Stability Analysis of a Flow Control System for a Combined Input-Crosspoint Buffered Packet Switch

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Abstract *– Credit-based flow control is popularly used in buffered crossbar switches. However, it is difficult to analyze this flow control scheme when the transmission delay is non-negligible. In this paper, we propose an AQM-based flow control, by which we are able to investigate the sensitivity of the stability margin with respect to key switch parameters such as crosspoint buffer size and round-trip time (RTT). In addition, we design an input shaper to be added onto a proportional controller and a short response time is obtained.*

Keywords *– Buffered crossbar, AQM, Stability, Feedback, Delay*

I. INTRODUCTION

Combined input-crosspoint buffered (CICB) switches are an alternative to input-queued (IQ) packet switches to provide high-performance switching and to relax arbitration timing for packet switches with high-speed ports. CICB switches use time efficiently because input and output port selections are performed separately. CICB switches with FIFOs as input queues have been used to reduce the crosspoint-buffer size and to reduce packet loss ratio. However, a CICB switch with input FIFOs may have the throughput limited by the head-of-line blocking phenomena. We refer to a VOQ-CICB switch as a CICB switch for the sake of brevity in the remainder of this paper.

Plenty of research on congestion-avoidance for routers have been done recently. These works are mainly used to support TCP flows. The basic operating principle of an active queue management (AQM) mechanism is to provide early congestion notification to the sources in order to avoid packet loss due to the buffer overflow in the router. With this model, control theory can be applied to analyze and design AQM schemes. This paper focuses on applying this model to flow control in a CICB switch.

Credit-based flow control is popularly used in CICB switches [1]. Some previous works with credit-based flow control assume a negligible transmission delay in switches [3] in their arbitration schemes. As the buffered crossbar switch can be physically located far from the input ports in a multi-rack router implementation, actual round trip times (RTTs) can be non-negligible. The non-negligible round trip delays was studied in [4], [5]. However with credit-based flow control, it is difficult to furtherly investigate the relationship between transmission delay and crosspoint buffer size and its effect on the stability of the switch. In this paper, we designed a feedback controller for the flow control in a CICB switch, by which we analyzed the relationship between the round-trip time and crosspoint buffer size and their effect on the stability of the switch.

This paper is organized as follows. Section II briefly introduces the AQM-based flow control model in a CICB switch and presents the analysis on the relationship between the system stability and crosspoint buffer size as well as round-trip time. Section III discusses the design of an input shaper to obtain short transient response time of a flow control system. Section IV presents the conclusions.

II. FLOW CONTROL MODEL AND STABILITY ANALYSIS

Figure 1 shows a buffered crossbar switch with M inputs and outputs. In this switch model, there are M VOQs at each input. A VOQ at input i that stores cells for output j is denoted as $VOQ_{i,j}$. A crosspoint element in the buffered crossbar that connects input port i, where $0 \leq i \leq M-1$, to output port j, where $0 \leq j \leq M-1$, is denoted as $XP_{i,j}$. The buffer at $XP_{i,j}$ is denoted as $XPB_{i,j}$, and it is considered of k-cell size,where $k \geq 1$.

The observations on the CICB switch show us that each VOQ and its corresponding XPB comprise a closed loop. In order to avoid overflow in XPB, feedback is needed to tell the VOQ to control the sending data rate. Based on the similar concept of TCP windowing, we use a frame size to control the VOQ's sending rate. The frame size is the amount of packets transferring into the switch fabric [6],[7].

In TCP, the congestion window size $W(t)$ is increased by one every round trip time if no congestion is detected, and is halfed upon a congestion detection. This additive-increase multiplicative-decrease (AIMD) behavior of TCP

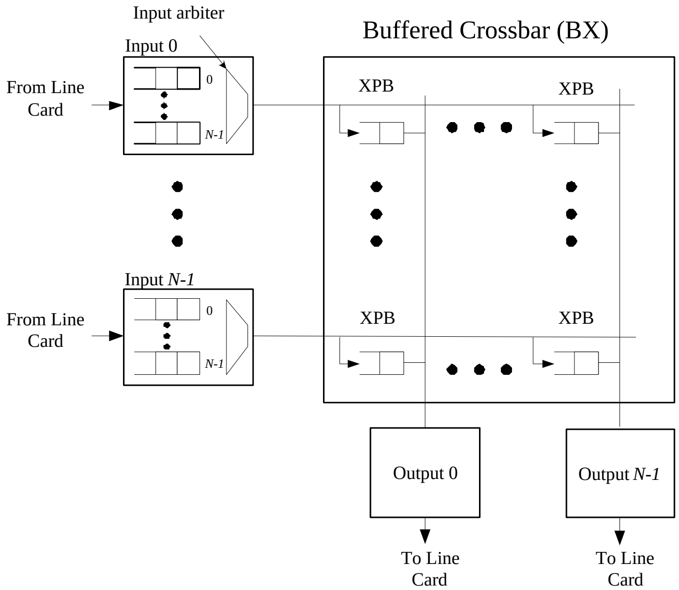

Fig. 1. Combine input-crosspoint buffered crossbar switch.

