERLAN/2) [1]–[4]. The basic idea of OFDM is that of converting a frequency-selective channel into a parallel collection of frequency flat subchannels. Hence one can easily recover the signal by a one-tap equalizer on each flat subchannel. Since the different subcarriers' signal spectrum can overlap in frequency, the available bandwidth is used very efficiently.

Single-carrier block transmission (SCBT) have gained a lot of attention in the literature recently [5]. When combined with frequency domain equalization (SC-FDE) [6], [7], this approach delivers performance similar to OFDM, with essentially the same overall complexity. In addition, SCBT uses only one carrier, instead of the many typically used in OFDM, so the peak-to-average transmitted power ratio for SC-modulated signals is smaller. It was also proposed as a promising candidate for IEEE802.16 [8].

However, the performance comparison between OFDM and SCBT in terms of the other issue, sensitivity to carrier frequency offset (CFO), turns out to be controversial. In an early contribution, Sari *et al.* [6] claimed that SCBT is much more robust to CFO than OFDM systems with the same amount of frequency offset. This was questioned by Polley *et al.* [9], who pointed out that the frequency offset error in a SC receiver due to the inaccurate estimation of the carrier frequency could actually be reduced by a factor of N in the OFDM receiver with any tracking algorithms of the comparable precision. At the same time, Pollet *et al.* [10] gave an insightful quantitative measure for the performance degradation of OFDM and SC in AWGN channels, concluding that the values of the CFO and the linewidth of the oscillators allowed for OFDM are orders of magnitude smaller than for SC carrying the same bit rate. This was recently inquired by Ma *et al.* [11], who declare that it was an unfair comparison because different sample rates were assumed for two candidates.

Actually, all conclusions mentioned above are correct, based on assumptions they made. However, these works did not really discuss the CFO issue under the same environment, which may be the partial reason for the controversy. In this paper, we intend to give a comprehensive understanding and systematic survey of the effect of CFO on both OFDM and SCBT systems in AWGN channels. We first present the effect of CFO on both systems, then by carefully investigating and comparing previous works, we put them into two categories: CFO analysis with compensation and without compensation. Finally we show in agreement with [6] and [10] that OFDM is more sensitive to CFO than SCBT in AWGN channels, whereas, if fair assumptions are assumed, the BER difference between two systems is much smaller than the theoretically predicted by them for the BPSK modulation and almost negligible for the QPSK modulation with or without the compensation of CFO.

One may argue that it is more practical to compare the CFO effect of these two candidates in fading channels with coded data. However, since all previous discussions and derivations are mainly focused on AWGN channels with the uncoded case, we believe it is more reasonable that we present similar analysis to facilitate a fair comparison. Moreover, it is difficult to qualitatively derive and explicitly compare the effect of CFO in the fading channel with coding schemes for OFDM and SCBT systems, since there are too many other parameters which also have important effects on the system performance.

The rest of the paper is organized as following: Section II presents the system models with CFO for OFDM and SCBT systems in AWGN channels respectively. The comparison between these two systems in the presence of CFO is presented in Section III, where the analytical and numerical results are also shown for both BPSK and QPSK modulations. Finally, Section IV draws conclusions.

II. System Models

A. OFDM System Model

Fig. 1(a) shows the general structure of OFDM systems in the presence of CFO. At the output of the modulation block, IDFT is used on the current symbol, which includes N data samples $X_o(k), k = 0, 1, ..., N-1$, to produce the corresponding signals in the time domain. Then N_g cyclic prefix (CP) samples are added to the beginning of the output stream. The time domain signal is then given by

$$x_o(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_o(k) e^{j2\pi nk/N}, \qquad (1)$$

where n ranges from $-N_g$ to $N-1$.

