Opportunistic Relaying in Wireless Networks

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Abstract*— We analyze fading interference relay networks with n ad hoc nodes and m half-duplex relays, all operating in the same frequency band. This setup has attracted significant attention and several schemes have been reported in the literature. However, most of the proposed solutions require either centrally coordinated scheduling or detailed channel state information (CSI) at the source nodes. We propose an opportunistic relaying scheme that alleviates these limitations, without sacrificing the system throughput scaling in the regime of large n. The scheme entails a two-hop communication protocol, where sources communicate with destinations only through half-duplex relays. The key idea is to schedule at each hop only the subset of nodes that can benefit from *multiuser diversity*. To select the source and destination nodes for each hop, only integer-value CSI feedback is required from the receivers (relays for the first hop, and destination nodes for the second hop). Moreover, the relays operate in a completely distributed fashion, with no cooperation. For the case when n is large and m is fixed, we show that the proposed scheme achieves a system throughput of $m/2$ bits/sec/Hz. In contrast, the upper bound of $(m/2)\log\log n$ bits/sec/Hz is achievable with only more demanding CSI assumptions and full cooperation between the relays. Furthermore, we show that (by allowing m to grow as a function of n, and then finding the optimal order of m that maximizes throughput) the system throughput of the proposed scheme scales as $m = \Theta(\log n)$.

I. Introduction

The design and analysis of transmission protocols for wireless networks have recently attracted a lot of interest in the research community. Examples include multihop schemes [1], [2], and two-hop relaying schemes [3], [4], just to name a few. While these works have made great strides towards understanding wireless ad hoc network capacity, implementation requires either central coordination between nodes [1], [2] or some level of CSI awarness (channel amplitude and/or phase) at the transmitter side [3], [4]. In a large system, obtaining such level of CSI, especially at the transmit side, may not be practical. Likewise, the cooperation between wireless relays does not come for free, since the overhead to set up the cooperation may drastically reduce the useful throughput [5].

A. Main Contributions and Related Work

The main contributions of this work can be summarized as follows:

- A two-hop opportunistic relaying scheme in fading channels is proposed and analyzed. The scheme has the following features.
  - — The scheme operates in a completely decentralized fashion. No cooperation between relays is assumed or required.
  - — Only practical CSI requirements are imposed. At each hop, receivers are assumed to have knowledge of incoming channel realizations; transmitters have access to only integer-value CSI via a low-rate feedback from the receivers.
- The throughput of the proposed scheme is characterized.
  - — In the regime where the number of nodes, n, is large and the number of relay nodes, m, is fixed, the proposed scheme is shown to achieve a system throughput of $m/2$ bits/sec/Hz. Contrast that with the scaling upper bound $(m/2)\log\log n$, achievable only with more demanding cooperation and CSI assumptions.
  - — We show that m can grow (as a function of n) as fast as $\Theta(\log n)$, while still guaranteeing the linear throughput scaling in m. The linearity breaks down if m grows beyond $\Theta(\log n)$. Therefore, the throughput scaling of the proposed opportunistic relaying scheme is given by $\Theta(\log n)$.

The key idea behind the proposed scheme is to schedule at each hop only the subset of nodes, which can benefit from *multiuser diversity gain*. The concept of multiuser diversity gain was originally studied in the context of cellular systems [6], [7]. At any given time, in a system with a large number of independently fading users, with a high probability there is a strong user to whom resources (bandwidth and power) can be allocated. By doing so, the system performance can be significantly boosted. The concept is by now well understood in the cellular community and has been adopted in 3G cellular systems and other emerging wireless standards. However, to the best of our knowledge, it has received less attention for wireless access networks, with some exceptions [8], where the potential of opportunistic relaying is reported in the framework of a diversity-multiplexing trade-off. In this work, we show another example where multiuser diversity can be exploited in a wireless access network, with focus on simplified network operations and analysis of throughput scaling. The opportunistic scheme proposed here is in the spirit of [9], where mobility of the ad hoc nodes is assumed, and the random channel gains due to distance-dependent path-loss model is exploited in scheduling.

This work was supported in part by the National Science Foundation under Grant CNS-0626611.

