Phase Noise Mitigation for V-BLAST OFDM system

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***Abstract*— Future wireless mobile systems are required to transport multimedia traffics at much higher bit rates. Among all feasible implementations, V-BLAST OFDM system is one of the most promising candidates for such goal. However, similar to SISO-OFDM, V-BLAST OFDM suffers significant performance degradation due to the presence of phase noise. Although several phase noise mitigation method have been proposed in the literature for SISO channel, none has addressed this MIMO scenario, in which the performance degradation is more severe than SISO as it faces more complicated channel with little diversity gain.**

Moreover, in the original work of V-BLAST, a large number of antennas (8 transmit and 12 receive antennas) were proposed to obtain high bit rate. This may imply that multiple oscillators would be used to support such system. Most phase noise mitigation methods deal with one oscillator case, while in this paper, by considering the general case for any number of oscillators, we proposed a Phase Noise Mitigation (PNM) scheme, which efficiently reduces the effect of multiple phase noise generated by different oscillators in V-BLAST OFDM system. Although the algorithm developed is based on independent MIMO channel, its effectiveness in correlated channel is also numerically evaluated.

***Index Terms*— Phase Noise, MIMO, OFDM, V-BLAST**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an attractive modulation scheme used in broadband wireless systems which encounter large delay spread [1]. It has been adopted in a variety applications, e.g., digital subscriber line (DSL), digital video/audio broadcasting (DVB/DAB), IEEE 802.11a wireless local area network (HIPERLAN/2) [2], [3]. The basic principle of OFDM is to convert a frequency-selective channel into a parallel collection of frequency flat subchannels. Hence we can easily recover the signal by a one-tap equalizer on each flat subchannel. Since the different subcarriers' signal spectrum overlap in frequency, the available bandwidth is used very efficiently.

Multiple Input Multiple Output (MIMO) technique uses two main category: space-time coding [4] and spatial multiplexing [5]. In space time coding case, since the spatial rate of space-time codes is unity or less, these codes did not have any multiplexing gain with respect to a SISO (Single Input Single Output) channel, but, unlike a SISO channel, it ideally possesses a diversity order of $M_T M_R$. Therefore, these codes are excellent for improving the link quality by combating deep fades. On the other hand with spatial multiplexing, like V-BLAST (Vertical-Bell Lab Layered Space-Time), multiplexing gain of M_T is obtained when transmitting M_T independent data streams through the channel, whereas the diversity order reduces to $M_R - M_T + 1$. Depending on the frequency and bandwidth of the transmitted signals, each data stream takes its own paths. These paths by their very nature have some degree of orthogonality to other paths. Hence we do not need to deliberately make the data streams orthogonal from each antenna as it is required in space-time coding. In a sense, spatial multiplexing is similar to SDMA (Space Division Multiple Access), since both of them exploit the spacial separation existing in nature.

The combined V-BLAST OFDM scheme has an advantage over conventional systems due to its much improved system capacity introduced by MIMO technique, and its robustness to channel frequency selectivity due to OFDM technique [6]. However, similar to SISO-OFDM, V-BLAST OFDM suffers severe performance degradation due to the presence of phase noise [7], [8]. Although various phase noise correction methods were proposed for single phase noise SISO systems [9], [10], little has been done for MIMO case. Moreover, in V-BLAST, as the number of antennas is relatively large, multiple phase noise might be generated. In this paper, we considered a single user uplink scenario (usually we assume base station has high quality oscillators) and the worst multiple phase noise case: each antenna has its own oscillator and independent phase noise. We examined the effect of phase noise and proposed a phase noise mitigation (PNM) scheme based on Multiple Phase Noise Estimator (MPNE) for V-BLAST OFDM system, which in general can contain any number of oscillators and any number of antennas. Furthermore, the robustness of the proposed PNM to the correlated channel is also numerically examined.

This paper is organized as follows: In Section II, the V-BLAST OFDM system model is introduced. Section III specifies the effects of phase noise and then derives the proposed PNM scheme. Numerical results are presented in Section IV to show the advantages of the proposed scheme. Section V gives the conclusion.

II. SYSTEM MODEL

A. Notations and Assumptions

In this paper, normal letters indicate scalar quantities, boldface letters represent vectors, and boldface capitals indicate matrices. Moreover, we assume an uplink case where both the transmitter and receiver have a large number of antennas.