At the receiver side, down-converting RF introduces carrier frequency offset Δf into the received signals. Assuming the system design satisfies the Nyquist criterion and we take samples at time $t = nT/N$. By removing CP, the discrete signal expression of the nth sample is shown to be:

$$y_o(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_o(k) e^{j2\pi n(k+\epsilon)/N} + z_o(n) \qquad (2)$$

where $\epsilon = \Delta f T$ is the normalized CFO of the system and $z_o(n)$ denotes the AWGN samples. Hence the output of DFT is expressed by

$$Y_o(k) = X_o(k)C(0) + \sum_{l=0, l\neq k}^{N-1} X_o(l)C(l-k) + Z_o(k) \qquad (3)$$

where $Z_o(k)$ is the DFT output of $z_o(n)$ and

$$C(k) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi n(k+\epsilon)/N}. \qquad (4)$$

B. SCBT System Model

The system structure of SCBT is shown in Fig. 1(b)[1]. Assuming similar system parameters to those of OFDM, especially the same normalized CFO ϵ, hence the SCBT system can be described as

$$y_s(n) = x_s(n) e^{j2\pi n\epsilon/N} + z_s(k) \qquad 0 \leq n \leq N-1 \qquad (5)$$

[1]We omit the frequency domain equalization part for both systems because the AWGN channel is assumed through this paper.

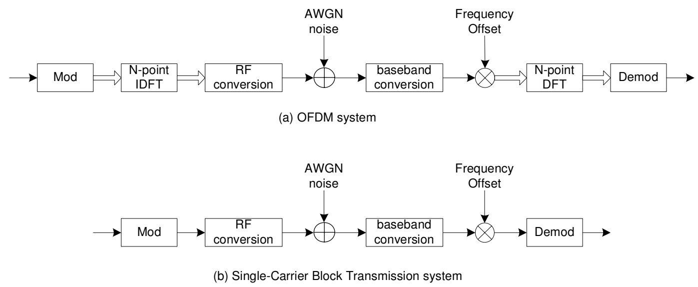

Fig. 1. OFDM and SCBT systems in the presence of carrier frequency offset.

where $x_s(n), y_s(n), z_s(n)$ are the transmitted signal, received signal and AWGN sample respectively. In this paper, same CP implementation as in OFDM is used in SCBT systems to facilitate a fair comparison [6].

Note that the major difference between SCBT and a regular SC system is that the former makes decision after collecting data of the whole block (say, N samples), while the latter does it sample by sample.

III. Comparison between OFDM and Single-Carrier Block Transmission Systems

In this section, we will place OFDM and SCBT under a common denominator and compare them in various practical and theoretical aspects.

A. Assumptions

There are several assumptions to be made for a fair comparison between OFDM and SCBT systems in AWGN channels:

1) The same bandwidth is allocated.
2) ϵ, N_g and N are the same, i.e., the SCBT system also has a CP of length N_g [6], which leads to the same throughput and frame structure as OFDM.
3) The sample rate is the same, i.e., $R_o = R_s = N/T_o = 1/T_s$, where T_o represents the OFDM symbol duration and T_s denotes the SCBT sample duration. Note that the SCBT system has N samples in one block, which makes the block duration the same as OFDM symbol duration.

Under these assumptions, the only difference between OFDM and SCBT systems in AWGN channels is the former makes decision in the frequency domain (based on (3)) while the latter does it in the time domain (based on (5)).

B. Effects of Carrier Frequency Offset

A major problem in block transmission systems is the carrier frequency offset, which exhibits different impacts on OFDM and SCBT systems.

1) OFDM: It is noticed from (3) that CFO contributes to (see Fig. 2(a))

- common phase shift (CPS), indicated by $C(0)$, causes the rotation of the desired signals. By the definition given in (4), it is easy to show that CPS does not change within one OFDM symbol.