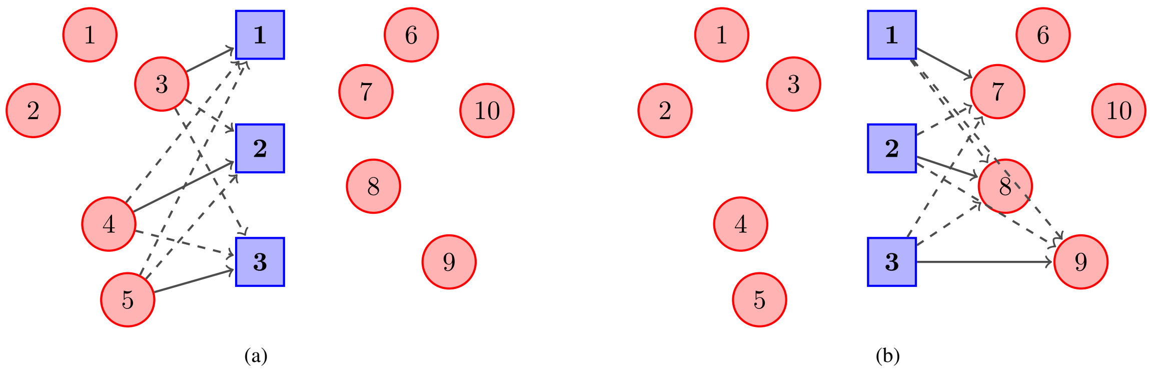

Fig. 1. Illustration of wireless network with $n = 10$ ad hoc nodes (in red circle) and $m = 3$ relay nodes (in blue square). (a) In Phase 1 (source nodes transmitting to relays), nodes $\{3, 4, 5\}$ are scheduled, i.e., $\mathcal{K} = \{3, 4, 5\}$. (b) In Phase 2 (relays transmitting to destination nodes), the signal corresponding to nodes $\{7, 8, 9\}$ are transmitted by relays. In the figures, the solid lines indicate the scheduled sender–receiver mappings. For example, in (a) source node 3 is scheduled, and accordingly will be decoded, by relay 1.

B. Organization of the Paper

The rest of the paper is organized as follows. The system model and the proposed two-phase relay protocol are introduced in Section II. Section III characterizes the system throughput in the regime where n is large and m fixed. The throughput scaling of the proposed scheme is evaluated in Section IV. Finally, Section V, contains a discussion and conclusion.

Notation: $|\mathcal{X}|$ stands for the cardinality of the set $\mathcal{X}$. $\log(\cdot)$ indicates the natural logarithm. We write $X \sim \text{Exp}(1)$ as random variable X follows the standard exponentially distribution with probability density function (pdf) given by $f_X(x) = \exp(-x)$, $x > 0$. Indicator function is denoted as $\mathbf{1}(\cdot)$ and we use "$\chi^2(2p)$" to denote a chi-square random variable with $2p$ degrees of freedom. Let $f(n)$ and $g(n)$ be two positive functions. We write $f(n) = \Theta(g(n))$ if $f(n)$ grows exactly at the same asymptotic rate as $g(n)$. Strictly, $f(n) = \Theta(g(n))$ if there exist positive constants c_1, c_2, n_0 such that $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ for all $n \ge n_0$. Similarly, we write $f(n) = O(g(n))$ if there exist positive constants c, n_0 such that $0 \le f(n) \le cg(n)$ for all $n \ge n_0$, $f(n) = \Omega(g(n))$ if there exist positive constants c, n_0 such that $0 \le cg(n) \le f(n)$ for all $n \ge n_0$, and $f(n) = o(g(n))$ if for any positive c there exists a positive n_0 such that $0 \le f(n) \le cg(n)$ for all $n \ge n_0$.

II. System Model

Consider a wireless network with n ad hoc nodes and m relay nodes. We assume that only the ad hoc nodes have data traffic to send (referred as source nodes in what follows) or receive (referred to as destination nodes), while relay nodes have no traffic demand on their own. Since asymptotic behavior is concerned, without loss of generality, we do not distinguish between source and destination nodes. We consider a two-hop decode-and-forward communication protocol, where sources can communicate with their destinations only through half-duplex relays. Specifically, in Phase 1 of the protocol a subset of sources is scheduled for transmission to relays. The relays decode and buffer the packets. During Phase 2 of the protocol, the relays forward packets to a subset of destinations (not necessarily the set of destinations associated with the source set in Phase 1). These two phases are interleaved: in the even-indexed time-slots, Phase 1 is run; in the odd-indexed time-slots, Phase 2 is run. The selection of source/destination sets is of opportunistic nature and will be elaborated on in the sequel. Once scheduled for transmission, the transmission rate is fixed at 1 bit/sec/Hz, and therefore the communication is successful if the corresponding signal to interference and noise ratio (SINR) is greater than or equal to 1. (Generalization to higher SINR threshold and corresponding higher transmission rate is straightforward, but working with $\mathsf{SINR} = 1$ as threshold simplifies notation.) An example of the two-hop relay protocol is depicted in Fig. 1.

We first describe the channel model. We assume that the wireless network has i.i.d. Rayleigh connections $h_{i,r}$ from source node i, $1 \le i \le n$, to relay node r, $1 \le r \le m$. Thus, the channel gains follow an exponential distribution, i.e., $\gamma_{i,r} = |h_{i,r}|^2 \sim \text{Exp}(1)$. Let the link strengths $\xi_{k,j}$ from relay k, $1 \le k \le m$, to destination node j, $1 \le j \le n$, be also i.i.d., governed by the $\text{Exp}(1)$ distribution. Furthermore, we assume that $\{\gamma_{i,r}\}$ and $\{\xi_{k,j}\}$ are independent over i, r, k, and j. Quasi-static fading is assumed, with channel gains fixed during the transmission of each hop, and taking on independent values at different transmission times. In this model, channel gains are dominated by the effects of small-scale fading, making it possible to neglect path loss effects. Such model applies to networks that are physically small, as well as scenarios where large-scale fading effects are compensated by a power control mechanism. Regarding the assumption of CSI availability, we assume implicitly that at both hops, the receivers have detailed CSI of all senders. As to senders, they have access only to an *integer-value* representing CSI and obtained via receiver feedback. Note that these assumptions are practical as most wireless access network standards incorporate some form of pilot signals.

