Simulating Multiphase Systems

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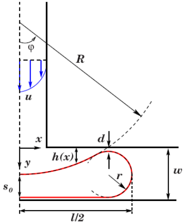
Droplet Breakup in a T-junction

Experiments, PoF 2010:
oil droplets in de-ionized water
(a) $\sigma = 15.4$ mN/m, viscosity ratio 1.67
range of speed: 5.8 to 300 μm/s
(b) $\sigma = 5$ mN/m, viscosity ratio 0.11
range of speed: 2.4 to 165 μm/s

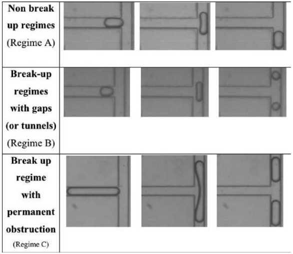


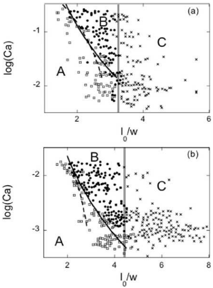
$$\frac{d^3h}{dx^3} = \frac{3\eta_e\bar{U}}{\sigma}\frac{w}{h^3}$$

$$d/w \approx 0.923(3\eta_e\bar{U}/\sigma)^{2/5}$$

$$\ell/w \approx 1.3(\eta_e\bar{U}/\sigma)^{-0.21}$$

cross-sectional dimensions of 80 μm high and 80 μm wide





Non-Dimensionalization

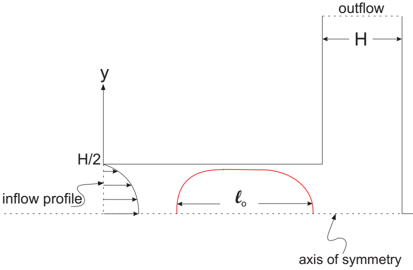
$$\mathbf{x}^* = \mathbf{x}/H\,,\ \mathbf{u}^* = \mathbf{u}/\bar{U}\,,\ p^* = pH/(\eta_e \bar{U})$$
$$t^* = t\bar{U}/H\,,\ \eta^* = \eta/\eta_e\,,\ \rho^* = \rho/\rho_e$$

The equations of motion then become

$$\mathrm{Re}\left(\frac{\partial}{\partial t^*}(\rho^*\mathbf{u}^*) + \nabla^*\cdot(\rho^*\mathbf{u}^*\mathbf{u}^*)\right) =$$
$$-\nabla^* p^* + \nabla^*\cdot\tau^* + \frac{1}{\mathrm{Ca}}\mathbf{F}^*_{st}$$

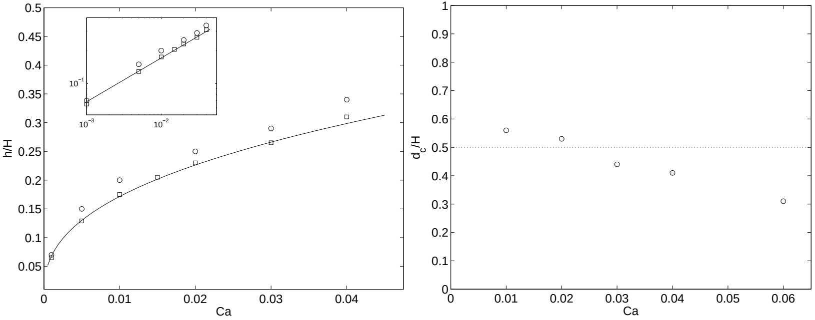
$$\mathrm{Re} = \rho_e \bar{U} H/\eta_e, \quad \mathrm{Ca} = \eta_e \bar{U}/\sigma$$

$$h/H \approx 0.923(3\eta_e\bar{U}/\sigma)^{2/5}$$
$$\ell/H \approx 1.3(\eta_e\bar{U}/\sigma)^{-0.21}$$
$$\ell_o/H \approx 0.98(\eta_e\bar{U}/\sigma)^{-0.21}$$



