Closed-Form Expressions for BER Performance in OFDM Systems with Phase Noise

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***Abstract*— OFDM system suffers significant performance degradation due to phase noise. Many phase noise analysis methods have been proposed in the literature showing its effect as a loss in the ratio of the signal power to the statistical average of the intercarrier interference (ICI). It would be more desirable, however, to predict phase noise effect on bit error rate (BER) directly.**

In this paper, by investigating the phase noise properties and the impacts caused by phase noise on an OFDM system, we derive approximate closed-form expressions for BER performance in AWGN channel with BPSK and QPSK modulations. In general, it is impossible to get such expression for multipath Rayleigh fading channel. Hence we establish upper and lower bounds for the degradation caused by phase noise, which give insightful view of the system performance. Simulation results show that the proposed simple analytical forms are quite accurate for different phase noise levels, which lead to the conclusion that a phase noise mitigation mechanism is needed to obtain acceptable performance.

***Index Terms*— Phase Noise, BER, ICI, OFDM**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been widely adopted and implemented in wire and wireless communication, such as digital subscriber line (DSL), digital terrestrial TV broadcasting (DVB-T), IEEE 802.11a wireless local area networks (WLANs) and European high performance local area network (HIPERLAN/2) [1]–[4]. It is, however, sensitive to carrier frequency offset and phase noise [5], [6].

Frequency offset is a deterministic phenomenon, which is usually caused by the different carrier frequencies of the transmitter and the receiver, or by Doppler shift. On the other hand, phase noise is a random process caused by frequency fluctuation at the receiver and transmitter oscillators. The principles of analyzing the frequency offset effects have been described in many papers [5], [7], hence in our contribution, only the phase noise problem will be discussed.

Because of the random nature of phase noise (PN), its effect is harder to be analyzed. In the literature, two important types of PN was considered. When the system uses phase-locked loops the resulting PN has low power and is modeled as a zero-mean, stationary, finite-power random process [8]. When the system is only frequency-locked the resulting PN is slowly varying but not limited in power, hence is modeled as a zero-mean, nonstationary, infinite-power Wiener process [5], [6], [9], [10]. We will focus on the Wiener process assumption in this paper.

The effect of Wiener phase noise has been expressed as a loss in the ratio between the signal power and the statistical average of the ICI [5], [6], [11], wherein AWGN channel was considered in [11], while [5], [6] included fading channel. However, it is both interesting and useful to find the precise relation between the bit error rate (BER) or symbol error rate (SER) and the phase noise. The original work done by Tomba [9], which is based on [10], considers only the BER for the AWGN channel and ignores the comparison with simulation results.

In this paper, we propose a new method to derive closed-form expressions for BER performance in AWGN channel with the presence of phase noise. Although we only study BPSK and QPSK modulations in our work, the results can be extended to a higher modulation constellation. Moreover, we also investigate two extreme cases in Rayleigh fading channel, which give rise to the upper and lower bounds of the BER performance. Simulation results validate all expressions we have derived and show clearly the necessity of phase noise mitigation mechanism.

The rest of the paper is organized as follows. We describe the system model in section II, where phase noise, OFDM system and simulation models are presented. In section III, different effects of phase noise are discussed. The closed form expressions for BER performance in AWGN channel are shown in section IV. section V gives the analytical results for Rayleigh fading channel. Finally we draw some conclusions in section VI.

II. SYSTEM MODEL

A. Phase Noise Model

Phase noise $\phi(t)$, generated at both transmitter and receiver oscillators, can be described as a continuous Brownian motion process with zero mean and variance $2\pi\beta t$, where β denotes the phase noise linewidth, i.e., frequency spacing between 3dB points of its Lorentzian power spectral density function [5], [10]. Such noise has independent Gaussian increments [10], which, from a spectral point of view, can be presented as a Wiener process [12], [13]. To better characterize phase noise, Demir et al. [14] developed a unifying theory using a nonlinear method that provides more accurate description. With such a method [14, Remark 7.1], phase noise $\phi(t)$ is shown to converge, asymptotically with time, to a Gaussian random process having a constant mean, a variance increasing linearly with time, and a correlation function that satisfies $E[\phi(t)\phi(t+\tau)] = min[E(\phi(t)^2), E(\phi(t+\tau)^2)]$. Furthermore, as indicated in [15], the aforementioned properties suggests a discrete Markov process which, in an OFDM system, can describe the phase noise on the nth sample of the mth symbol as