We now describe the scheduling at each hop. We start with the first hop (Phase 1). All relays operate independently. Thus, without loss of generality, let us focus on any specific relay, say r. Relay r measures the channels $\gamma_{i,r}$, $1 \leq i \leq n$, and schedules the transmission of the strongest source node, say i, by feeding back the index i. The overhead of this phase of the protocol is an integer number per relay node. Each of the m relays schedules one source node, whereby enabling the scheduling of up to m source nodes. Suppose the scheduled nodes constitute a set $\mathcal{K} \subset \{1, \ldots, n\}$, then these source nodes transmit simultaneously at rate of 1 bit/sec/Hz. Thus, the communication from source i to relay r is successful if the corresponding SINR

$$\mathsf{SINR}_{i,r}^{\mathrm{P1}} = \frac{\gamma_{i,r}}{1/\rho + \sum_{\substack{t \in \mathcal{K} \\ t \neq i}} \gamma_{t,r}} \geq 1, \tag{1}$$

where ρ is the average signal to noise ratio (SNR) of the source-relay link. It is worth mentioning that it is possible that multiple relays schedule the same source. In such cases, $|\mathcal{K}| < m$.

The scheduling at the second hop (Phase 2) works as follows. Each destination node j, $1 \leq j \leq n$, measures the forward channel strengths, $\xi_{k,j}$, $1 \leq k \leq m$, and computes m SINRs by assuming that relay k is the desired sender and the other relays are interference as follows:

$$\mathsf{SINR}_{k,j}^{\mathrm{P2}} = \frac{\xi_{k,j}}{1/\rho_R + \sum_{\substack{1 \leq \ell \leq m \\ \ell \neq k}} \xi_{\ell,j}}, \tag{2}$$

where ρ_R denotes the average SNR of a relay–destination link. If the destination node j captures one good SINR, say, $\mathsf{SINR}_{k,j}^{\mathrm{P2}} \geq 1$ for some k, it instructs relay k to send data by feeding back the relay index k. Otherwise, the node j does not provide a feedback. It follows that the overhead of the second hop is also at most an integer value per destination node. Relay k, upon receiving feedback, relays the data to the destination node at rate 1 bit/sec/Hz. In case a relay receives multiple feedback messages, it randomly picks one destination for transmission. It is noted that in steady-sate operation of the system, the relays have the ability to buffer the data received from source nodes, such that it is available when the opportunity arises to transmit it to the intended destination nodes over the second hop of the protocol. This ensures that relays always have packets destined to the nodes that are scheduled.

III. Throughput: Large n and Fixed m

Motivated by the fact that as communication devices become more and more pervasive, the number of infrastructure nodes is not likely to keep pace, the throughput analysis in this paper pays special attention to the regime in which the number of source and destination nodes, n, is large, while the number of relay nodes, m, is relatively small. We will show that both Phase 1 and Phase 2 achieve throughputs of m bits/sec/Hz, yielding a $m/2$ bits/sec/Hz throughput for the whole two-hop scheme. Comparing this result to the upper bound on scaling, $(m/2) \log \log n$, the proposed scheme succeeds in capturing the leading linear term.

A. Phase 1: Source Nodes to Relays

In Phase 1, m relays operate independently, each scheduling one source node. Hence, the total number of scheduled source nodes can be any integer between 1 and m, i.e., $|\mathcal{K}| \leq m$. In cases when $|\mathcal{K}| < m$, the analysis of the probability of successful transmission should consider explicitly those links with multiple receivers. Due to these various possible combinations, the exact characterization of the throughput of Phase 1, R_1, is mathematically involved. Fortunately, in order to show the achievability of m successful concurrent transmissions, it suffices to lower-bound R_1 by considering only the case when the m scheduled source nodes are distinct (hence neglecting the probability of other combinations).

By symmetry, each source node has a probability of $1/n$ to be the best node with respect to a relay. Then the probability that the scheduled users are distinct, i.e., no source node is scheduled by more than one relay, is given by $\Pr[N_m] = n(n-1)\cdots(n-m+1)/n^m$, where N_m denotes the event "m distinct source nodes are scheduled." Now, the lower bound on R_1 is written

$$R_1 \geq m \cdot \Pr[N_m] \cdot \Pr[S_m], \tag{3}$$

where S_m denotes the event of a successful transmission, i.e., $\mathsf{SINR} \geq 1$. For convenience, we drop the source node and relay indices from the description of the link strengths $\gamma_{i,r}$. From (1), it follows that the probability of a successful transmission between a source and a relay can be written

$$\begin{aligned} \Pr[S_m] &= \Pr\left[\mathsf{SINR}^{\mathrm{P1}} \geq 1\right] \\ &= \Pr\left[\frac{X}{1/\rho + Y} \geq 1\right], \end{aligned} \tag{4}$$

where X is the power gain of the strongest link between all source nodes to the relay, and Y is the aggregate interference from all other $(m-1)$ concurrent transmitting source nodes. Since the link strengths are i.i.d. $\mathrm{Exp}(1)$ random variables, the cumulative distribution function (cdf) of X (largest of n i.i.d. $\mathrm{Exp}(1)$ random variables) can be written explicitly as