has been modeled by the equation (1) and (2):

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2R(t-R(t))}p(t-R(t)) \tag{1}$$

In a network topology of N homogeneous TCP sources and one router, the equation for the queue dynamics in [2] is given as

$$\dot{q}(t) = \frac{W(t)}{R(t)}N(t) - C. \tag{2}$$

After linearization of (1) and (2), we have the transfer function of the target plant.

$$P(s) = \frac{\frac{C^2}{2N}}{(s+\frac{2N}{R_0^2 C})(s+\frac{1}{R_0})}$$

The following terms are as defined in [2] :

W is the average VOQ frame size (packets)

q is the average crosspoint buffer size (packets)

$R_0(t)$ is the round-trip time (secs)

C is the link capacity (packets/sec)

N is the load factor.

However in a CICB switch, some parameters need to be adapted. Since in each closed loop of VOQ and its corresponding XPB there is only one session, so the load factor is N=1. Also, the link capacity C is set to $\frac{C}{M}\frac{k}{R_0}$, where M is the switch size. The minimum rate that one crosspoint buffer supports for uniform traffic is $\frac{C}{M}\frac{k}{R_0}$. Therefore, we discuss the feedback control on the following equations:

$$P(s) = \frac{\frac{C^2 k^2}{2M^2 R_0^2}}{(s+\frac{2M}{R_0 Ck})(s+\frac{1}{R_0})} \tag{3}$$

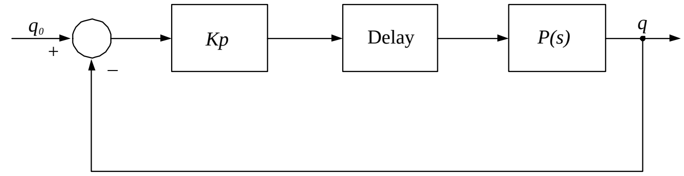

Fig. 2. Block Diagram of feedback control in a VOQ-XPB closed loop.

Instead of using the RED (Random Early Detection) [8] mechanism, we use P (Proportional) control for the flow control in a CICB switch (as Figure 2 shows). The feedback signal is the regulated output (crosspoint buffer occupancy) multiplied by a gain factor K_p.

The nominal loop transfer function in the proportional controller case is

$$L(s) = \frac{K_p \frac{C^2 k^2}{2M^2 R_0^2} e^{-sR_0}}{(s+\frac{2M}{R_0 Ck})(s+\frac{1}{R_0})} \tag{4}$$

We can take the loop's unity-gain crossover frequency as the geometric mean of corner frequency

$$w_g = \sqrt{\frac{2M}{R_0^2 Ck}} \tag{5}$$

The phase margin (PM) is

$$PM = 180 - \arctan w_g R_0 - \arctan \frac{w_g R_0 Ck}{2M} - \frac{180}{\pi} w_g R_0. \tag{6}$$

Let's consider the following example.

Example 1: Consider the following setup in a CICB switch. $C = 3750\ packets/s$, switch size $M = 32$, round-trip time R_0=0.0246 s, and the value of k is the crosspoint buffer size in time slots.

TABLE I
PHASE MARGIN AND THE CROSSPOINT BUFFER SIZE

k	k/RTT	PM
10	$\frac{1}{10}$	-54.85°
50	$\frac{1}{2}$	25.23°
100	1	44.19°
1000	10	75.52°

TABLE II
PHASE MARGIN AND THE CROSSPOINT BUFFER SIZE

k	k/RTT	PM
10	$\frac{1}{10}$	64.39°
50	$\frac{1}{2}$	78.56°
100	1	81.90°
1000	10	87.43°

Since $C = 3750$ $packets/s$, one time slot is 0.267 ms. For example, a buffer size is 10-packet long, then k=10 $\times$0.267ms= 2.67ms.

Assume that the round-trip time is fixed. When the crosspoint buffer size increases, the system's phase margin increases as Table I shows. However, when crosspoint buffer size decreases to a certain ratio, the system becomes unstable.

