Math 331-004 Practice Problems for the Final Exam

1. Diffusion Equation in a finite box.

   PDE: $u_t = k(u_{xx} + u_{yy})$, for $0 < x < L$, $0 < y < H$, $t > 0$.

   BC: $u_y(x, 0) = 0$, $u_y(x, H) = 0$, $u(0, y) = 0$, $u(L, y) = 0$

   IC: $u(x, y, 0) = f(x, y)$.

   Simplify your answer in the case that the initial condition has no y dependence, i.e. $f(x, y) = g(x)$. Does the solution in this case look familiar? Explain.

2. Diffusion Equation on a semi-infinite strip.

   PDE: $u_t = k(u_{xx} + u_{yy})$, for $0 < x < +\infty$, $0 < y < +H$, $t > 0$.

   BC: $u_y(x, 0, t) = 0$, $u_y(x, H, t) = 0$, $u(0, y) = 0$.

   IC: $u(x, y, 0) = f(x, y)$.

   Try to solve in two ways: (1) straight separation of variables (2) cos or sin transform in x followed by separation of variables.

   Next change the boundary condition at $x = 0$ to $u_x(0, y) = 0$ and again try to solve either by straight separation of variables or by a transform followed by separation of variables.

3. Diffusion Equation on a quarter-disk.

   PDE: $u_t = \nabla^2 u$ on $0 \le r < a$, $0 < \theta/2$.

   BC: $u(a, \theta) = 0$ for $0 \le \theta \le \pi/2$, $u(r, 0) = 0$, $u(r, \pi/2) = 0$

   IC: $u(r, \theta) = f(r, \theta)$.

   Solve this problem by separation of variables. Be sure to complete the problem by obtaining formulas for the expansion coefficients. What is the long time behavior?

4. The Nonhomogeneous Wave Equation.

   PDE: $u_{tt} - c^2 u_{xx} = p(x, y)$, $-\infty < x < +\infty$, $t > 0$.

   IC: $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ for $-\infty < x < +\infty$.

   Solve this problem using the Fourier Transform.