$$F_X(x) = (1 - e^{-x})^n. \tag{5}$$

The interference term Y is a chi-square distributed random variable with $2(m-1)$ degrees of freedom, whose cdf reads

$$F_Y(y) = 1 - e^{-y} \sum_{k=0}^{m-2} \frac{1}{k!} y^k. \tag{6}$$

Since random variables X and Y are not independent, there is no apparent way to proceed with the computation of $\Pr[S_m]$. Instead, with the help of (5) and (6), we further lower-bound (4) by introducing a real variable s ($s > 0$). By total probability, we have

$$\begin{aligned}
\Pr[S_m] &= \Pr\left[\frac{X}{1/\rho+Y} \geq 1\right] \\
&= \Pr[X \geq s] \cdot \Pr\left[\frac{X}{1/\rho+Y} \geq 1 \middle| X \geq s\right] \\
&\quad + \Pr[X \leq s] \cdot \Pr\left[\frac{X}{1/\rho+Y} \geq 1 \middle| X \leq s\right] \qquad (7) \\
&\geq \Pr[X \geq s] \cdot \Pr\left[\frac{X}{1/\rho+Y} \geq 1 \middle| X \geq s\right] \\
&\geq \Pr[X \geq s] \cdot \Pr\left[\frac{s}{1/\rho+Y} \geq 1\right] \qquad (8) \\
&= \big(1-(1-e^{-s})^n\big) F_Y(s-1/\rho). \qquad (9)
\end{aligned}$$

The operational meaning of (9) can be interpreted as follows: Each relay sets a threshold s and only schedules the transmission of the source when it is the strongest *and* the power gain of the link exceeds the threshold. The probability of such event is given by $1-(1-e^{-s})^n$. Upon being scheduled and transmitting at 1 bit/sec/Hz, the probability of successful communication with the relay is at least $F_Y(s-1/\rho)$.

Substituting (9) into (3) yields the following lower bound on the throughput of Phase 1.

Lemma 1: For any ρ, $n \geq m$ and $s > 0$, the achievable throughput of the opportunistic relay scheme in Phase 1 is lower-bounded by

$$R_1 \geq m \tfrac{n(n-1)\cdots(n-m+1)}{n^m} \big(1-(1-e^{-s})^n\big) F_Y(s-\tfrac{1}{\rho}). \quad (10)$$

For the regime of interest where n is large and m fixed, it is trivially, e.g., empirically set $s = \log n - \log\log n$, to see following result.

Corollary 1: For the opportunistic relaying scheme, the lower bound on the throughput for the first hop is $R_1 \to m$ for a number of source nodes $n \to \infty$ and fixed number of relays m.

A tighter lower bound can be found by maximizing (9) over s, but we found that little insight can be gained from this exercise. The tightness of lower bound (10) is substantiated by numerical results in Fig. 2. The two simulations curves were obtained by averaging throughputs over $2,000$ channel realizations. The "simulated R_1" curve was obtained utilizing all assignments of source nodes, while the curve marked "(3)" represents only assignments of distinct source nodes (cf. (3)). The figure shows that both the simulated throughput and the analytical lower bound (10) exhibit linearity with respect to m, consistent with Corollary 1.

Remark 1: Inspecting (10), we note that the lower bound R_1 exhibits a tradeoff between the quantity and the quality of the scheduled link. By making m large, one can schedule more simultaneous transmissions, which is beneficial from throughput perspective. However, more transmissions results in more interference, which deteriorates SINR. Thus, the probability of successful transmissions decreases. What is interesting is that from Fig. 2 we observe that the simulated R_1 also demonstrates a similar tradeoff and there exists one

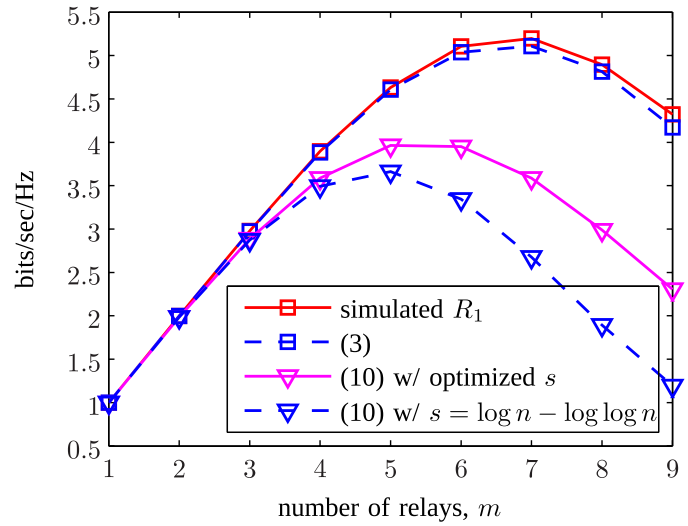

Fig. 2. Simulated R_1, simulated lower bound of R_1 (by only counting the $|\mathcal{K}| = m$ case), and two analytical lower bound of R_1 of (10). $n = 1200$ for all cases. $\rho = 10$ dB.

optimal m which delivers maximum throughput. Section IV continues this line of discussion on the optimal value of m.

