Numerical Modeling of Ferrofluid Droplets in Magnetic Fields

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Abstract. The motion and shape of a ferrofluid droplet placed in a non-magnetic viscous fluid and driven by a magnetic field are modeled numerically. The governing equations are the Maxwell equations, momentum equation and incompressibility. The numerical simulation uses a volume-of-fluid algorithm with a continuum-surface-force formulation for an axisymmetric geometry. The deformations in the drop under non-uniform magnetic fields are simulated. Droplets exhibit shape changes along the applied magnetic field. We have found that the ferrofluid droplet forms a prolate ellipsoid in the presence of a non-uniform magnetic field. For higher magnetic field strengths, the droplet undergoes dramatic deformation. This in turn influences the motion of the droplet through the viscous medium.

Keywords: ferrofluid drop, VOF, computational fluid mechanics
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INTRODUCTION

Ferrofluids consist of magnetic nanoparticles in a colloidal solution. Recent developments in the synthesis and characterization of ferrofluids are motivated by biomedical applications [1, 2]. A small amount of ferrofluid is injected into the vitreous cavity of the eye and guided by a permanent magnet inserted outside the scleral wall of the eye. The drop travels toward the side of the eye, until it can seal a retinal hole. The understanding of the dynamics of a droplet under an applied magnetic field is important for the efficient manipulation of the procedure. For instance, the size and the shape of the ferrofluid droplet can influence the motion of the droplet as it travels in a viscous medium. To investigate the response of a ferrofluid droplet to an applied magnetic field or to the capillary effects requires a thorough understanding of magnetohydrodynamics in such a system. The mathematical formulation of the flow of a ferrofluid is described by Rosensweig [3]. In this paper, we present a methodology for the numerical modeling of a two-phase system of a ferrofluid liquid and an immiscible viscous medium. The magnetic force competes with the interfacial tension force and viscous drag to deform the drop. Here, we develop a numerical model to incorporate the magnetic forces at the interface between a ferrofluid and an immiscible surrounding liquid in the context of the volume-of-fluid method and the continuum-surface-force model and to simulate the field-induced motion of a ferrofluid droplet in a viscous medium.

NUMERICAL METHODOLOGY

In the absence of an initially imposed velocity and the gravity, we non-dimensionalize the variables as follows:

$$\mathbf{x}^* = \mathbf{x}/R_0 , \; t^* = t\eta_0/(\rho_0 R_0^2) , \; \eta^* = \eta/\eta_0 , \; \rho^* = \rho/\rho_0,$$
$$\mathbf{u}^* = \mathbf{u}\rho_0 R_0/\eta_0 , \; p^* = p\rho_0 R_0^2/(\eta_0)^2 , \; \mathbf{H}^* = \mathbf{H}/H_0,$$

where R_0 is the initial droplet radius, ρ_0 and η_0 are the ferrofluid density and viscosity, respectively, and H_0 is the characteristic scale of the magnetic field strength. The equations of motion then become

$$\rho^*\frac{d\mathbf{u}^*}{dt^*} = -\nabla^* p^* + \nabla^* \cdot \tau^* + La\,\mathbf{F_s}^* + La_m\,\nabla^* \cdot \sigma_m^*, \tag{1}$$

$$\nabla^* \cdot \mathbf{u}^* = 0, \tag{2}$$

where $\tau^* = \eta^*(\nabla^*\mathbf{u}^* + \nabla^*\mathbf{u}^{*T})$ represents the dimensionless shear stress tensor, $\mathbf{F}_{st}^*$ denotes the dimensionless body force due to surface tension, σ_m^* is the dimensionless magnetic stress tensor. The Laplace number $La = \gamma\rho_0 R_0/\eta_0^2$ is the ratio of the surface tension to the viscous dissipation (note that $La = 1/Oh^2$ where Oh is the Ohnesorge number) and the magnetic Laplace number $La_m = \mu_0 H_0^2 \rho_0 R_0^2/\eta_0^2$ is the ratio of the magnetic force to the viscous dissipation.