B. *V-BLAST MIMO-OFDM system setup*

For an arbitrary subcarrier k, the received OFDM signal vector $\mathbf{r}_k$ can be written as

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{s}_k + \mathbf{n}_k \qquad 0 \le k \le N-1 \tag{1}$$

where $\mathbf{s}_k = [s_1(k), s_2(k), ..., s_{M_T}(k)]^T$ denotes the transmitted signal, which is assumed to be independent and have identity power. $\mathbf{H}_k$ is a $M_R \times M_T$ matrix with element $\{H_{ij}(k)\}$ denotes the channel frequency response between jth transmit antenna and ith receiver antenna. We assume $E\left[|h_{ij}(k)|^2\right] = 1$. And $\mathbf{n}_k = [n_1(k), n_2(k), ..., n_{M_R}(k)]^T$ denotes the corresponding noise vector with zero mean and variance σ^2.

The ordered successive cancellation (OSUC) algorithm is usually applied in V-BLAST receivers. This provides improved performance at the cost of increased computational complexity. In this paper, the OSUC is combined with the MMSE algorithm to yield a performance superior to the ZF with OSUC combination. The MMSE receiver optimally suppresses both the interference and noise components, whereas the ZF receiver removes only the interference components. In the presence of noise, MMSE receiver minimizes the mean square error between the transmitted symbols and the estimates detected at the receiver. By modifying the ZF method[5], the MMSE algorithm for any specific subcarrier k is given by:

Initialization

$$\begin{aligned} i &\leftarrow 1 \\ \mathbf{r}_k^1 &= \mathbf{r}_k \\ \mathbf{G}_1 &= (\mathbf{H}_k^H \mathbf{H}_k + \sigma^2 I_{M_T})^{-1} \mathbf{H}_k^H \\ u_1 &= \arg\min_j \|(\mathbf{G}_i)_j\|^2 \end{aligned}$$

Recursion

$$\begin{aligned} \mathbf{w}_{u_i} &= (\mathbf{G}_i)_{u_i} \\ y_{u_i} &= \mathbf{w}_{u_i}^T \mathbf{r}_k^i \\ \hat{b}_{u_i} &= Q(y_{u_i}) \\ \mathbf{r}_k^{i+1} &= \mathbf{r}_k^i - \hat{b}_{u_i}(\mathbf{H}_k)_{u_i} \\ \mathbf{G}_{i+1} &= (\mathbf{H}_k^H \mathbf{H}_k + \sigma^2 I_{M_T})^{-1} \mathbf{H}_k^H \\ u_{i+1} &= \arg\min_{j \notin (u_1, ..., u_i)} \|(\mathbf{G}_{i+1})_j\|^2 \\ i &= i + 1 \end{aligned} \tag{2}$$

where i is an indicator of steps, $(\mathbf{G}_i)_j$ denotes the jth column of matrix $\mathbf{G}_i$. $Q(\cdot)$ denotes the quantization (slicing) operation appropriate to the constellation in use. σ^2 is the variance of i.i.d. complex Gaussian random noise with zero mean. It can be seen that the detection ordering is based on the SINR and u_ith transmitted symbol on subcarrier k has a detection order i.

III. PHASE NOISE MITIGATION FOR V-BLAST OFDM SYSTEM

A. *The Effects of Phase Noise*

The term phase noise is widely used for describing short term random frequency fluctuations of a signal. It is caused by both transmitter and receiver oscillators and can be described as a continuous Brownian motion process with zero mean and variance $2\pi\beta t$, where β denotes the phase noise linewidth [7], [11]. In spatial multiplexing, multiple phase noise might be applied. Therefore, the expression of (1) is subsequently modified to

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{C}_0 \mathbf{s}_k + \underbrace{\sum_{n=0, n\neq k}^{N-1} \mathbf{H}_n \mathbf{C}_{k-n} \mathbf{s}_n}_{\mathbf{ICI}_k} + \mathbf{n}_k \tag{3}$$

where

$$\begin{aligned} \mathbf{C}_k &= diag[c_1(k), c_2(k), ..., c_{M_T}(k)] \\ c_m(k) &= \frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi nk + j\phi_m(n)} \qquad 1 \le m \le M_T \end{aligned}$$

$\phi_m(n)$ denotes the phase noise of antenna m at discrete time n. The variance of $\phi_m(n)$ is given by $2\pi\beta_m Tn/N$, where β_m and T denote the phase noise linewidth of antenna m and the OFDM symbol duration (assume T is the same for different datastreams) respectively. We assume ϕ_m is independent with ϕ_n, when $m \neq n$, hence c_m is independent to c_n. It is noticed from (3) that phase noise contributes to

1. Common Phase Error (CPE), indicated by $\mathbf{C}_0$, which causes the rotation of the desired signals. It's invariant within one OFDM symbol;

2. Intercarrier Interference (ICI), indicated by the term $\mathbf{ICI}_k$, which causes interference on the desired signals.