To summarize Phase 1, the core idea is that, even through the relay does not have SINR information to guarantee successful transmission, the relay does have knowledge about the instantaneous channel strength. Therefore, the relay can schedule the best source node and thanks to multiuser diversity gain, this strong channel strength can compensate the interference and noise.

B. Phase 2: Relays to Destination Nodes

We proceed to develop an expression for the throughput of the relay-destination links. This is done by first determining that only a single relay per destination can produce a required SINR larger than one, and then computing the probability of the event that the SINR is indeed ≥ 1. $\mathsf{SINR}^{\mathrm{P2}}_{k,j}$ of (2) are i.i.d. random variables for $j = 1, \ldots, n$ (but not independent over $k = 1, \ldots, m$) with the pdf [10]

$$f(x) = \frac{e^{-x/\rho_R}}{(1+x)^m}\left(\frac{1}{\rho_R}(1+x)+m-1\right). \qquad (11)$$

The corresponding cdf is

$$F(x) = 1 - \frac{e^{-x/\rho_R}}{(1+x)^{m-1}}, \quad x \geq 0. \qquad (12)$$

In contrast to Phase 1, where we had to be satisfied with a lower bound on the throughput, for Phase 2 of the communication protocol, we are able to write the exact expression of the system throughput:

Lemma 2: For any ρ_R, m and n, the achievable throughput of the opportunistic relay scheme at Phase 2 satisfies

$$R_2 = m\left(1-\left(1-\frac{e^{-1/\rho_R}}{2^{m-1}}\right)^n\right). \qquad (13)$$

Proof: First of all, we observe that each destination node j has at most one $\mathsf{SINR}^{\mathrm{P2}}_{\ell,j} \geq 1$ for all relays $1 \leq \ell \leq m$. Suppose that for some relay k, $\mathsf{SINR}^{\mathrm{P2}}_{k,j} \geq 1$. This implies that there is no other relay k' for which $\mathsf{SINR}^{\mathrm{P2}}_{k',j} \geq 1$. Indeed, from (2), we have

$$\xi_{k,j} \geq 1/\rho_R + \sum_{\substack{1 \leq \ell \leq m \\ \ell \neq k}} \xi_{\ell,j},$$

from which it follows

$$\xi_{k,j} > \xi_{k',j}, \quad \forall\, k' \neq k.$$

Therefore

$$\mathsf{SINR}^{\mathrm{P2}}_{k',j} = \frac{\xi_{k',j}}{1/\rho_R + \sum_{\substack{1 \leq \ell \leq m \\ \ell \neq k'}} \xi_{\ell,j}} < \frac{\xi_{k',j}}{\xi_{k,j}} < 1.$$

Thus, each destination node can have at most one *good* relay as its sender.

The probability for a destination node that no $\mathsf{SINR} \geq 1$ is[1]

$$\begin{aligned} &\Pr[\text{relay } k \text{ does not transmit data}] \\ &\quad = \Pr\left[\forall\, j,\ \mathsf{SINR}^{\mathrm{P2}}_{k,j} \leq 1\right] \\ &\quad = \left((F(1)\right)^n \\ &\quad = \left(1 - \frac{e^{-1/\rho_R}}{2^{m-1}}\right)^n. \end{aligned} \tag{14}$$

The throughput of the relay-destination links is given by summing the probabilities of the relays being engaged in transmission. Accounting for the 1 bit/sec/Hz rate per relay, we have for the average throughput of the second hop:

$$\begin{aligned} R_2 &= \sum_{k=1}^{m} \Pr[\text{relay } k \text{ transmits data}] \cdot 1 \\ &= m\left(1 - \left(1 - \frac{e^{-1/\rho_R}}{2^{m-1}}\right)^n\right). \end{aligned} \tag{15}$$

□

The following corollary follows:

Corollary 2: $R_2 \to m$ for $n \to \infty$ destination nodes and for fixed m.

C. Two-Hop Communication

With the help of Corollary 1 and 2, and by taking into account the $1/2$ penalty due to the two hops, the whole system throughput, defined as $\frac{1}{2}\min(R_1, R_2)$, can be readily shown to be as follows.

Theorem 1: The two-hop opportunistic relaying scheme achieves a system throughput of $m/2$ bits/sec/Hz in the regime $n \to \infty$ and fixed m.

Since the proposed scheme works in a completely decentralized fashion and with a low rate CSI feedback, it is natural to expect some throughput degradation compared to more intensive schemes. Fortunately, we show the throughput loss is negligible. To see this, we derive a simple upper bound on the throughput scaling by making optimistic assumptions of full relaying cooperation and full CSI. With these assumptions, the two hops communication can be interpreted as MIMO (multiple-input multiple-output) multiple access channels (MAC) followed by a MIMO-BC, each known to have a scaling of $m \log\log n$ (the BC scaling is established in [10], and the MAC scaling follows from he MAC–BC duality [11]).

Lemma 3: For *any* two-hop relaying architecture, with fixed m and SNR, and large n, the sum rate capacity scales at most like $\frac{1}{2} m \log\log n$.