Here, we present a summary of the numerical approach. The reader is referred to [4, 5] for details. A volume-of-fluid algorithm on a marker-and-cell grid of equidistant mesh Δ is used. Each liquid is identified with a color function $C(r,z,t)$ which is 0 in the outer liquid and 1 in the drop liquid. The color function is advected with the flow, and the location of its discontinuity yields the interface position. Interfacial tension is discretized using the continuum-surface-force model. The new aspect is the extension of the algorithm to the ferrofluid. The magnetic stress tensor σ_m is $\mathbf{BH}^T + k\mathbf{I}$, where k denotes a constant (the latter term is absorbed into a redefined pressure field) and $\mathbf{B}$ and $\mathbf{H}$ are the magnetic induction and the magnetic field, respectively. The Maxwell equations are $\nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{H} = 0$, where

$$\mathbf{B} = \begin{cases} \mu_0(1+\chi_m)\mathbf{H} & \text{in the ferrofluid} \\ \mu_0\mathbf{H} & \text{in the viscous medium} \end{cases}, \tag{3}$$

where μ_0 is the permeability of vacuum and χ_m is magnetic susceptibility. The Maxwell equations can be described in terms of a magnetic scalar potential ψ as $\nabla \cdot (1+\chi_m)\nabla\psi = 0$, where $\mathbf{H} = \nabla\psi$. The susceptibility jumps in value across the interface, so that ψ changes as the interface evolves. The boundary conditions on the magnetic field are reconstructed from the experimental measurements of [2]. A multigrid Poisson solver is then used to obtain the solution of $\nabla \cdot (1+\chi_m)\nabla\psi = 0$.

RESULTS

We first present the results of the simulated applied magnetic field and the imposed magnetic field boundary conditions. Figure 1 shows the magnetic field contour lines plotted for cases of a 2mm diameter droplet centered at distances 8mm and 4mm from the bottom of the computational domain. The magnetic field lines in the viscous medium that is non-magnetizable are disoriented in the presence of the ferrofluid droplet due to having different permeability.
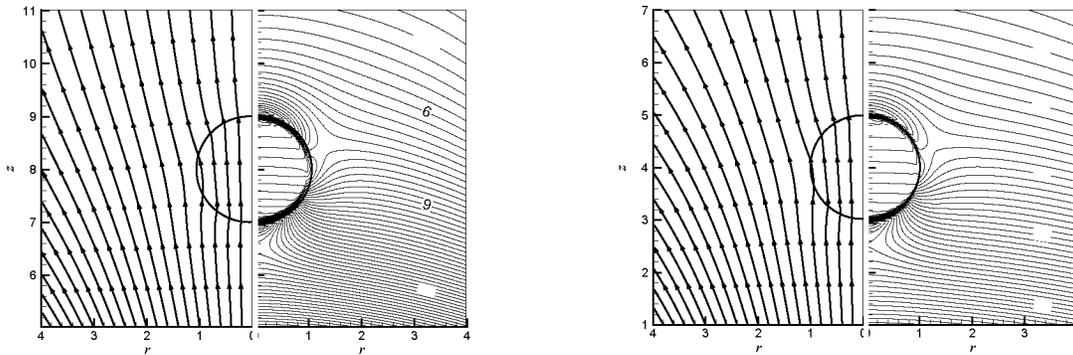

FIGURE 1. Magnetic field lines (left) and magnetic field contours (right) in the presence of a ferrofluid droplet in a non-magnetizable medium. A 2mm droplet is centered at 8mm and 4mm distances away from the bottom of the computational domain (from left to right), $\chi_m = 1$.