Droplet in T-junction: Numerical Simulations

$\gamma = 0.1$ (□) and $\gamma = 1$ (○)

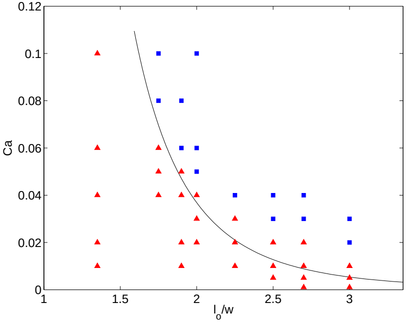


$$h/H \approx 0.923(3\eta_e \bar{U}/\sigma)^{2/5}$$

Droplet in T-junction: Numerical Simulations

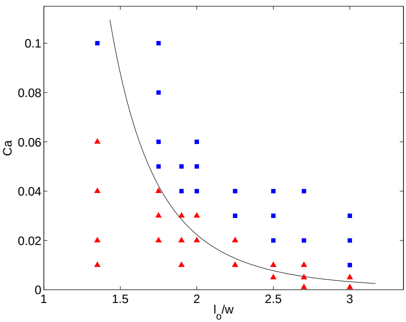
nonbreaking (△) and breaking (▫)

$\gamma = 0.1$



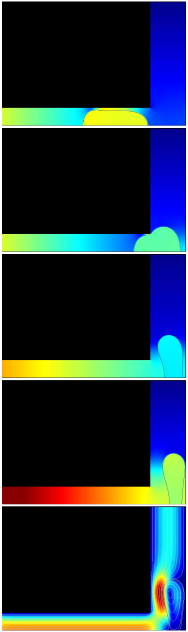
$\ell_o/w \approx 0.98(\eta_e \bar{U}/\sigma)^{-0.21}$