B. *Phase Noise Mitigation (PNM)*

We assume a block fading channel (channel is constant for several OFDM symbols) and channel state information (CSI) is obtained by using the preamble[1]. Note that $\mathbf{C}_0$ varies from symbol to symbol due to the random phase noise, and therefore needs to be estimated per symbol basis, hence it prevents us from combining the estimation of channel and phase noise together. As such, a per symbol based phase noise estimation method has to be implemented.

For each received pilot subcarrier (the pilot set of each antenna is assumed to be $\mathbf{P} = \{k_1, k_2, ..., k_{N_p}\}$), (3) is equivalent to

$$\mathbf{r}_{k_i} = \mathbf{H}_{k_i} \mathbf{S}_{k_i} \mathbf{c}_0 + \overline{\mathbf{n}}_{k_i} \quad 1 \le i \le N_p \tag{4}$$

where

$$\begin{aligned} \mathbf{c}_k &= [c_1(k), c_2(k), ..., c_{M_T}(k)]^T \\ \mathbf{S}_{k_i} &= diag[s_1(k), s_2(k), ..., s_{M_T}(k)] \\ \overline{\mathbf{n}}_{k_i} &= \sum_{n=0, n\neq k_i}^{N-1} \mathbf{H}_n \mathbf{S}_n \mathbf{c}_{k_i - n} + \mathbf{n}_{k_i} \end{aligned}$$

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[1] Since a more sophiscated procedure can be applied in the channel estimation during preamble, we assume the presence of phase noise will not affect the accuracy of CSI.

The MMSE-based phase noise mitigation method requires to find an appropriate coefficient vector $\mathbf{W}$ that minimizes $E\left[\left\|\mathbf{c}_0 - \mathbf{W}^H\mathbf{r}_{k_i}\right\|^2\right]$. With some algebraic manipulation, it is readily shown that the optimal coefficient is given by

$$\mathbf{W} = (\mathbf{H}_{k_i}\mathbf{S}_{k_i}\mathbf{R}_{\mathbf{c}_0}\mathbf{S}_{k_i}^H\mathbf{H}_{k_i}^H + \mathbf{R}_{\bar{\mathbf{n}}_{k_i}})^{-1}\mathbf{H}_{k_i}\mathbf{S}_{k_i} \tag{5}$$

which gives rise to the MMSE estimate of CPE

$$\mathbf{e}_0^{MMSE} = \mathbf{W}^H\mathbf{r}_{k_i} = \mathbf{S}_{k_i}^H\mathbf{H}_{k_i}^H(\mathbf{H}_{k_i}\mathbf{S}_{k_i}\mathbf{R}_{\mathbf{c}_0}\mathbf{S}_{k_i}^H\mathbf{H}_{k_i}^H+\mathbf{R}_{\bar{\mathbf{n}}_{k_i}})^{-1}\mathbf{r}_{k_i} \tag{6}$$

with

$$\begin{aligned}\mathbf{R}_{\mathbf{c}_0} &= E\left[\mathbf{c}_0\mathbf{c}_0^H\right] \\ &= diag[E(|c_1(0)|^2),\ldots,E(|c_{M_T}(0)|^2)] && (7)\\ \mathbf{R}_{\bar{\mathbf{n}}_{k_i}} &= E\left[\bar{\mathbf{n}}_{k_i}\bar{\mathbf{n}}_{k_i}^H\right] = E\left(\mathbf{ICI}_k\mathbf{ICI}_k^H\right) + \sigma_n^2\mathbf{I}_{M_R} && (8)\end{aligned}$$

where $\mathbf{ICI}_k = \left(\sum_{n=0,n\neq k_i}^{N-1}\mathbf{H}_n\mathbf{S}_n\mathbf{c}_{k_i-n}\right)$, and $\mathbf{I}_{M_R}$ denotes the $M_R \times M_R$ identity matrix.