$$\phi_m(n) = \phi_{m-1}(N-1) + \sum_{i=-N_g}^{n} u[m(N+N_g)+i] \quad (1)$$

where u(i)'s denote mutually independent Gaussian random variables with zero mean and variance $\sigma_u^2 = 2\pi\beta T/N$, and T denotes the OFDM symbol length. In particular, (1) reduces to $\phi_0(n) = \sum_{i=-N_g}^{n} u(i)$ for $m = 0$ ($\phi_m(n) = 0$, when $m < 0$, and $\phi_0(n) = 0$ when $n < -N_g$).

B. OFDM System Model

The OFDM system model over a slowly time varying multipath fading channel with phase noise can be described as

$$y_m(n) = e^{j\phi(n)} \sum_{l=0}^{L-1} h_l x_m(n-l) + n(n) \qquad 0 \le n \le N-1 \tag{2}$$

where h_l's ($0 \le l \le L-1$) are i.i.d. complex-valued Rayleigh fading random variables, and $n(n)$'s ($0 \le n \le N-1$) are independent complex-valued Gaussian random variables with zero mean and variance σ_n^2 for both real and imaginary components, while $x_m(n)$'s are the time domain transmitted symbols. L is the length of the time-domain channel impulse response (CIR).

After discarding the cyclic prefix (CP) and performing an FFT at the receiver, we obtain the received data frame in the frequency domain:

$$Y_m(k) = X_m(k)H_m(k)C_m(0) + ICI_m(k) + N(k) \tag{3}$$

with

$$C_m(k) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j[\frac{2\pi nk}{N} + \phi_m(n)]} \tag{4}$$

$ICI_m(k)$ is defined as $\sum_{l=0,l\neq k}^{N-1} X_m(l)H_m(l)C_m(k-l)$, and $N(k)$ is the DFT transform of $n(n)$. Since DFT is linear and does not change the noise energy, $N(k)$ is still a Gaussian random variable with zero mean and variance σ_n^2.

Note that common phase error (CPE) and ICI are represented by $C_m(0)$ and $ICI_m(k)$ respectively. If phase noise does not exist, Eqn. (3) and (4) reduce to $Y_m(k) = X_m(k)H_m(k) + N(k)$ and $C_m(k) = \delta(k)$, respectively.

C. Simulation Model

Simulations are carried out for the IEEE 802.11a standard, with 64 subcarriers for each OFDM symbol. The length of cyclic prefix is assumed to be larger than channel delay spread. For fading channel, a multipath fading channel is modeled as

$$h(t, \tau) = \sum_{r=0}^{L-1} \alpha_r(t)\delta(\tau - \frac{\tau_r(t)T}{N}) \tag{5}$$

where $\delta(\tau)$ and $\{\alpha_r(t)\}_{r=0}^{L-1}$ are a Dirac delta function and zero-mean complex Gaussian random variables, respectively. For block fading whose response does not vary within one block (M OFDM symbols), the parameter t can be omitted, which is the case in this paper. Delay spread $\{\tau_r\}_{r=0}^{L-1}$ is assumed to be uniformly and independently distributed within $[0, N_g]$.

III. Effects of Phase Noise

There are three major problems affected by phase noise: phase shift, effective power degradation, and ICI. For the purpose of convenience, we will drop the subscript m, i.e., our discussion is restricted in one OFDM symbol.

A. the Effect on Phase Shift

Common Phase Error (CPE), indicated by $C(0)$, causes the rotation of the desired signals, is invariant within one OFDM symbol, although it is varying from symbol to symbol. From Eqn. (4), we have $C(0) = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\phi(n)} = Ae^{j\theta}$, where A denotes the amplitude of CPE, which will be discussed later, and θ is a random variable denoting phase shift, which is the major effect of CPE when phase noise is small. The probability density function (pdf) of θ which will be needed for analyzing its effect on BER performance, is derived in Appendix A as

$$f_\theta(x) \approx \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{-\frac{x^2}{2\sigma_\theta^2}} \tag{6}$$

where $\sigma_\theta^2 = \frac{7}{12}N\sigma_u^2 = \frac{7}{6}\pi\beta T$.

