# Performance of Multi-Relay Collaborative Hybrid-ARQ Protocols over Fading Channels

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***Abstract*—According to collaborative Hybrid Automatic-Repeat-reQuest (ARQ) protocols, relay stations that have been able to decode the original message from previous transmissions, collaborate with the source in future retransmissions by jointly sending a space-time codeword. In this letter, analysis of the average number of retransmissions and throughput of Collaborative Hybrid-ARQ Type I (i.e., without memory) and Chase Combining (i.e., with memory) protocols is provided for any number of relays.**

***Index Terms*—Automatic-repeat-request (ARQ), block fading, collaborative networks, Hybrid ARQ, relay channel.**

## I. INTRODUCTION

COLLABORATIVE communications technology aims at providing spatial diversity through deployment of single-antenna radio terminals, mimicking the ideal scenario where terminals form a fully cooperative antenna array (Multiple Input Single Output, or MISO, case) [1]. In practice, realistic constraints, such as half-duplex transceivers at relay (or cooperating) stations, introduce performance degradation as compared to the ideal reference system [2]. Most of the proposed collaborative schemes exploit a fixed TDMA frame structure for terminal cooperation, where at least one time-slot is dedicated to transmission by relays [2]. This lack of flexibility can induce reduction of system throughput, that can be overcome by exploiting collaboration only *if needed*, as proposed in [3]. Therein, according to the Automatic Repeat reQuest (ARQ) principle, terminals (source and relays) are assumed to collaborate through Space-Time-Block-Coding (STBC) [4] during retransmissions, upon request by the destination. Notice that no channel state information (CSI) at the transmit side and perfect CSI at the receiver side are assumed. In this letter, we derive analytical expression for expected number of retransmissions and achievable throughput of Collaborative Hybrid-ARQ (HARQ) protocols. Analysis is performed for multi-relay networks employing: (*i*) HARQ-Type I (HARQ-TI) protocol, that prescribes memoryless detection, i.e., erroneous packets are discarded at the receivers; and (*ii*) Chase Combining protocol (HARQ-CC) [5], whereby erroneous packets at the destination are preserved for soft combing with the currently received packet.

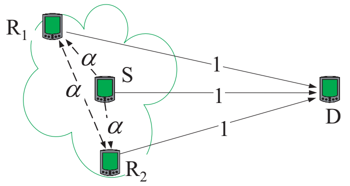

Fig. 1. Illustration of the system model ($M = 2$ relays).

## II. SYSTEM OVERVIEW

### A. Protocol overview

In the first time-slot, the source broadcasts a packet that is decoded at the destination and available relays. If erroneous decoding at the destination is detected through Cyclic Redundancy Check (CRC), a Not-Acknowlegde (NACK) message is sent by the destination requesting retransmission. In this case, relays that have successfully decoded the packet signal their availability to the source and switch to the transmitting mode. During the retransmission slot, the activated relays and the source form a distributed transmitting antenna array, sending a STBC codeword that is decoded at the destination and the remaining receiving relays according to the chosen HARQ protocol. This procedure repeats with possible activation of currently receiving relays, until final successful transmission is confirmed via an Acknowledge (ACK) message from the destination, or until predefined maximum number of retransmission is reached.

### B. System model

We consider a system with $M + 2$ single-antenna stations, consisting in a source, destination and $M$ relays, as shown in Fig. 1. A block Rayleigh fading model is assumed, where the fading channel gain $h_{ij}^{(n)}$ between terminals $i$ and $j$ at transmission $n$ (i.e., retransmission $n - 1$), stays constant during the transmission slot, but changes independently with each retransmission (i.e., with $n$). The channels between any two terminals are mutually independent circularly symmetric complex Gaussian variables with power normalized to unity. As depicted in Fig. 1, the average power received by the relay stations from the source and from other relays is assumed to be larger than the average power received by the destination from any node by a factor $\alpha > 1$. This model accounts for a scenario where source and relays are relatively close and at approximately the same distance from the destination. All nodes transmit with the same power $P$ and all receivers are impaired by Gaussian noise with one-sided power spectral density $N_0$. To complete the set of assumptions, ACK and NACK messages are considered to be received reliably.

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Moreover, their transmission time, as well as propagation and processing delays are considered negligible as compared to the time needed for the packet transmission.

## III. PERFORMANCE ANALYSIS

The average delay, i.e., the expected number of transmissions (original transmission plus retransmission), of a given HARQ protocol is given by

$$E[T]=\sum_{n=1}^{\infty} nP\left\{T=n\right\}, \tag{1}$$

where the probability that $n$ attempts (i.e., $n-1$ retransmissions) are necessary, $P\left\{T=n\right\}$, reads

$$P\left\{T=n\right\}=\left[1-p_e\left(n\right)\right]\prod_{k=1}^{n-1}p_e(k), \tag{2}$$

with $p_e(k)$ denoting the probability that $k$-th transmission is erroneously decoded at the destination, given that the previous transmissions were also unsuccessful. The ratio $C_0/E[T]$, where $C_0$ $[bit/s/Hz]$ is the transmission rate, determines the throughput of the system. In the following, the probability of error $p_e\left(n\right)$ is derived for the collaborative HARQ schemes of interest. The results will be expressed in terms of $F(x,\nu)$, that is the cumulative distribution function of a chi-square variable with $\nu$ degrees of freedom, taken at value $x$. Analysis is corroborated by numerical results in Sec. IV.