As we can see from (7) and (8), the MMSE-based estimator is almost practically intractable. Moreover, even if we can get all the information we need for the estimator in (6), it is still too complicated to be implemented. Fortunately, since the oscillators at the transmitter (user) should have similar properties (i.e., same type oscillators), we approximate the phase noise linewidth of each oscillator as $\beta = \frac{1}{M_T}\sum_{i=1}^{M_T}\beta_i$, where β_i denotes the phase noise linewidth of ith transmit antenna. Inspired by [7], [9], we have

$$\begin{aligned}\mathbf{R}_{\mathbf{c}_0} &\approx \left(1 - \frac{\pi\beta T}{3}\right)\mathbf{I}_{M_R}\\ \mathbf{R}_{\bar{\mathbf{n}}_{k_i}} &\approx \left(\frac{M_T\pi\beta T E_X}{3} + \sigma_n^2\right)\mathbf{I}_{M_R}\end{aligned}$$

By using these approximations, the linear Multiple Phase Noise Estimator (MPNE) becomes:

$$\begin{aligned}\mathbf{e}_0^{MPNE} &= \mathbf{S}_{k_i}^H\mathbf{H}_{k_i}^H(\mathbf{H}_{k_i}\mathbf{S}_{k_i}\mathbf{S}_{k_i}^H\mathbf{H}_{k_i}^H \\ &\quad + \left(\frac{M_T\pi\beta T E_X + 3\sigma_n^2}{3 - \pi\beta T}\right)\mathbf{I}_{M_R})^{-1}\mathbf{r}_{k_i} && (9)\end{aligned}$$

where β can be easily obtained from the specification of the oscillators or estimated by the unused subcarrier in OFDM system[9]. Note that, the structure of equation (9) shows that MPNE can be used for any number of oscillators and any number of antennas.

As $\mathbf{e}_0$ is invariant within one OFDM symbol, it is possible to average CPE by more available pilots:

$$\mathbf{b} = \frac{1}{N_p}\sum_{k\in P}\mathbf{e}_0^{MPNE}$$

With the estimate of $\mathbf{b}$, the corresponding Eqn (2) in V-BLAST algorithm is modified to: $y_{u_i} = \mathbf{w}_{u_i}^T\mathbf{r}_k^i/\mathbf{b}_{u_i}$

IV. NUMERICAL RESULTS

Numerical results given in this section demonstrate the effectiveness of the proposed method. An uncoded (since we focus on symbol error rate (SER)) OFDM system is chosen with symbol size N=64. An MIMO channel with 8 transmit antennas and 12 receive antennas is used as [5]. A frequency selective Rayleigh fading channel between any antenna pairs is assumed in the simulation. A cyclic prefix, larger than channel maximum delay spread, is added to the head of each OFDM symbol in order to eliminate ISI caused by channel frequency selectivity. Due to IEEE 802.11a standard, 4 pilots are assigned to each data symbol.

Fig.1 shows the SER performance of the proposed scheme in comparison to no-phase-noise and phase-noise-without-correction cases. The performance of an MMSE estimator with perfect knowledge is also added as reference (we assume $|\beta_i - \beta| < 10\%\beta$). As shown in this figure, even for small phase noise variance of 10^{-2} with no correction, there is an apparent error floor, which must be mitigated. On the other hand, PNM results in performance closed to that perfect MMSE estimator which is the best estimator among all linear ones.

The robustness of the proposed algorithm to different phase noise variance levels is shown in Fig.2. It demonstrates that the proposed scheme efficiently correct the phase noise error if the variance is less than 10^{-1}. It is also shown that the PNM is not needed when the variance is less than 10^{-4}. Note that, this phase noise variance working range $\left[10^{-4}, 10^{-1}\right]$ fits the practical consideration of phase noise levels in any OFDM systems.

In practice, the MIMO channel is always correlated. Therefore, it would be more adequate to make our PNM based on correlated channel matrix $\mathbf{H}$. Instead, for simplicity, we keep our estimation based on uncorrelated channel as in (9), but examine the performance of PNM with a correlated $\mathbf{H}$. We assume that the fading at transmitting and receiving ends is correlated with correlation matrices $\mathbf{R}_t$ and $\mathbf{R}_r$ respectively, where both of them follow Jake's model (i.e., $\mathbf{R}(i,j) = J_0\left(\frac{2\pi|i-j|d}{\lambda}\right)$, where d denotes the antenna spacing, J_0 denotes the zeroth-order Bessel function of the first kind). It is further assumed that both ends are independent from each other, which allows one to write for the $\mathbf{R} = \mathbf{R}_t^T \otimes \mathbf{R}_r$ covariance matrix , where $\otimes$ denotes the Kronecker product and T denotes the transpose. In the simulation, the receive antenna spacing in base station is uniformly set as λ, where λ denotes the carrier wavelength, and the transmit antenna spacing is set to be 0.5λ, 0.4λ and 0.3λ respectively.