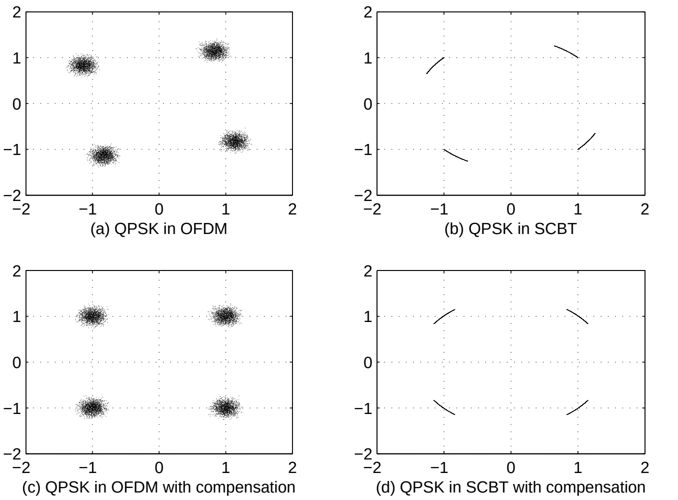

Fig. 2. OFDM and SCBT systems in the presence of normalized CFO $\epsilon = 0.05$, SNR=10, without AWGN noise.

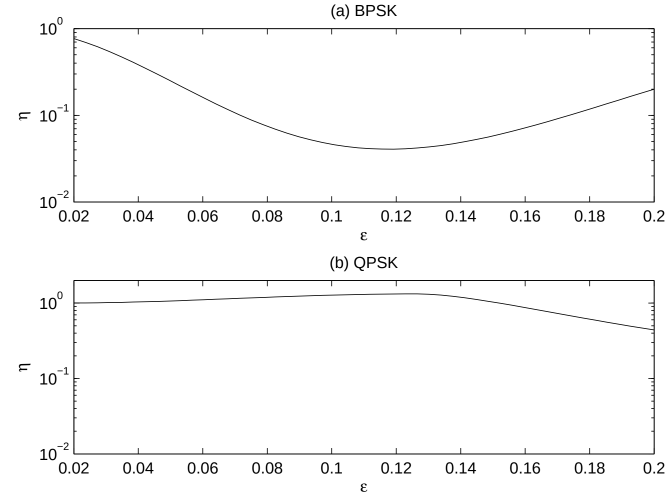

Fig. 3. BER ratio η versus normalized CFO ϵ, without compensation, SNR=10dB.

- intercarrier interference (ICI), indicated by the second term at the right side of (3), causes interference on the desired signals and destroys orthogonality between OFDM subcarriers.

2) SCBT: The constellation points of individual symbols experience a rotation alone effect (RAE) in the presence of CFO, which is a linearly increased rotation along the block (see Fig. 2(b) and Eqn. (5)).

Intuitively, the OFDM system should be more sensitive to CFO than SCBT since CFO introduces both rotation and interference in OFDM while SCBT only suffers from the former [5]. However, since CPS is constant while RAE is linearly increasing in a block, the actual impact on both systems due to the presence of CFO is not straightforward and should be examined carefully.

Existing works have compared the CFO effect on both OFDM and SCBT systems, but they are not actually done under the same set of assumptions. Hereinafter, we divide these works into two categories: carrier frequency offset analysis with compensation and without compensation. The later is discussed in next section.

C. BER Performance without Carrier Frequency Offset Compensation

The closed-forms of BER performances in the presence of CFO for the BPSK modulation without the compensation were derived in [11] as

$$P_{\text{OFDM}} \approx Q\left(\sqrt{\frac{\frac{\sin^2 \pi\epsilon}{N^2 \sin^2 \frac{\pi\epsilon}{N}} \cos^2 \frac{\pi\epsilon(N-1)}{N}}{\left(1 - \frac{\sin^2 \pi\epsilon}{N^2 \sin^2 \frac{\pi\epsilon}{N}}\right) + \frac{1}{\text{SNR}}}}\right) \tag{6}$$

and

$$P_{\text{SCBT}} = \frac{1}{N} \sum_{k=0}^{N-1} Q\left(\sqrt{\text{SNR} \cos^2 \frac{2k\pi\epsilon}{N}}\right) \tag{7}$$