Comparing Theorem 1 and Lemma 3, we see that the proposed scheme comes with the loss of the $\log\log n$ term. However, it succeeds in achieving the leading linear term of the upper bound.

Remark 2: Contrasting Theorem 1 to Lemma 3 reveals different ways of utilizing multiuser diversity gain. Fundamentally, multiuser diversity gain is a power gain, e.g., in the Rayleigh fading case, multiuser diversity gain will boost the average power by a factor of $\log n$ [12]. With the assumption of relay cooperation, the m spatial multiplexing gain can be readily achieved (e.g., even by suboptimal zero-forcing receiver [13]). Then, multiuser diversity can further boost the rate of each parallel channel by $\log\log n$, as shown by Lemma 3. In contrast, in our scheme, multiuser diversity gain is used to achieve the linear scaling in m. Only with multiuser diversity gain can the SINR of each *noncooperative* link have the chance to meet the threshold.

As discussed in Remark 1, there exists a tradeoff in increasing m and therefore we cannot arbitrarily increase m, while still supporting m transmissions simultaneously and successfully. *What is the optimal* m*?* We address this natural question next.

IV. How Fast Can m Grow?

In this section, we show that m can grow (as a function of n) at most as fast as $\Theta(\log n)$ and still retain linearity of throughput in m. We will show that if m grows faster than $\Theta(\log n)$ the linearity of the throughput in m breaks down. In light of Theorem 1, it is equivalent to conclude that the throughput scaling of the proposed two-hop opportunistic relaying scheme is $\Theta(\log n)$.

The proof for Phase 2 can be shown straightforwardly by examining the asymptotic behavior of (15). The proof of Phase 1, on the other hand, is more involved.

A. Phase 1

1) Scaling Upper Bound of Phase 1: First, we characterize the upper bound on the throughput scaling by assuming that there exists a genie, who has access to the full CSI of the network, and can coordinate the operation of the whole network. We will prove the throughput scaling of this genie-aided scheduling scheme by the probabilistic method [14]. The

[1] While it may seem logical to shut off a relay for which the highest SINR is still ≤ 1, we still allow such relay to transmit (say, control information). This is because, as shown by numerical results, the performance bottleneck is in the source-relay link.

basic idea of the probabilistic method is that in order to prove the existence of a structure with certain properties, one defines an appropriate probability space of structures and then shows that the desired properties hold in this space with positive probability. In the present context, let $X(m)$ be the number of groups of m nodes that can be scheduled simultaneously such that all m nodes in the group transmissions are successful (i.e., $\mathsf{SINR}^{\mathrm{P1}} \geq 1$). We will show that, $\Pr\left[X(m) \geq 1\right] \to 0$, when $m = \frac{\log n}{\log 2} + 2$, while $\Pr\left[X(m) = 0\right] \to 0$ goes to zero when $m = (1-\epsilon)\frac{\log n}{\log 2} + 1$.

Theorem 2: Under the two-hop opportunistic relaying scheme, and assuming genie-aided scheduling, then with probability approaching 1, there is no set of $\frac{\log n}{\log 2} + 2$ nodes whose simultaneous transmissions to relays are all successful. The converse is also true: there is a set of $(1-\epsilon)\frac{\log n}{\log 2} + 1$ nodes whose simultaneous transmissions to relays are all successful.

Proof: The nonexistence part of the proof can be trivially proved by the first moment method (Markov's inequality). For the existence part of the proof, we use the result in [15, Ch. 8] as the lower bound of our setup.

We begin with the nonexistence part. Define the space $\Omega = \big\{(\mathcal{A}, \pi) : \mathcal{A} \subset \{1, \ldots, n\}, |\mathcal{A}| = m,\ \pi \text{ is a permutation on } \{1, \ldots, m\}\big\}$. Let $\mathcal{A}$ be a set of m source nodes. The permutation π denotes all possible one-to-one mappings from all the nodes in $\mathcal{A}$ to relays. Let $B_{\mathcal{A}}^{\pi}$ be the event "all nodes in $\mathcal{A}$ can transmit simultaneously and successfully under mapping rule π" and $X_{\mathcal{A}}^{\pi}$ the corresponding indicator random variable, i.e.,

$$\begin{aligned} X_{\mathcal{A}}^{\pi} &= \mathbf{1}\Big(\mathsf{SINR}_{i,R(i)}^{\mathrm{P1}} \geq 1, \quad \forall i \in \mathcal{A}\Big) \qquad (16)\\ &= \mathbf{1}\left(\frac{\gamma_{i,R(i)}}{1/\rho + \sum_{\substack{t \in \mathcal{A}\\ t \neq i}} \gamma_{t,R(i)}} \geq 1, \quad \forall i \in \mathcal{A}\right), \end{aligned}$$

where the subscript $R(i)$ in (16) denotes the corresponding relay for source i under mapping rule π. In other words, for all $i \in \mathcal{A}$, the $R(i)$'s are uniquely determined by the given π.