The results on the deformation behavior of the ferrofluid droplet under applied magnetic fields are presented next. Past theoretical studies have shown that microscopic ferrofluid droplets ($2\text{-}20\mu$m) deform to prolate droplets in the direction of the uniform applied magnetic field [6]. Here we also numerically observe that drops elongate in the presence of non-uniform magnetic fields. The computational domain is $1.6R_0 \times 12.8R_0$. A freely suspended ferrofluid droplet of radius R_0 (1.25mm) is initially centered at $(0, 10.4R_0)$. The permanent magnet is at the bottom of the domain. At the walls, the velocities satisfy no slip. Due to symmetry, only half of the domain is simulated. The mesh size is $\Delta = R_0/20$. The value of the magnetic susceptibility is χ_m=0.25. The density ratio ($\rho_{droplet}/\rho_{viscous}$) is 1.32 and the viscosity ratio ($\eta_{droplet}/\eta_{surrounding}$) is 1.5. The Laplace number La is 5.15. Figures 2 shows droplet shapes for

magnetic Laplace numbers $La_m \approx 0.3$, 1.5, 3, and 6 at non-dimensional times $\tau = t\eta_0/(\rho_0 R_0^2)$. These figures show that the increase of the magnetic field results in a drop elongation in the direction of the applied magnetic field. While for $La_m \approx 0.3$, the shape of the droplet remains almost rounded for all time, higher magnetic Laplace numbers result in a dramatic deviation from rounded shapes to further elongated shapes forming columnar configurations. Figures 2 also shows that increase of the magnetic Laplace number La_m results in a continuous drop prolation accompanied by a deformation from a rounded shape to a tear-drop shape. At the highest magnetic Laplace number ($La_m \approx 0.3$), small surface undulations begin to appear on flat sides of the front of the droplet.
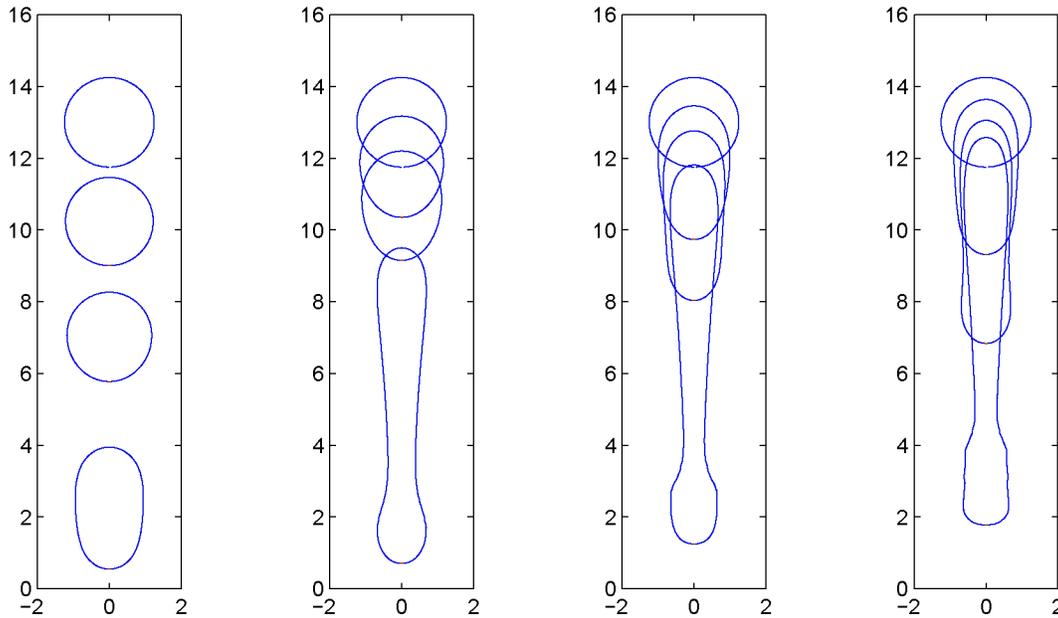

FIGURE 2. Droplet shapes at different magnetic Laplace numbers: $La_m \approx 0.3$ ($\tau = 0, 410, 500, 560$), $La_m \approx 1.5$ ($\tau = 0, 7.5, 11.2, 15$), $La_m \approx 3$ ($\tau = 0, 1.9, 2.6, 3.3$), and $La_m \approx 6$ ($\tau = 0, 0.6, 0.9, 1$) (from left to right).

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