where P_{OFDM} and P_{SCBT} denote the BER of OFDM and SCBT systems respectively; SNR represents the signal to noise ratio. Define $\eta = P_{\text{SCBT}}/P_{\text{OFDM}}$ as the BER ratio between the two systems which is shown in Fig. 3(a) as a function of ϵ. Clearly, if there is no CFO, η equals one. Note that within the regular normalized CFO range of $[0.02, 0.2]$ (used in [11]), η is not monotonously decreasing and has a minimum value in the middle of that range. The reason is as follows: when CFO is small, its rotation effect of CPS in OFDM and of RAE in SCBT, is small and causes little impact on the system performance. Also the extra ICI effect of OFDM systems is almost negligible. Hence both have similar BER performance. On the other hand, if ϵ is large, ICI causes severe performance degradation in OFDM systems, despite the invariance of CPS effect during the whole block. While the SCBT system also suffers from the increased RAE effect, as the samples at the end of the SCBT blocks have relatively large rotations, that may force the received signal rotating outside the correct decision zone and induce significant performance loss. Therefore there exists a certain ϵ in the range of [0.02 0.2], for which the performance difference between OFDM and SCBT systems is the largest. In the figure, the threshold value of η occurs when ϵ is around 0.1, which corresponds to the largest performance difference between two systems.

Fig. 4 illustrates the BER performance for both OFDM and SCBT systems in AWGN channels for the BPSK modulation without compensation, which is the case discussed in [11]. Clearly, SCBT outperforms OFDM through the normalized CFO range, but the gap between the performance of two systems confirms the theoretical results in Fig. 3(a), i.e., closer to one another at the two extreme values of ϵ than at middle.

However, in the QPSK case, by using similar formulas as (6) and (7) from [11], Fig. 3(b) shows that the performance ratio almost equals one within the most of the range of ϵ. This phenomenon implies that CFO causes similar performance loss in both systems, which is different from the BPSK case. The reason is that since a smaller decision range is allocated for a desired signal in QPSK than BPSK, it is much easier for the RAE effect in SCBT to cause a prominent impact on the system performance. Hence the threshold value of η discussed

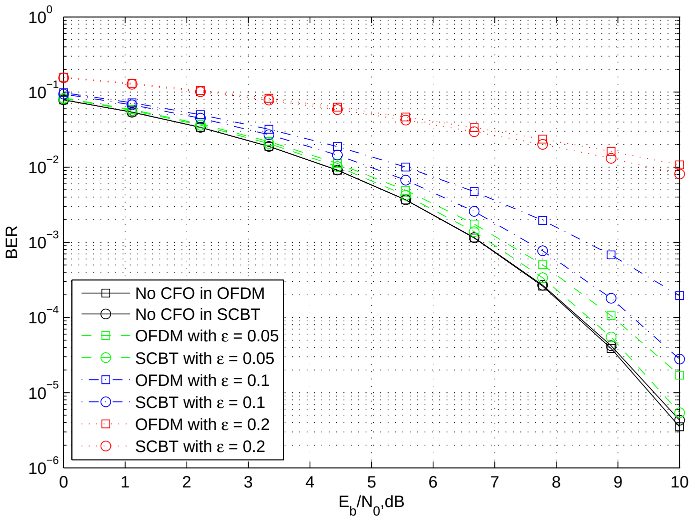

Fig. 4. OFDM and SCBT systems in the presence of different carrier frequency offset, BPSK, without compensation.

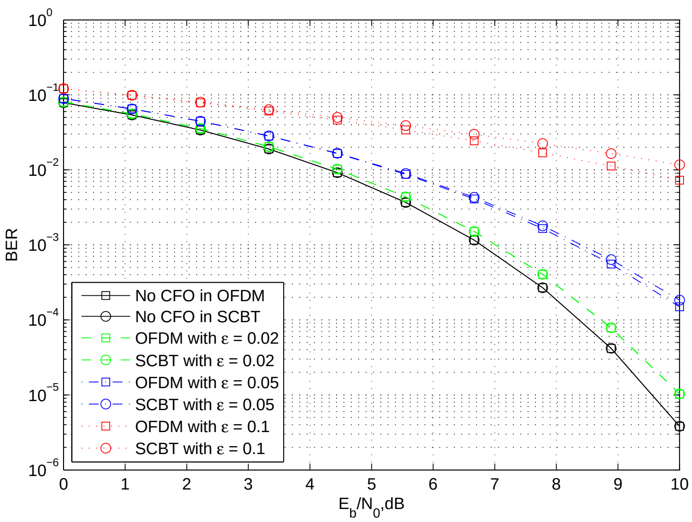

Fig. 5. OFDM and SCBT systems in the presence of different carrier frequency offset, QPSK, without compensation.

in the BPSK case is almost vanished in the QPSK scenario.