Then

$$\begin{aligned} \mathbb{E}[X_{\mathcal{A}}^{\pi}] &= \Pr[B_{\mathcal{A}}^{\pi}] \\ &= \Pr\Big[\mathsf{SINR}_{i,R(i)}^{\mathrm{P1}} \geq 1, \quad \forall i \in \mathcal{A}\Big] \\ &= \left(\Pr\Big[\mathsf{SINR}_{i,R(i)}^{\mathrm{P1}} \geq 1\Big]\right)^m \qquad (17)\\ &= p_s^m, \qquad (18) \end{aligned}$$

where (17) follows from the fact that for $r \neq r'$ ($1 \leq r, r' \leq m$) and any i, i', $\mathsf{SINR}_{i,r}^{\mathrm{P1}}$ and $\mathsf{SINR}_{i',r'}^{\mathrm{P1}}$ are i.i.d., and $p_s = 1 - F(1)$ in (18) is the probability of a single transmission being successful, with $F(\cdot)$ the cdf of the SINR computed in (12).

The number of valid sets that satisfy the SINR threshold is given by

$$X(m) = \sum_{\mathcal{A}} \sum_{\pi} X_{\mathcal{A}}^{\pi}. \qquad (19)$$

Linearity of the expectation operator yields

$$\mathbb{E}\left[X(m)\right] = \binom{n}{m} m!\, p_s^m. \qquad (20)$$

We can upper bound $\Pr\big[X(m) \geq 1\big]$ using the first moment method (Markov's inequality) as

$$\begin{aligned} \Pr[X(m) \geq 1] &\leq \mathbb{E}\left[X(m)\right] \\ &= \frac{n!}{(n-m)!} p_s^m \\ &\leq (np_s)^m \leq \left(\tfrac{n}{2^{m-1}}\right)^m \\ &= e^{m(\log n - (m-1)\log 2)} \qquad (21) \end{aligned}$$

Substituting $m = \frac{\log n}{\log 2} + 2$ into (21),

$$\begin{aligned} \Pr[X(m) \geq 1] &\leq e^{-\log n + o(\log n)} \\ &= O\left(\tfrac{1}{n}\right). \qquad (22) \end{aligned}$$

What (22) tells us is that when $m = \frac{\log n}{\log 2} + 2$, the probability of finding a set of m nodes for supporting m concurrent and successful transmissions decreases to zero.

Now we turn to look at the existence part. We will apply the approach employed in [15] for a problem that can be viewed as a special case of ours. In [15], Etkin characterized the bandwidth scaling of large networks. There, the network has n *systems* where each system consists of a *designated* S–D pair. The problem is formulated as to find the minimum number of channels that is needed to maintain a fixed rate of communication for each S-D pair. It turns out that for i.i.d. Rayleigh channels, the minimum number of channels required is given by $\sim \frac{n}{2\log n}$ asymptotically almost sure (property holds with probability approaching 1 as $n \to \infty$). This implies that each channel can allow $2\log n$ S–D pairs to communicate simultaneously by properly selecting them opportunistically by exploiting channel diversity. Thus, if we focus on one specific frequency, then the setup of [15] can be viewed as a special case of our problem, since our setup has additional freedom of choosing S–D mappings, i.e., the permutation π. We first quote the following results from [15] (couched in the notation of the present paper and applying an exact expression for p_s of (18), instead of the approximation in [15]:

Lemma 4: In the case where the number of *systems*, n, goes to infinity, under the i.i.d. Rayleigh fading model and declare a group is valid if and only if all systems in the group can communicate successful in a shared spectrum. Let $X'(m)$ denote the number of total valid groups, then for any $\epsilon > \epsilon' > 0$ and $m = (1-\epsilon)\frac{\log n}{\log 2} + 1$

$$\Pr[X'(m) = 0] < e^{-n^{\epsilon'}}$$

Note that our setting consumes the setup of Lemma 4 as a special case. Thus, in our setup the probability of nonexistence of valid group must be smaller that of Lemma 4, i.e., $\Pr[X(m) = 0] \leq \Pr[X'(m) = 0]$.

Corollary 3: Under the assumptions described in Section II, then we have

$$\Pr[X(m) = 0] < e^{-n^{\epsilon'}}$$

for any $\epsilon > \epsilon' > 0$ and $m = (1-\epsilon)\frac{\log n}{\log 2}+1$, with the definition of $X(m)$ given in (19).

Corollary 3, together with (22), completes the proof. □

2) Achievablity of Scaling Upper Bound: Theorem 2 states that, with high probability, there exists a valid set when $m = (1-\epsilon)\frac{\log n}{\log 2}+1$. However, the proof is nonconstructive: it does not afford an insight into how to find such a set in practice. The proof also assumes that there is a genie with global channel information, who can enumerate all $\binom{n}{(1-\epsilon)\frac{\log n}{\log 2}+1}$ possibilities and pick a good one for scheduling. Our opportunistic relaying scheme operates in a completely decentralized manner, and it is not clear if this operational simplification incurs a loss in the scaling order of the throughput. It can be shown that, fortunately, the upper bound of scaling $\log n$ can be met by the lower bound of (10). To see this, we examine the asymptotic behavior of (10).

