A Phase Noise Mitigation Scheme for MIMO WLANs with Spatially Correlated and Imperfectly Estimated Channels

Pan Liu, *Student Member, IEEE,* Songping Wu, *Member, IEEE,* and Yeheskel Bar-Ness, *Fellow, IEEE*

***Abstract*— Applying multiple input multiple output (MIMO) technique to OFDM-based wireless local area networks (WLANs) promises impressive high capacity and spectral efficiency compared with conventional systems. However, similar to SISO-OFDM, MIMO-OFDM suffers significant performance degradation due to the presence of phase noise.**

Many methods have been developed to mitigate phase noise for a single antenna system with perfect channel estimation, whereas none has been proposed for correlated MIMO-OFDM scenarios. Therefore, in this letter, by using the phase noise correlation function, a new phase noise mitigation scheme is proposed for the general $M_T \times M_R$ MIMO WLANs system with channel estimation errors. Numerical results show that, compared with conventional approaches, the proposed scheme achieves significant performance gain with high spectral efficiency, requiring few pilots, and is robust to spatial correlation and channel estimation errors, which makes it very attractive for practical applications.

***Index Terms*— Phase Noise, WLANs, MIMO, OFDM, MMSE**

I. INTRODUCTION

The combined MIMO-OFDM scheme has an advantage over conventional SISO systems for its much improved system capacity [1] or BER performance [2] introduced by the MIMO technique, and its robustness to channel frequency selectivity due to the OFDM technique. However, similar to SISO-OFDM [3], MIMO-OFDM is also very sensitive to phase noise. Various phase noise correction methods were proposed for single -antenna systems [4], [5] with perfect channel estimation, but nothing has been suggested for the correlated multi-antenna case. In this letter, we proposed a new phase noise mitigation method for the general $M_T \times M_R$ spatially correlated MIMO WLANs with channel estimation errors.

II. MIMO-OFDM SYSTEM MODEL

Consider a frequency selective MIMO channel with M_T transmit antennas, M_R receive antennas. By using Cyclic Prefix (CP) in the data sequence, the time domain linear convolution with the channel response is turned into the cyclic convolution. Let $\mathbf{x}_r(k) = [x_r^1(k), x_r^2(k), ..., x_r^{M_T}(k)]^T$ and $\mathbf{y}_r(k) = [y_r^1(k), y_r^2(k), ..., y_r^{M_R}(k)]^T$ denote the transmitted and received data for all antennas on subcarrier k of OFDM symbol r respectively, where $0 \leq k \leq N-1$ and $0 \leq r \leq B-1$ with N and B denoting the number of OFDM subcarriers and OFDM symbols block size respectively. Hence the general form of MIMO-OFDM process can be summarized as

$$\mathbf{y}_r(k) = \mathbf{H}(k)\mathbf{x}_r(k) + \mathbf{n}(k) \quad (1)$$

where $\mathbf{H}(k)$ is an $M_R \times M_T$ matrix with element $\{H_{ij}(k)\}$ denotes the channel frequency response between jth transmit antenna and ith receive antenna. We assume as is customarily done that the estimated channel response remains constant over the OFDM symbols block (i.e., quasistatic fading). $\mathbf{n}(k)$ is an $M_R \times 1$ vector denotes the noise samples for all antennas on subcarrier k which are zero-mean AWGN with the variance σ^2.

III. PHASE NOISE MITIGATION

A. The Effects of Phase Noise

The term phase noise is widely used for describing short term random frequency fluctuations of a signal. It is caused by both transmitter and receiver oscillators and can be described as a continuous Brownian motion process with zero mean and variance $2\pi\beta t$, where β denotes the phase noise linewidth [3]. In a MIMO system, for cost reasons, one might use a single oscillator to support multiple antennas, and hence the expression of (1) is subsequently modified to

$$\mathbf{y}_r(k) = \mathbf{H}(k)\mathbf{x}_r(k)c_r(0) + \overline{\mathbf{n}}_r(k) \quad (2)$$

where $\overline{\mathbf{n}}_r(k) = \sum_{n=0,n\neq k}^{N-1} \mathbf{H}(n)\mathbf{x}_r(n)c_r(n-k) + \mathbf{n}(k)$ and $c_r(n) = \frac{1}{N}\sum_{k=0}^{N-1} e^{j2\pi kn + j\phi_r(k)}$, with $\phi_r(k)$ denoting the phase noise in kth sample of rth OFDM symbol. The variance of $\phi_r(k)$ is given by $2\pi\beta Tk/N$, where β and T denote the phase noise linewidth and the OFDM symbol duration respectively [4]. It is noticed from (2) that the phase noise effect comprises a common phase error (CPE), indicated by $c_r(0)$, which causes the rotation of the desired signals, and the intercarrier interference (ICI), depicted by the second term, which causes interference on the desired signals. When phase noise is small, CPE accounts for over 90% of the phase noise energy while ICI is relatively small [4], therefore our approach in the following sections will focus on CPE reduction only.