B. the Effect on Power Degradation

Power degradation effect was thoroughly investigated in [5], [6]. When phase noise is small, the power of CPE and ICI was found to be approximated by,

$$\sigma_{CPE}^2 = E\left[|A|^2\right] \approx 1 - \frac{\pi\beta T}{3} \tag{7}$$

and

$$\sigma_{ICI}^2 = E\left[|ICI(k)|^2\right] \approx \frac{\pi\beta T E_s E_H}{3} \tag{8}$$

respectively. In this paper, we assume the channel response is normalized, i.e., $E_H = E[|H(k)|^2] = 1$. Hence with small phase noise, the signal to interference plus noise ratio (SINR) was derived in [6] as

$$\Gamma = \frac{\sigma_{CPE}^2 E_s}{\sigma_{ICI}^2 + \sigma_n^2} = \frac{1 - \frac{\pi\beta T}{3}}{\frac{\pi\beta T}{3} + \frac{1}{\gamma_s}} \tag{9}$$

where $\gamma_s = E_s/\sigma_n^2$ denotes SNR, E_s and σ_n^2 denote symbol and noise power respectively.

C. the Effect on ICI

The effect of ICI power has been included in the previous discussion. However, the probability distribution of ICI also plays an important part in BER analysis. It is usually treated in AWGN channel as a Gaussian distributed random variable [9], [13], whereas in fading channel, such approximation is not appropriate in some scenario. Details will be given in section V.

IV. BER performance in AWGN channel

In this section, We study the BER performance in the presence of phase noise for OFDM in AWGN channel, i.e., $H(k) = 1$ for all k. Hereinafter, we assume coherent receiver is implemented and phase noise is unknown at the receiver, which causes degradation of the BER performance.

A. Closed-Form Expressions for BER Performance

By using the pdf given in Eqn. (6) and SINR given in Eqn. (9), it is possible to obtain a closed-form expression for BER, which would serve as an attractive simple alternative to previously derived SNR degradation results [5], [6]. Two typical modulations are analyzed as following.

BPSK: To derive the exact expression of BER, we write the system model as:

$$Y(k) = X(k)C(0) + N'(k) \tag{10}$$

with

$$ICI(k) = \sum_{l=0, l\neq k}^{N-1} X(l)C(k-l) \tag{11}$$

where $N'(k) = ICI(k) + N(k)$ can be approximated by Gaussian random variable ([9], [12]) with zero mean and variance $\pi\beta NT/3+\sigma_n^2$. Therefore, the analytical BER_A form in AWGN channel can be derived as

$$\begin{aligned} BER_A^B(\gamma_s) =& \frac{1}{2}P\{\Re[Y(k)] < 0 | X(k) = +1\} \\ &+ \frac{1}{2}P\{\Re[Y(k)] > 0 | X(k) = -1\} \\ &\approx \int_{-\infty}^{\infty} P_b(\Gamma; x) f_\theta(x) dx \\ &= 2\int_0^{\infty} \frac{1}{\sqrt{2\pi}\sigma_\theta} Q\left(\sqrt{\frac{1-\frac{\pi\beta T}{3}}{\frac{\pi\beta T}{3}+\frac{1}{\gamma_s}}\cos^2 x}\right) e^{-\frac{x^2}{2\sigma_\theta^2}} dx \end{aligned} \tag{12}$$

where $\sigma_\theta^2 = \frac{7}{6}\pi\beta T$ and Q denotes the Gaussian probability integral, $P_b(\Gamma; x)$ is the average probability of error for each realization of θ at given Γ.

QPSK: In this case, we assume detection is implemented by evaluating the real and imaginary components independently, hence BER_A is readily derived as

$$\begin{aligned} BER_A^Q(\gamma_s) \approx& \int_0^{\infty} Q\left(\sqrt{\frac{1-\frac{\pi\beta T}{3}}{\frac{\pi\beta T}{3}+\frac{1}{\gamma_s}}\cos^2(\frac{\pi}{4}+x)}\right) f_\theta(x) dx \\ &+ \int_0^{\infty} Q\left(\sqrt{\frac{1-\frac{\pi\beta T}{3}}{\frac{\pi\beta T}{3}+\frac{1}{\gamma_s}}\sin^2(\frac{\pi}{4}+x)}\right) f_\theta(x) dx \end{aligned} \tag{13}$$

Note that in both cases, the integral variable x denotes the realization of phase shift and can be greater than 2π, although the possibility of such event is negligible when phase noise variance is small.