Fig. 5, which compares the performance of the two systems using QPSK, shows two major results:

- By relating to Fig. 4, it can be observed that QPSK is more sensitive to CFO than BPSK. Note that different ranges of ϵ are used in Fig. 4-5, since $\epsilon < 0.05$ is too small to show the performance difference between two candidates in BPSK (Fig. 4) while there is almost no useful information can be recovered in QPSK when $\epsilon > 0.1$ (Fig. 5).
- As also indicated in Fig. 3(b), the gap between two systems almost vanishes when we consider the impact of CFO for the QPSK scenario.

D. BER Performance with Carrier Frequency Offset Compensation

If the compensation mechanism of the carrier frequency offset is implemented at receiver [9], [10], the situation is different with the no compensation case we mentioned above. We first recall the existing results for comparing single carrier and OFDM systems with CFO. Then by pointing out the limitations and assumptions of previous works, we propose a fair comparison between SCBT and OFDM systems in the rest of this section.

When the CFO is small relative to the subcarrier spacing, the SNR degradation for OFDM and single-carrier transmissions with the same sample rate can be approximated respectively as in [10] by

$$D_o \approx \frac{10}{3\ln 10}(\pi\epsilon N)^2\frac{E_s}{N_0}\text{dB} \tag{8}$$

and

$$D_s \approx \frac{10}{3\ln 10}(\pi\epsilon)^2\text{dB}, \tag{9}$$

where D_o and D_s denote the SNR degradation in dB for OFDM and SC systems respectively, E_s is the transmitted symbol power while N_0 represents the variance of AWGN. We deduce from (8) and (9) that the degradation for both systems is proportional to the square of the CFO. For OFDM, the degradation is also proportional to the SNR, and the square of the number of subcarriers. In fact, the SNR degradation for OFDM is N^2E_s/N_0 (a quite big number) times larger than SC systems, which implies a huge BER difference under the same ϵ. However, these results are based on the assumption that arg(I_0), where

$$I_0 = \frac{1}{T}\int_0^T e^{j2\pi\Delta ft}dt \tag{10}$$

with $T = T_s$ in SC and $T = T_o$ in OFDM, is known to the receiver and can be compensated during the detection. In fact, I_0 can be treated as the average of the CFO effect over one symbol duration T. Since $R_o = R_s$ was assumed in [10], we have $T_o = NT_s$, which implies that the SC system has the information of the average CFO effect over each sample duration T_s, while the OFDM system knows only that information over the block duration T_o. This assumption is reasonable, since the SC receiver makes decision sample by sample, hence can track that average and compensate it at each sample. However, it is unjustified to use these results directly in SCBT systems, because the structure of SCBT is similar to OFDM, where the receiver has to wait for the whole block before the detection. For a fair evaluation, the same information on CFO should be assumed in the comparison of these two systems. Therefore in this paper, we assume the average of CFO effect over T_o is known in both systems, while the average of CFO over T_s is unknown for them.

Fig. 2(c)-(d) depict the results of the compensation by using the same information of the average of CFO effect during a block. This causes the whole cluster in both systems to rotate back to its original place. Note that if an assumption is made that the average of CFO effect over T_s is known to the receiver and can be compensated during the detection, Fig. 2(d) would have almost reduced to dots instead of curves, resulting in a huge performance difference as predicted by [10].