Moreover, the discrete Brownian motion process can be expressed as

$$\phi_r(k) = \sum_{i=0}^{r(N+N_g)+k} u(i) \quad (3)$$

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P. Liu and Y. Bar-Ness are with the Center for Wireless Communications and Signal Processing Research (CWCSPR), Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ 07102, USA (e-mail:{pl7, barness}@njit.edu).

S. Wu is with Marvell Semiconductor Inc., Sunnyvale, CA, USA.

where $u(i)$'s denote mutually independent Gaussian random variables with zero mean and variance $\sigma_u^2 = 2\pi\beta T/N$, and N_g denotes the length of CP. Hence, the correlation function of the phase noise effect on OFDM symbols, $e^{j\phi_r(k)}$, is given by (for $m \geq n$)

$$E\left[e^{j\phi_m(i)}e^{-j\phi_n(k)}\right] = E\left[e^{j\sum_{p=n(N+N_g)+k+1}^{m(N+N_g)+i} v_p u(p)}\right]_{v_p=1} = \prod_{p=1}^{(m-n)(N+N_g)+i-k} \mathbf{\Psi}(v_p|_{v_p=1}) \quad (4)$$

where $\mathbf{\Psi}(v_p)$ denotes the characteristic function of the random Gaussian variable $u(p)$. It is straightforward to show that $\mathbf{\Psi}(v_p) = E\left[e^{jv_p u(p)}\right] = e^{-\frac{v_p\sigma_u^2}{2}} = e^{-\pi\beta T v_p/N}$, hence (4) can be further simplified for arbitrary m and n as

$$E\left[e^{j\phi_m(i)}e^{-j\phi_n(k)}\right] = e^{-\pi\beta T|(m-n)(N+N_g)+i-k|/N}. \quad (5)$$

We can evaluate (5) for some typical values: in IEEE802.11a, $N = 64$, $N_g = 16$, for $2\pi\beta T = 0.01$ [4], $i = k$, $m - n = 10$ and 110, the correlation value equals 0.9394 and 0.5028 respectively. Define B_C as the coherent block size, i.e., $\phi_m(i)$ and $\phi_n(k)$ are considered uncorrelated (correlation less than 0.5) if $|m-n| > B_C$, therefore $B_C \approx 110$ in this scenario. Therefore, we can conclude that the phase noise effect, $e^{j\phi_r(k)}$, is highly correlated within a small block in WLANs, as is its linear combination $c_r(0)$.

B. Estimation of Channel Response and Correlation

For IEEE802.11a [6], there are two kinds of packets: preamble and data. Preamble consists of 10 short symbols and 2 long symbols which are used for channel estimation, synchronization and frequency offset. Many channel estimation methods for the MIMO-OFDM have been proposed in the literature. Since the preamble signals are also contaminated by phase noise, by using the most commonly used least squares (LS) solution, channel estimates can be obtained as $\mathbf{U}_0(k) = \widehat{\mathbf{H}(k)c_0(0)}$, $0 \leq k \leq N-1$ (where without loss of generality we assume $r = 0$ in the preamble (Fig. 1a)). Furthermore, the correlation of ICI plus noise term in (2) can be estimated by the null subcarriers in short symbols of the preamble (the feasibility of the usage of such subcarriers for estimation in OFDM system has been discussed in [4]). Recalling that in IEEE802.11a there are 80 null subcarriers (not including the guard band) in short symbols of the preamble, we assume $\mathcal{K}_Q^m = \left\{k_1^m, k_2^m, ..., k_Q^m\right\}$ $(1 \leq m \leq M)$ denotes the null subcarriers set in each short symbol, where Q denotes the number of null subcarriers in one short symbol and M denotes the number of short symbols. Hence, the correlation matrix of $\overline{\mathbf{n}}_0(k)$ in (2) can be estimated as (Fig. 1a),

$$\widehat{\mathbf{R}}_0 = \frac{1}{MQ}\sum_{m=1}^{M}\sum_{k\in\mathcal{K}_Q^m} \mathbf{y}_m(k)\mathbf{y}_m^H(k). \quad (6)$$

Note that we dropped the index k from $\widehat{\mathbf{R}}_0$, since the spatial correlation information is the same for each subcarrier and the variance of each element in $\overline{\mathbf{n}}_0(k)$ is almost identical [4].