B. Phase Noise Effects on BER performance

Eqn. (12) and (13) indicate that in the presence of phase noise, the BER is a function of various system parameters β, T, γ_s, and their corresponding ratios:

1) When T, γ_s are fixed, larger β will cause worse BER performance. Hence as expected, the system can benefit a lot from a small linewidth oscillator.

2) Note that $T = N/R$, where R denotes the transmission data rate, and N, the number of data point in OFDM symbol, is usually fixed for a certain application, hence the higher transmission data rate R the better is the system performance. In fact, the phase noise linewidth to transmission data rate ratio $\eta = \beta/R = \beta T/N$ is of interest. Its effects can be seen in two ways: a higher η means a smaller effective SINR Γ, which increases $P_b(\Gamma; x)$; at the same time, higher η introduces larger variance of phase shift σ_θ^2, which raises the whole integral result in (12). Therefore, both ways lead to a higher BER and vice versa.

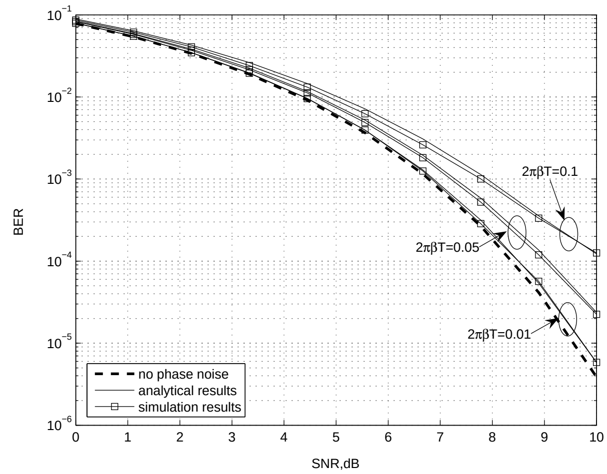

Fig. 1. BER performance in AWGN for BPSK modulation with different phase noise variance $2\pi\beta T$

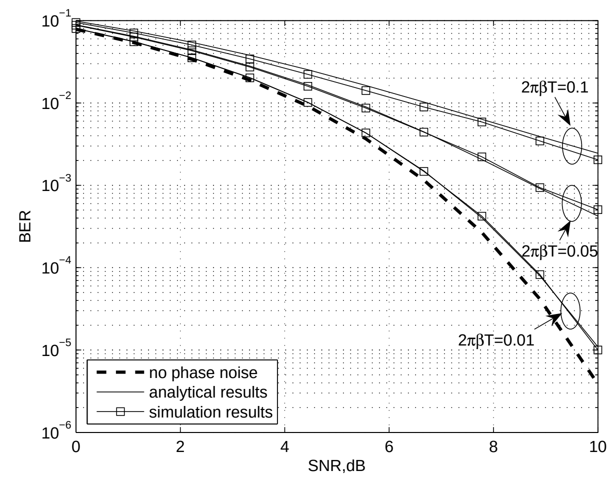

Fig. 2. BER performance in AWGN for QPSK modulation with different phase noise variance $2\pi\beta T$

These analytical and their corresponding simulation results are shown in Fig.1 and Fig.2 for BPSK and QPSK respectively. Although the analytical solutions in (12) and (13) are derived with several approximations, they fit quite well with the simulation results. The figures also show that the QPSK modulation is more sensitive to phase noise than BPSK (particularly for higher phase noise variance), which can be attributed to the difference of the minimum distance between the points of these constellation.

Although the BER performance might seem acceptable when phase noise variance is 0.01, it is not the case in practice. As customarily done, the timing and frequency synchronization has always been performed in the preamble, and the results are used for the whole data block following that preamble. Since phase noise process is nonstationary with increasing variance, the BER performance deteriorates rapidly as the data block length increases. Consequently, a symbol by symbol phase noise tracking mechanism is needed even with a high quality oscillator, which is the essential motivation for phase noise mitigation studies.