$\gamma = 1$

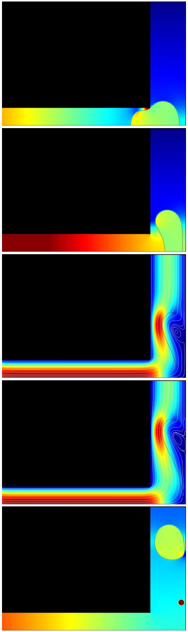


$\ell_o/w \approx 0.9(\eta_e \bar{U}/\sigma)^{-0.21}$

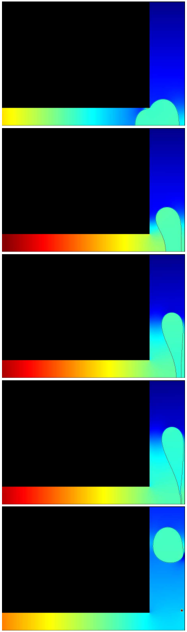
$\gamma = 0.1$, $\mathrm{Ca} = 0.06$



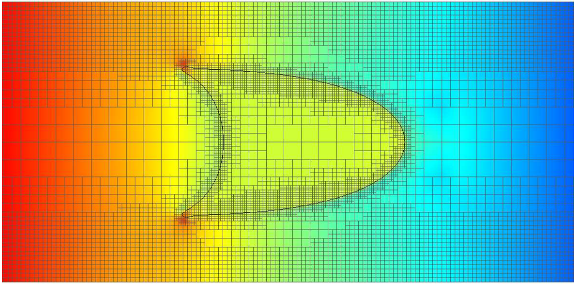
$\gamma = 1$, $\mathrm{Ca} = 0.06$



$\gamma = 0.1$, $\mathrm{Ca} = 0.08$



Richardson Bubble: 2D Bubble in Viscous Flows

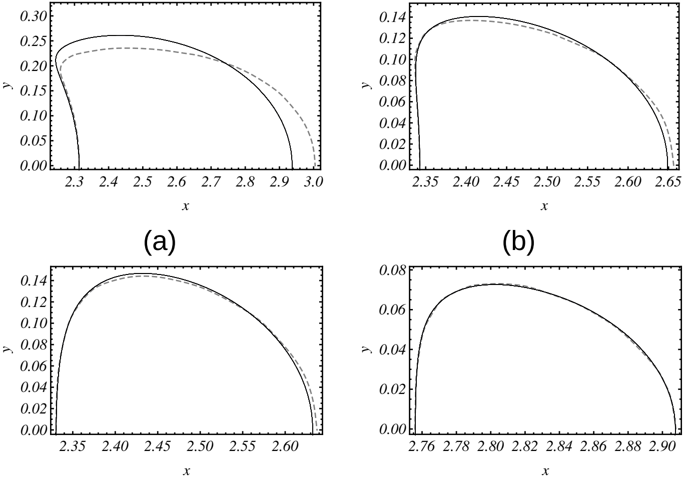


- One-parameter family of solutions governed by: $Z = \sigma/a^2 B\eta_e$, $\quad B = \Delta p/(2\eta_e L)$
- Bubble placed at center of parabolic flow profile moves faster than the undisturbed fluid velocity at the center [Richardson, JFM 1973], $U_m = H^2/8\eta_e(\Delta p/L)$.
- Rewrite:

$$Z = \frac{1}{4}(H/a)^2 \mathrm{Ca}^{-1}$$

where $\mathrm{Ca} = \eta_e U_m/\sigma$

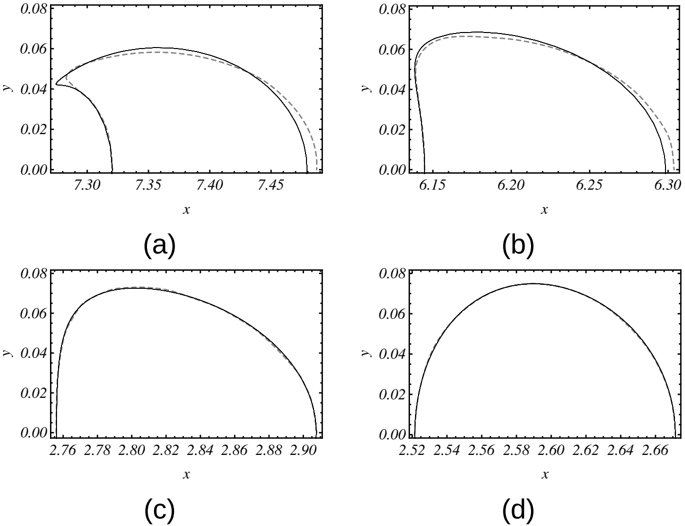
Richardson Bubble: 2D Bubble in Viscous Flows



(a) (b) (c) (d)

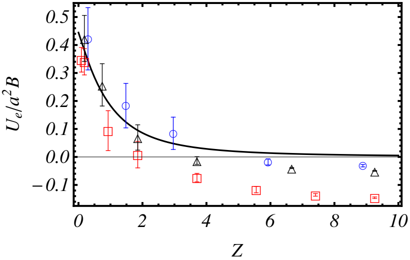
$a = 0.3$: $Z = 0.93$ (a), $a = 0.15$ $Z = 1.85$ (b), $a = 0.15$ $Z = 3.7$ (c), $a = 0.075$ $Z = 2.96$ (d)

Richardson Bubble: 2D Bubble in Viscous Flows



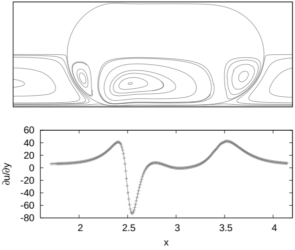
$a = 0.075$: $Z = 0.3$ ($\mathrm{Ca} = 100$) (a), $Z = 1.48$ ($\mathrm{Ca} = 20$) (b), $Z = 2.96$ ($\mathrm{Ca} = 10$) (c), $Z = 14.8$ ($\mathrm{Ca} = 1$) (d)