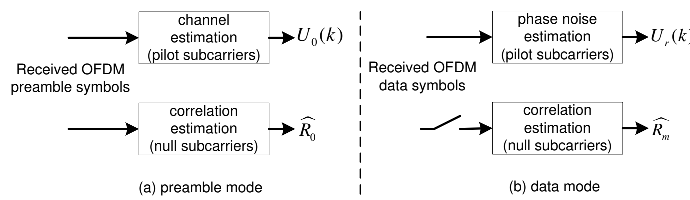

Fig. 1. Proposed scheme for phase noise and correlation estimation

As the statistic of $\mathbf{H}(n)\mathbf{x}_r(n)$ remains constant within one block, and phase noise effect is highly correlated, $\widehat{\mathbf{R}}_0$ doesn't change much during the preamble and can be used as statistic information of $\overline{\mathbf{n}}_r(k)$ within the coherent block size.

C. Phase Noise Mitigation (PNM) Scheme

Channel coefficients require constant tracking to combat phase drift after the estimation during the preamble. This is aided by inserting known pilot symbols at fixed or variable subcarrier positions. Rewriting (2) for pilot signals (the pilot set for each antenna is assumed to be $\mathcal{K}_p = \{k_1, k_2, ..., k_p\}$), we have

$$\mathbf{y}_r(k_i) = \mathbf{s}_r(k_i)v_r + \overline{\mathbf{n}}_r(k_i), \qquad 1 \leq i \leq p \quad (7)$$

where

$$\mathbf{s}_r(k_i) = \mathbf{H}(k_i)\mathbf{x}_r(k_i)c_0(0) \approx \mathbf{U}_0(k_i)\mathbf{x}_r(k_i) \quad (8)$$

and

$$v_r = \frac{c_r(0)}{c_0(0)}. \quad (9)$$

If the channel is spatially independent, intuitively, we can apply the traditional phase noise mitigation method to the current scenario, since a MIMO channel can be easily decomposed to a set of parallel SISO channels. However, in practice, the MIMO channel is always correlated, which indicates that the ICI of each receive antenna is also correlated. Therefore, it would be more adequate to consider the correlation of $\overline{\mathbf{n}}_r(k)$ in our algorithm. To estimate v_r in (7), we use in this letter a minimal mean square error (MMSE) based scheme, which minimizes the cost function $E\left[\left\|v_r - \mathbf{w}^H\mathbf{y}_r(k_i)\right\|^2\right]$ by finding an appropriate coefficient $\mathbf{w}$. With some algebraic manipulations, it is readily shown that the optimal coefficient is given by

$$\mathbf{w} = (\mathbf{s}_r(k_i)\mathbf{s}_r^H(k_i)\mathbf{R}_r/E_v)^{-1}\mathbf{s}_r(k_i) \quad (10)$$

where E_v denotes the average energy of v_r, and $\mathbf{R}_r$ denotes the correlation matrix of $\overline{\mathbf{n}}_r(k_i)$. Since phase noise is highly correlated in a small block, $E_v \approx 1$, $\mathbf{R}_r \approx \mathbf{R}_0$ (whose estimate was found in (6)). As $\widetilde{v}_r$ is invariant within an OFDM symbol [3], and if several pilots are available within a symbol, then by using the estimate of $\mathbf{R}_0$, the MMSE estimate of v_r is given by

$$\widetilde{v}_r = \frac{1}{p}\sum_{k_i\in\mathcal{K}_p} \mathbf{s}_r^H(k_i)(\mathbf{s}_r(k_i)\mathbf{s}_r^H(k_i) + \widehat{\mathbf{R}}_0)^{-1}\mathbf{y}_r(k_i). \quad (11)$$

When the block size B is larger than B_C, the original estimate $\widehat{\mathbf{R}}_0$ in (11) can be replaced by $\widehat{\mathbf{R}}_m$, where $m =$

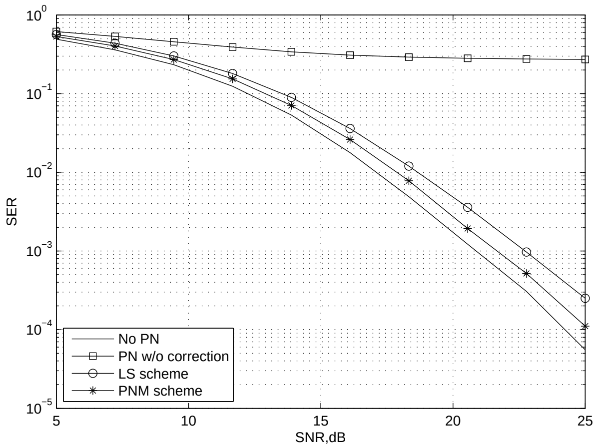

Fig. 2. SFBC, 2 transmit antennas and 2 receive antennas, $2\pi\beta T = 10^{-2}$, p=4, 16QAM