V. BER PERFORMANCE IN RAYLEIGH FADING CHANNEL

Usually, OFDM is used to combat the multipath effect in a slow fading environment. Hereinafter, we divide our analysis into two groups: frequency flat fading and frequency selective fading. Moreover, we assume the channel is ergodic, which allows us to average the BER over all possible channel realizations. As such, an analytical BER for OFDM in the Rayleigh fading channel with BPSK modulation can be derived as

$$\begin{aligned} BER_R^B(\gamma_s) &\approx \int_0^\infty\int_{-\infty}^\infty P_b(\Gamma(\gamma);x) f_\gamma(\gamma) f_\theta(x) dx d\gamma \\ &= \int_0^\infty\int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}\sigma_\theta} Q\left(\sqrt{\Gamma(\gamma)\cos^2 x}\right) e^{-\gamma} e^{-\frac{x^2}{2\sigma_\theta^2}} dx d\gamma \end{aligned} \quad (14)$$

with

$$\Gamma(\gamma) = \frac{\sigma_{CPE}^2 E_s \gamma}{\sigma_{ICI}^2 + \sigma_n^2} \quad (15)$$

where γ denotes the instantaneous power of frequency response on a certain subcarrier. Note that $\Gamma(\gamma)$ represents the instantaneous SINR in fading channels.

A. Frequency Flat Fading

In this case, the frequency responses for all subcarriers are the same (i.e., $H(l) = H(k)$ for all k) and there is only one resolvable path in the channel. Hence

$$\begin{aligned} ICI(k) &= \sum_{l=0, l\neq k}^{N-1} X(l)H(l)C(k-l) \quad (16) \\ &= H(k)\left[\sum_{l=0, l\neq k}^{N-1} X(l)C(k-l)\right] \end{aligned}$$

Note that, for each channel realization, $H(k)$ is a constant and the expression in the bracket is exactly the same as AWGN case (Eqn. (11)) which can be modeled as a Gaussian distributed random variable. Therefore, the instantaneous variance of ICI term in each channel realization can be calculated as the variance of AWGN part multipled by the instantaneous power of $H(k)$, i.e., $\sigma_{ICI}^2 = \frac{\pi\beta T E_s}{3}\gamma$. Hence

$$\Gamma(\gamma) = \frac{1 - \frac{\pi\beta T}{3}}{\frac{\pi\beta T}{3} + \frac{1}{\gamma_s \gamma}} \quad (17)$$

substituting (17) into (14), we obtained the exact BER form for frequency flat fading channel.

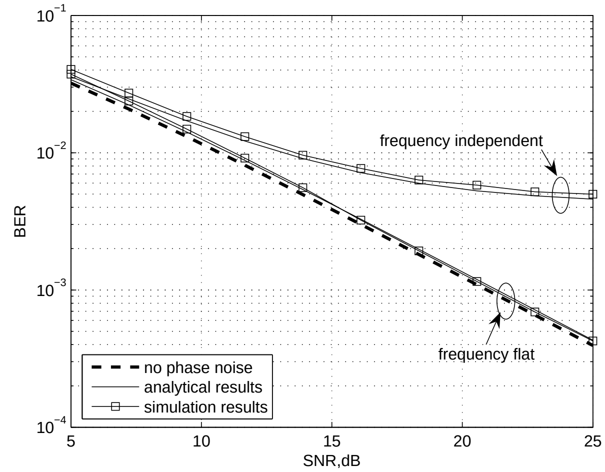

Fig. 3. BER performance in Rayleigh frequency flat and independent channel for BPSK modulation with phase noise variance $2\pi\beta T$=0.1

B. Frequency Selective Fading

Because of the lack of knowledge on the ICI distribution, it is quite difficult to analyze the BER in a frequency selective channel. Due to the correlation between frequency channel response on different subcarriers, it is inappropriate to assume ICI has Gaussian distribution. However, a special case can be analyzed and treated as an upper bound of this scenario: a frequency selective channels with N independent paths and uniform power delay profile (PDP), which can also be considered as a frequency independent fading channel. Apparently, this is impractical assumption for regular channel since the path number L is usually far less than the subcarrier number N. Nevertheless, such assumption will give us an insightful view of the upper bound of BER performance in a frequency selective channel. To this end, since $H(k)_{k=1}^N$ is assumed as i.i.d. distributed, ICI still can be approximated to Gaussian distribution as what has been done in AWGN channel. Recalling that the channel response is normalized, hence the variance of ICI in frequency independent channel equals to the AWGN case (Eqn.(8)), which leads to

$$\Gamma(\gamma) = \frac{(1 - \frac{\pi\beta T}{3})\gamma}{\frac{\pi\beta T}{3} + \frac{1}{\gamma_s}} \quad (18)$$

substituting (18) into (14), we obtained the exact BER expression for frequency independent fading channel. Note that such bound is loose since the real Rayleigh fading channel can't be such stringent.

simulation results are carried out for frequency flat and the described independent channel in Fig.3, which shows that our analytical results are quite accurate with the Gaussian distribution assumption. Apparently, certain mechanism must be implemented to mitigate the catastrophic effects of phase noise in a Rayleigh fading channel.