As shown in Fig.3, despite using uncorrelated H to find the estimate of the CPE, PNM efficiently corrects the phase noise error even in a highly correlated channel, though the gap between MPNE and no-phase-noise case becomes larger compared with Fig.1 due to the absence of the correlation information in the proposed algorithm.

V. CONCLUSIONS

V-BLAST OFDM system suffers severe performance degradation in the presence of phase noise. This scenario is more severe than a SISO channel due to the lack of diversity gain.

In this paper, we proposed a new low-complexity phase noise correction method for V-BLAST OFDM system. By using only 4 pilots, the proposed PNM algorithm efficiently mitigates the phase noise effect, such that the resulting performance comes close to the no phase noise scenario, and is

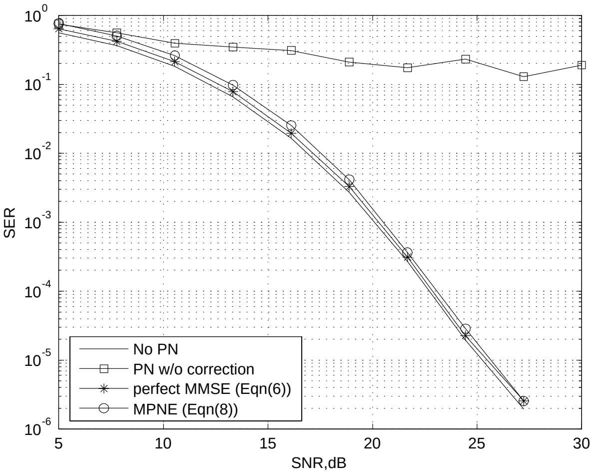

Fig. 1. MMSE V-BLAST system with 8 transmit antennas and 12 receive antennas, QPSK, pn variance is 10^{-2}

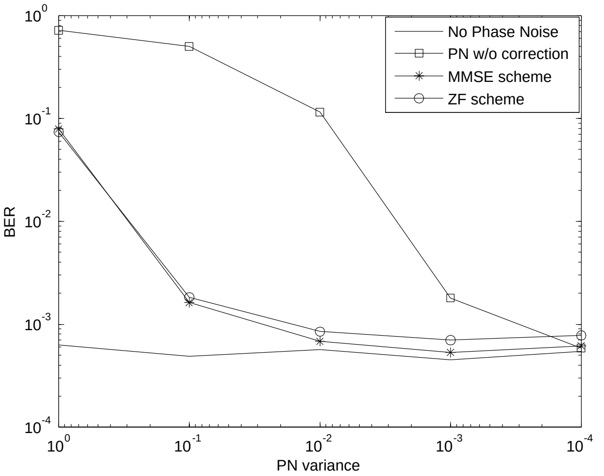

Fig. 2. MMSE V-BLAST system with 8 transmit antennas and 12 receive antennas, QPSK, SNR=20dB

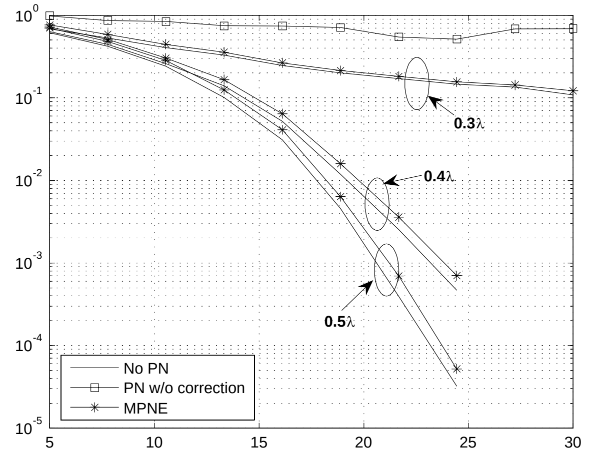

Fig. 3. MMSE V-BLAST system with 8 transmit antennas and 12 receive antennas, QPSK, correlated channel

a general form for any number of antennas and oscillators. Moreover, the algorithm is robust to different phase noise variance levels. The PNM which is based on uncorrelated channel is also suitable for the correlated channel environment, although the gap between the performance of the proposed algorithm and no phase noise case becomes larger when compared with the case in which H is uncorrelated. It is also expected that the aforementioned gap will be reduced when our PNM will take into consideration the correlation in channel.

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