Richardson Bubble: 2D Bubble in Viscous Flows

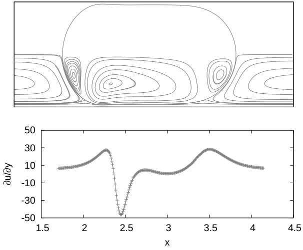


$a = 0.075$ (○), 0.15 (△), and 0.3 (□).

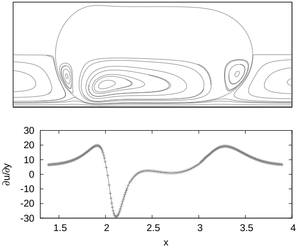
Full Numerical Solutions, $\gamma = 0.01$



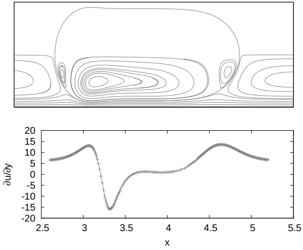
Ca $= 0.005$



Ca $= 0.01$

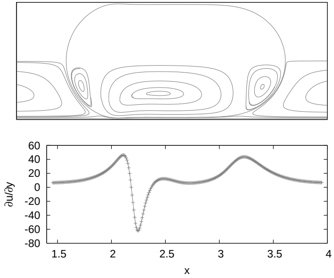


Ca $= 0.02$

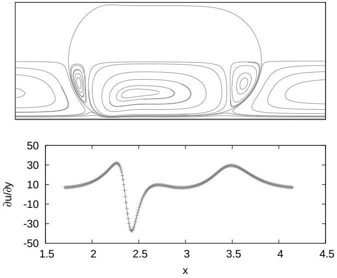


Ca $= 0.045$

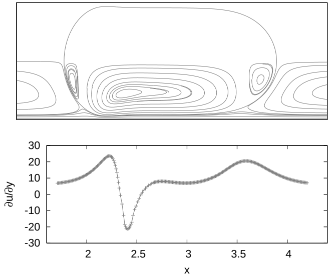
Full Numerical Solutions, $\gamma = 1$

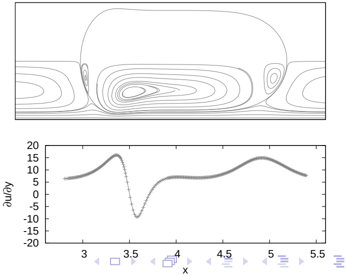


Ca $=$ 0.005

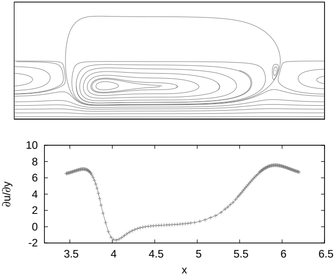


Ca $=$ 0.01

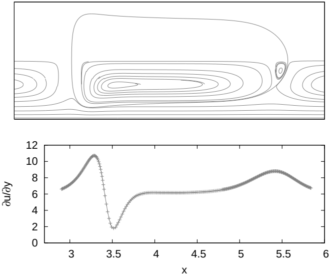




Full Numerical Solutions



$\mathrm{Ca} = 0.27, \gamma = 0.01$



$\mathrm{Ca} = 0.27, \gamma = 1$

Summary

- We have developed accurate surface tension model for numerical simulation of drops and bubbles in microchannels.
- Asymptotic solutions are ascertained with numerical study of breakup of drops in a T-junction.
- Numerical simulations predict estimated critical conditions for breakup.
- Obstructed breakup observed experimentally is simulated numerically.
- Need to develop theory for breakup with obstruction.

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