Finally, Fig.4 gives simulation results of a multipath channel with different number of paths and uniform power delay profile (PDP). It shows that the frequency flat and aforementioned frequency independent channels serve as the lower and upper

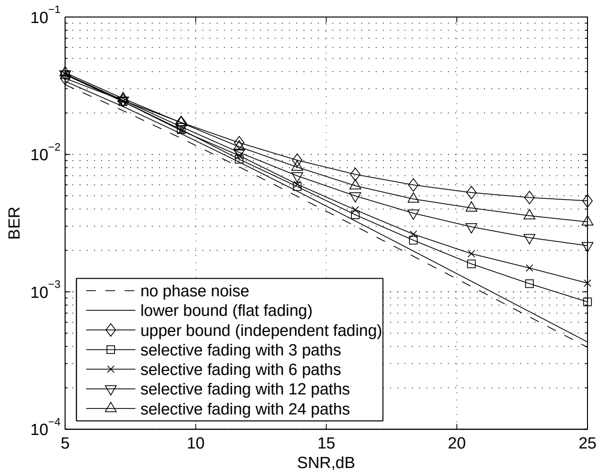

Fig. 4. Simulation of BER performance in Rayleigh fading channel for BPSK modulation with analytical upper and lower bounds, phase noise variance $2\pi\beta T$=0.1

bound of BER, as they represent two extreme cases of the effect introduced by ICI term.

Effect of different phase noise levels on BER with Rayleigh fading has similar behavior as in AWGN case, and the formulas to compute the error probability for QPSK are a straightforward extension of the one for BPSK. Hence, they are omitted in this paper.

VI. Conclusions

Phase noise causes severe performance degradation in OFDM systems. Although several analysis methods have been developed in the literature by considering the SNR loss, it is more desirable to evaluate the phase noise effect through BER performance. In this paper, we first analyzed the BER expressions in AWGN channel, where both BPSK and QPSK cases are considered, and then extend these expressions to different fading scenarios. Simulation results show that the methodology presented in this work is accurate and can be used as references in practice.

Appendix A

The discrete Brownian motion process can be expressed as

$$\phi(n) = \sum\nolimits_{i=-N_g}^{n} u(i) \tag{A-1}$$

We assume as customarily done that phase noise is small [5], [9], [12], [13], hence from (4), we have

$$\begin{aligned} C(0) &= \frac{1}{N}\sum_{n=0}^{N-1} e^{j\phi(n)} \approx \frac{1}{N}\sum_{n=0}^{N-1}(1+j\phi(n)) \\ &= 1 + j\frac{1}{N}\sum_{n=0}^{N-1}\sum_{i=-N_g}^{n} u(i) = Ae^{j\theta} \end{aligned} \tag{A-2}$$

where

$$\theta = \arctan\Theta = \arctan\frac{1}{N}\sum_{n=0}^{N-1}\sum_{i=-N_g}^{n} u(i) \tag{A-3}$$

since u(i)'s are mutually independent Gaussian random variables with zero mean and variance $\sigma_u^2 = 2\pi\beta T/N$, Θ is a Gaussian random variable with zero mean and variance $(\frac{(N+1)(2N+1)}{6N} + N_g)\sigma_u^2$. For purpose of simplicity, we take the practical assumption: $N \gg 1$, $N_g = N/4$, hence the variance of Θ can be simplified as $\frac{7}{12}N\sigma_u^2$.

When phase noise variance is small, $\theta = \arctan\Theta \approx \Theta$, hence

$$f_\theta(x) \approx \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{-\frac{x^2}{2\sigma_\theta^2}} \tag{A-4}$$

where $\sigma_\theta^2 = \frac{7}{12}N\sigma_u^2$. Note that, although the approximation is not hold when $\theta > 0.5rad$, the probability of such event is very small: $P(X > 0.5) = 3 \times 10^{-11}$ when $N\sigma_u^2 = 0.01$. The validation of our derivation is also shown in simulation results.

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