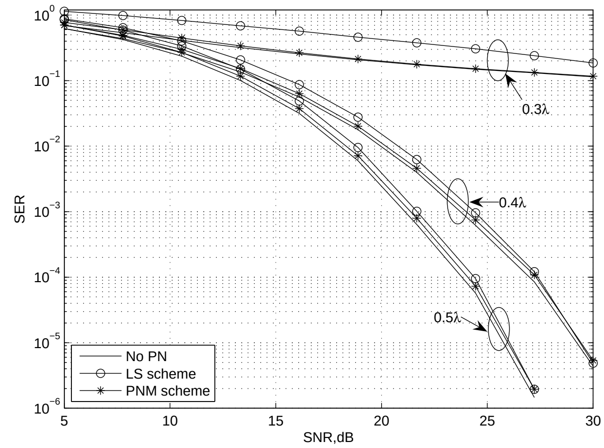

Fig. 3. MMSE V-BLAST, 8 transmit antennas and 12 receive antennas, $2\pi\beta T = 10^{-2}$, p=4, QPSK

$\lfloor r/B_C \rfloor \cdot B_C$ ($\lfloor\cdot\rfloor$ denotes the floor operation), which is obtained by applying (6) on null subcarriers in guard band of each OFDM symbols in WLANs [4] (Fig. 1b) . One may argue that such an estimation is not as accurate as (6) since there are only 12 null subcarriers used as guard band in one symbol in IEEE802.11a. However, since phase noise is highly correlated, the estimate of $\widehat{\mathbf{R}}_m$ can also be averaged by null subcarriers in guard band of Q symbols previous to symbol m.

Combining $\mathbf{U}_0(k)$ obtained during the preamble (Fig. 1a) and $\widehat{v_r}$ obtained from (11), the current channel estimate $\mathbf{U}_r(k) = \mathbf{U}_0(k)\widehat{v}_r = \mathbf{H}\widehat{(k)c_r}(0)$ eliminates the channel estimation error $c_0(0)$ and is ready to be used for various applications, such as space-frequency block coding (SFBC) [2] or in V-BLAST [1], which are both shown in next section.

IV. Numerical Results

Simulations are carried out for the IEEE 802.11a standard, with $N = 64$, $N_g = 16$, $p = 4$ in each OFDM symbol. Assume $B = 500$, and the length of cyclic prefix is larger than channel delay spread.

A spatially independent channel with SFBC[1] is implemented in Fig. 2 to facilitate a fair comparison with traditional SISO methods. One can see that with no correction, even for a small phase noise variance of 10^{-2}, there is an apparent error floor, which can not be tolerated. On the other hand, even with some channel estimation error induced by phase noise, the PNM scheme significantly reduces phase noise and its performance stays close to a no-phase-noise case. Also for comparison we apply the traditional SISO LS-based phase noise mitigation scheme [4] directly into this MIMO system with perfect channel estimation. The SER performance of the latter is still worse than the proposed algorithm under the same phase noise linewidth β since the PNM scheme not only enhances the signal but also suppresses the ICI plus noise.

Fig. 3 depicts the numerical performance for the spatially correlated V-BLAST system. We assume that the fading at transmitting and receiving ends has correlation matrices $\mathbf{R}_t$ and $\mathbf{R}_r$ respectively, where both follow Jake's model. It is further assumed that both ends are independent from each other, which allows one to write for the $\mathbf{R} = \mathbf{R}_t^T \otimes \mathbf{R}_r$ covariance matrix, where $\otimes$ denotes Kronecker product and T denotes the transpose. In the simulation, the receive antenna spacing at the base station is uniformly set as λ, where λ denotes the carrier wavelength. For different correlation levels, the transmit antenna spacing is set to be 0.5λ, 0.4λ and 0.3λ respectively. As shown in Fig. 3, since $\widehat{\mathbf{R}}_m$ captures the correlation information of ICI, which helps combat performance loss due to channel correlation, the PNM approach works well for a V-BLAST system with a spatially correlated channel and outperforms the LS scheme especially when the correlation is high.

[1]Similar results can be obtained for space time block coding (STBC).

V. Conclusions

Phase noise causes severe performance degradation in MIMO WLANs systems. In this letter, we first present the correlation function of the phase noise effect in OFDM systems, and then proposed a suitable phase noise mitigation scheme for any $M_T \times M_R$ MIMO WLANs system with channel estimation error and spatial correlation. It depicts better performance than the traditional method, and shows robustness to spatial correlation and SNR levels. Hence it can be used in various applications, such as V-BLAST and SFBC.

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