Two dimensional Solitons in shallow water

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Abstract

A soliton in a shallow water was first recognized by John Scott Russell in August 1834. He noted in his first paper (1838) on the subject that

> *I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the cahnnel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed .....*

He also said in his book (1865) that

> *This is a most beautiful and extraordinary phenomenon: the first day I saw it was the happiest day of my life. Nobody has ever had the good fortune to see it before or, at all events, to know what it meant. It is now known as the solitary wave of translation. No one before had fancied a solitary wave as a possible thing.*

This solitary wave is known as a "soliton", which may be identified as an exact solution of the Korteweg-de Vries (KdV) equation discovered in 1895. The equation describes one-dimensional wave propagation such as beach waves parallel to the coast line or waves in narrow canal. In 1970, Kadomtsev and Petviashvili (both plasma physicists) proposed a two-dimesional extension of the KdV equation to study the transverse stability of KdV soltons. Their equation is now known as the KP equation, and this equation is a most fundamental and beautiful equation in the research area of integrable systems. The KP equation also appears so many different branches of Physics as well as Mathematics.

In this talk, I will explain some of the exact solutions of the KP equation, and how those solutions can describe two-dimensional wave patterns observed in shallow water experiments. I hope I will convince you that *this (two-dimensional wave pattern) is a most beautiful and extraordinary phenomena* of two-dimensional shallow water waves, and try to make the time (of the talk) the *happiest* moment of your day. The talk will be elementary, and has a lot of pictures and movies.

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