Consider the case of $m = \log n$ and $s = \log n - \log\log n$. With $n \to \infty$, the term $\frac{n(n-1)\cdots(n-m+1)}{n^m} \to 1$. The term $\left(1-(1-e^{-s})^n\right)$ is independent of m and approaches 1 for $s = \log n - \log\log n$ and large n. Therefore, a throughput of $\Theta(\log n)$ can be achieved as long as $F_Y(s - \frac{1}{\rho}) = \Theta(1)$. This can be seen by noting that for $m = \log n$, the interference term Y in (6) is a chi-square distributed random variable with $2(\log n - 1)$ degrees of freedom. Then, Y can be approximated as a Gaussian random variable, with mean and variance both equal to $\log n$. Now, we have

$$F_Y(\log n - \log\log n - 1/\rho) \approx F_Y(\log n) = \frac{1}{2}, \qquad (23)$$

due to the symmetry of the Gaussian distribution. Consequently $R_1 \approx \frac{1}{2}\log n$. This result implies that for $m = \log n$ relays, each running the two-hop opportunistic relaying protocol, it is possible to schedule up to $\log n$ source nodes to transmit simultaneously, but half of them will fail to satisfy the SINR requirement due to the multiple access interference. In terms of throughput, this example yields $\frac{1}{2}\log n$, which confirms that the scheme is in fact order-optimal in achieving a throughput of $\Theta(\log n)$ at Phase 1.

B. Phase 2

It is straightforward to characterizing the throughput scaling of Phase 2 by directly studying the asymptotic behavior of (15) with m. The proofs are omitted here for the sake of saving pages.

Theorem 3: For Phase 2 of the two-hop opportunistic relaying scheme, if the number of relays $m = \Theta(\log n)$, then $R_2 = \Theta(m) = \Theta(\log n)$. Conversely, if $m = \Omega(\log n)$, then $R_2 = o(\log n)$.

C. Two-Hop Communications

From Theorems 2 and 3, it follows that there is a solution to the opportunistic relaying scheme as long as m is of the order of $\log n$. Thus, we can conclude that the m can not grow faster than $\log n$. As a by-product of this result, we conclude that the achievable throughput scaling of our proposed opportunistic relaying scheme is given by $\log n$.

Corollary 4: Under the setup of Section II, the proposed two-hop opportunistic relaying scheme yields a maximum achievable throughput of $\Theta(\log n)$.

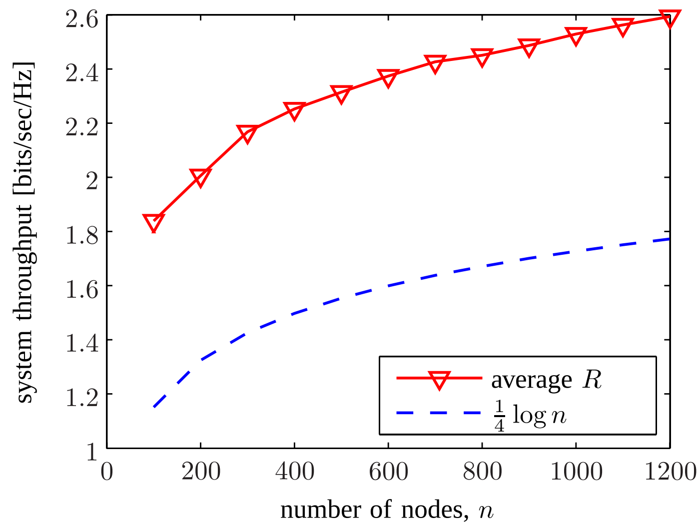

Fig. 3. Simulated system throughput of the proposed scheme with various n and optimal m, and the $\frac{1}{4}\log n$ curve for reference. $\rho = \rho_R = 10$ dB.

The total throughput of the two-hop opportunistic relaying scheme is shown in Fig. 3 for SNR of 10 dB at both the relays and at the destinations. We observe that the throughput exhibits the $\log n$ trend, as predicted by Corollary 4. For reference, we also plot the $\frac{1}{4}\log n$ curve, the lower bound in (23).

V. Discussion and Conclusion

A. Delay Considerations

There is a tension between the simplification of network operations and delay consideration. The delay issue is more salient in the two-hop scheme than in the cellular setup [12], because packets transmitted by one particular source in the Phase 1 have to be buffered at a relay until that relay schedules the original destination during Phase 2. While one can partially relieve the problem by, say, prioritizing the destination in case when the relay receive multiple requests from multiple destinations (including the destination of interest, of course), the delay may still be unbounded. The detailed study of end-to-end delay is the subject of future work.

B. Conclusion

In this work, we proposed an opportunistic relaying scheme that alleviates the demanding assumptions of central scheduling and CSI at transmitters. The scheme entails a two-hop communication protocol where sources can communicate with the destinations only through half-duplex relays. The key idea is at each hop to schedule only the subset of nodes that can benefit from multiuser diversity. To select the source and destination nodes for each hop, relays operate independently and only CSI at receiver, with one integer feedback to the transmitter, is required. The system throughput is characterized for the interesting operating regime where n is large and m

is relatively small. In this case, the proposed scheme achieves a system throughput of $m/2$ bits/sec/Hz, as opposed to the optimistic upper bound $(m/2)\log\log n$. Moreover, we further characterize the throughput scaling to be $\Theta(\log n)$.

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