Now we consider the case when the maximum rate that one crosspoint buffer supports for uniform traffic is $C\frac{k}{R_0}$. We discuss the feedback control on the following equations:

$$P(s) = \frac{\frac{C^2k^2}{2R_0^2}}{(s + \frac{2}{R_0Ck})(s + \frac{1}{R_0})} \tag{7}$$

The nominal loop transfer function in the proportional controller case is

$$L(s) = \frac{K_p\frac{C^2k^2}{2R_0^2}e^{-sR_0}}{(s + \frac{2}{R_0Ck})(s + \frac{1}{R_0})} \tag{8}$$

The phase margin (PM) is

$$PM = 180 - \arctan w_gR_0 - \arctan\frac{w_gR_0Ck}{2} - \frac{180}{\pi}w_gR_0. \tag{9}$$

Assume that the round-trip time is fixed. When the crosspoint buffer size increases, the system's phase margin increases as Table II shows.

As we know, to support long RTTs in a buffered crossbar switch, the crosspoint-buffer size need to be large enough, such that up to RTT cells can be buffered. However, the memory amount that can be allocated in a chip is limited. This is the motivation to investigate the relationship between the flow control system's stability and crosspoint buffer size as well as the round trip time. It is interesting to know the ratio of crosspoint buffer size k and round-trip time RTT by which phase margin of the system becomes negative and thus the flow control system becomes unstable.

III. PROPORTIONAL CONTROL WITH AN INPUT SHAPER

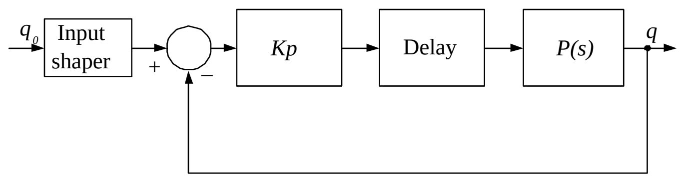

Fig. 3. Block Diagram of feedback control with input shaper in a VOQ-XPB closed loop.

Except for the robustness to variations in model parameters such as round-trip time and XPB size, the proportional controller must also have an acceptable transient response such that the crosspoint buffer occupancy can converge to a target value q_0 in a short time.

In order to obtain an acceptable transient response, we add an input shaper to the closed loop system. Compared with RED, PI (Proportional-Integral) [2], and PID (Proportional-Integral-Differential) [9], our P control with input shaping can speed up the responsiveness of the flow control system. With input shaping, we can avoid the long regulating time of PI control and the tuning difficulty of RED as well as the complexity of PID control.

Input shaper is a feed-forward pole-zero cancellation method. Ideally the input shaper uses its zeros to cancel the poles of a target system, a good performance is obtained.

As (3) and Figure 4 shows, the system has two poles. Because it is an underdamped system, there are some oscillations before the crosspoint buffer occupancy converges to q_0.

The input shaper might be designed as a step function. In our case, the step function is used to set up the value of q_0 and also the time to apply q_0 into the closed-loop system. Each step function has a step response. We use two step functions to cancel the two-pole system's overshoots. The key point is how to put these two step functions together.

Example 2: The transient response of two-pole system is shown in Figure 4. The maximum overshoot value q_{max} is measured as $q_{max} = 15.5$, the steady state value of queue occupancy q_{ss} is measured as $q_{ss} = 10$.

With the following equations, we can obtain the step value and step time for each step function.

$$V = \frac{q_{max} - q_{ss}}{q_{ss}} = 0.55 \tag{10}$$

The step value for the first step function is:

$$A_1 = \frac{1}{1+V} \tag{11}$$

The step value for the second step function is:

$$A_2 = 1 - A_1 \tag{12}$$

$$exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}) = V \tag{13}$$

From (13), the damping factor ζ is obtained as 0.1839. Since ζ is less than 1, the system is defined as an underdamping system. Finally the time interval between the first step function and the second step function is obtained.

$$\Delta T = \frac{\pi}{\omega\sqrt{1-\zeta^2}} = 1.035 \tag{14}$$

In summary, if the target crosspoint buffer occupancy is q_0, with a design of an input shaper, we first generate a step function whose step value is $q_0 \times A_1$ and its step time is 0. The step value of the second step function is $q_0 \times A_2$ and its step time is ΔT. Now, we get the input shaper. Before q_0 is input onto the proportional control system (Figure 2), it is shaped by the two step functions. That is why we call the two step functions as an input shaper. As it is shown in Figure 5, with an input shaper, the transient response time can be improved by almost 40%.

IV. CONCLUSION

In this paper we designed an AQM-based flow control for CICB switches. Our contribution is two-folded. First, we investigated the sensitivity of the stability margin with respect to key switch parameters such as crosspoint buffer size and round-trip time. Second, we improved the system's transient response time by almost 40 % with adding an input shaper to the closed-loop system.