Fig. 6 shows the BER performance in OFDM and SCBT systems for the BPSK modulation, under the assumption that

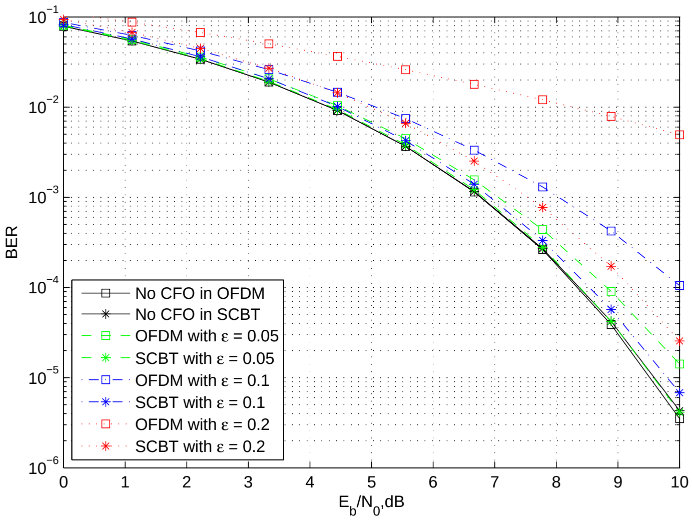

Fig. 6. OFDM and SCBT systems in the presence of different carrier frequency offset, BPSK, with compensation.

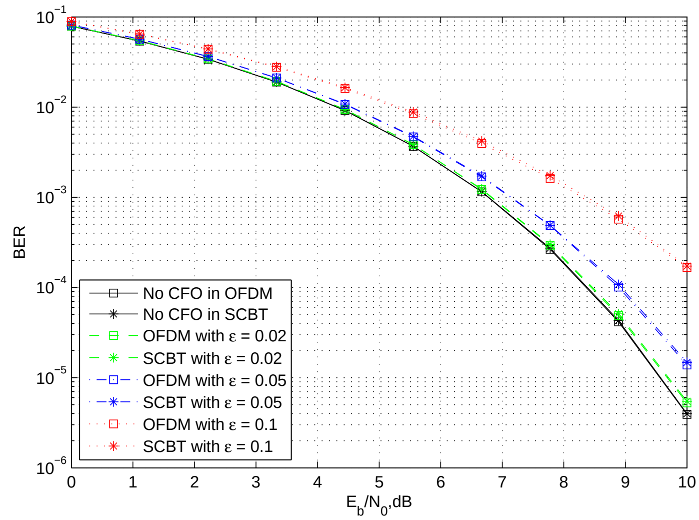

Fig. 7. OFDM and SCBT systems in the presence of different carrier frequency offset, QPSK, with compensation.

the average of CFO effect over T_o is known to the receiver and can be compensated during the detection. As we can see from the figure,

- Due to different set of assumptions used in the simulation as compared with [10], the performance gaps between two systems are much smaller than predicted by (8) and (9) for different ϵ.
- Compared with Fig. 4, we notice that for the same ϵ, the performance difference between two systems is larger in Fig. 6, which indicates that the compensation of the average CFO over the block is more effective in the BPSK case on SCBT than on OFDM systems .

Fig. 7 presents the BER performance for QPSK similar to Fig. 5, where both OFDM and SCBT systems exhibit almost identical performance after the compensation. This means that in a QPSK modulation system, with or without the compensation of the carrier frequency offset, the CFO effect causes similar impact on these two systems, if the same information of CFO is assumed in both cases.

IV. Conclusions

In this paper, we divide existing analytical works for CFO effect on OFDM and SCBT in AWGN channels into two categories: carrier frequency offset analysis with and without compensation.

For the BPSK modulation, OFDM is more sensitive to CFO than SCBT in both categories. However, the performance gaps between these two systems are much smaller than the theoretically predicted, as long as fair assumptions are made for both systems.

For the QPSK modulation, with or without the compensation, the BER performances of two candidates are almost the same according to both theoretical and numerical results. This implies that like the OFDM system, the CFO mitigation mechanism is also crucial in the SCBT system despite of its single carrier nature.

Furthermore, since phase noise, which is a random process caused by the fluctuation of the local oscillator [12], has similar impact on the system performance as the carrier frequency offset, the comparison between both OFDM and SCBT systems in the presence of phase noise [6], [10], is a topic worth further investigation.

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