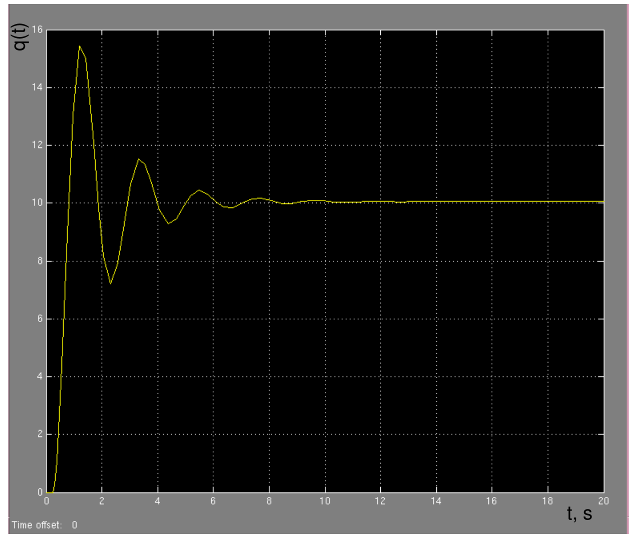

Fig. 4. Simulation result on feedback control without input shaper in a VOQ-XPB closed loop.

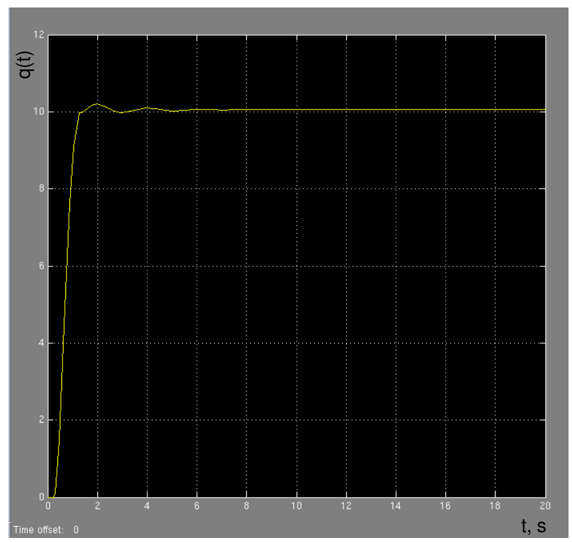

Fig. 5. Simulation result on feedback control with input shaper in a VOQ-XPB closed loop

References

[1] H. T. Kung, K. Chang, "Receiver-Oriented Adaptive Buffer Allocation in Credit-Based Flow Control for ATM Networks," *IEEE INFOCOM'95*, Vol. 1, pp. 239-252.

[2] C. V. Hollot, V. Misra, D. Towsley and W. Gong, "Analysis and Design of Controllers for AQM Routers Supporting TCP Flows," *IEEE Transactions on Automatic Control*, Vol. 47, No. 6, June 2002, pp. 945-959

[3] R. Rojas-Cessa, E. Oki, Z. Jing, and H. J. Chao "CIXB-1: Combined Input-One-Cell-Crosspoint Buffered Switch," *IEEE HPSR 2001*, pp. 324-329, May, 2001.

[4] R. Luijten, C. Minkenberg, and M. Gusat, "Reducing Memory Size in Buffered Crossbar with Large Internal Flow Control Latency, " *Proc. IEEE Global Telecommunications Conference*

2003, Vol. 3, pp. 3683-3687.

[5] F. Gramsamer, M. Gusat, and R. Luijten, "Optimizing Flow Control for Buffered Switches," *Proc. IEEE International Conference on Computer Communications and Networks 2002* , pp.438-443.

[6] A. Bianco, M. Franceschinis, et. al.,"Frame-Based Matching Algorithms for Input-Queued Switches," *Proc. IEEE HPSR 2002* May 2002, pp. 69-76.

[7] R. Rojas-Cessa, E. Oki, "Round-robin Selection with Adaptable-Size Frame in a Combined Input-Crosspoint Buffered Switch," *IEEE Communications Letter*, Vol. 7, No. 11, Nov., 2003, pp. 555-557.

[8] S. Floyd, V. Jacobson, "Random Early Detection Gateways for Congestion Avoidance," *IEEE/ACM Trans. Networking* , Vol. 1, pp. 397-413, Aug. 1993.

[9] F. Ren, C. Lin, "Speed up the Responsiveness of Active Queue Management System," *IEICE Trans. Commun.,* Vol. E86-B, No. 2, pp. 630-636, 2003.

[10] N. C. Singer, W. P. Seering, "Preshaping Command Inputs to Reduce System Vibrations," *ASME Journal of Dynamic Systems, Measurement, and Control,* Vol. 112, No. 1,pp. 76-82, 1990.