### *A. HARQ-TI*

In the first slot ($n=1$), the source ($S$) sends a packet containing a codeword with rate $C_0$. This is received correctly by the destination ($D$) only if the instantaneous achievable rate $C_D(1,0)=\log_2[1+|h_{SD}^{(1)}|^2P/N_0]$ is larger than $C_0$. Therefore, retransmission occurs with probability $P\{C_D(1,0)<C_0\}$. Similarly, a relay $R_i$ ($i=1,...,M$) decodes successfully in the first time-slot and thus collaborates in the possible retransmission if the achievable rate $C_{R_i}(1,0)=\log_2[1+|h_{SR_i}^{(1)}|^2\alpha P/N_0]$ is larger than the transmission rate $C_0$, i.e., with probability $P\{C_{R_i}(1,0)\geq C_0\}$. In general, the achievable rate at the destination for the $n$th transmission depends on the number $\tilde{k}_n$ of relays that have decoded correctly by transmission $n-1$ and therefore collaborate with the source via space-time coding in the $n$th retransmission

$$C_D(n,\tilde{k}_n)=\log_2\left[1+\left(\left|h_{SD}^{(n)}\right|^2+\sum_{j=1}^{\tilde{k}_n}\left|h_{R_jD}^{(n)}\right|^2\right)\frac{P}{N_0}\right]. \tag{3}$$

Notice that with HARQ-TI the achievable rate only depends on the current state of the fading channels since no combining of previously received packets is carried out. Furthermore, in enumerating the $M$ available relays, $R_1,...,R_M$, we have assumed without loss of generality that the indices of the active relays, i.e. the relays that decoded successfully, precede those of inactive.

From the above discussion, it follows that the probability of unsuccessful decoding at the $n$-th transmission (given that the previous $(n-1)$ transmissions were unsuccessful) can be written as (recall (2))

$$p_e(n)=\sum_{\mathcal{K}}P\{C_D(n;\tilde{k}_n)<C_0\}p_R\left(k_1,...,k_{n-1}\right) \tag{4}$$

where $p_R\left(k_1,...,k_{n-1}\right)$ is the probability that $k_1$ relays have decoded successfully in the first transmission, $k_2$ in second and so on. Notice that $\tilde{k}_n=\sum_{i=1}^{n-1}k_i$. Moreover, the sum in (4) is to be carried out over the set $\mathcal{K}$ of tuples $(k_1,...,k_{n-1})$: $\mathcal{K}=\{(k_1,...,k_{n-1})\,|\tilde{k}_n=\sum_{i=1}^{n-1}k_i\leq M\}$. This set can be easily shown to contain $\sum_{i=0}^{M}\binom{n-2+i}{i}$ terms. Since the fading term in (3) is the sum of $1+\tilde{k}_n$ independent exponentially distributed variables, the overall fading gain is a chi-square random variable with $2(1+\tilde{k}_n)$ degrees of freedom and the probability that the destination does not decode at the step $n$ with $\tilde{k}_n$ active relays reads

$$P\{C_D(n,\tilde{k}_n)<C_0\}=F[\mu,2(1+\tilde{k}_n)] \tag{5}$$

with $\mu=2\frac{e^{C_0}-1}{P/N_0}$. The probability $p_R\left(k_1,...,k_{n-1}\right)$ is evaluated next.

Toward this end, the rate achievable at relay $R_i$ at the transmission slot $n$, given that $\tilde{k}_n$ relays ($\tilde{k}_n<i$) turned active by the $(n-1)$th slot, is

$$C_{R_i}(n,\tilde{k}_n)=\log\left[1+\left(\left|h_{SR_i}^{(n)}\right|^2+\sum_{j=1}^{\tilde{k}_n}\left|h_{R_jR_i}^{(n)}\right|^2\right)\frac{\alpha P}{N_0}\right]. \tag{6}$$

Therefore, the probability that in the $n$-th trial the relay $R_i$ still does not successfully decode is

$$\bar{p}_{R_i}(n|\tilde{k}_n)=P\{C_{R_i}(n,\tilde{k}_n)<C_0\}=F[\mu/\alpha,2(\tilde{k}_n+1)]. \tag{7}$$

and the probability that $k_n$ relays successfully decode in the current slot is

$$p_R(k_n|\tilde{k}_n)=P_{bin}(\bar{p}_{R_i}(n,\tilde{k}_n),M-\tilde{k}_n,k_n) \tag{8}$$

where $P_{bin}(p,N,n)=\binom{N}{n}p^{N-n}(1-p)^n$ represents the binomial distribution. Finally, the probability $p_R\left(k_1,...,k_{n-1}\right)$ in (4) reads

$$p_R(k_1,k_2,...,k_{n-1})=\prod_{i=1}^{n}p_R(k_i|\tilde{k}_i), \tag{9}$$

where we have $\tilde{k}_1=0$.

### *B. HARQ-CC*

According to the HARQ-CC protocol, previously received packets are soft combined prior to detection. Therefore, the achievable rates depend not only on the number of transmission trials $n$ and total number of currently active relays $\tilde{k}_n$, but also on the exact time instants when these relays turned active, i.e., on $k_1,...,k_n$ (recall that $\tilde{k}_n=\sum_{i=1}^{n-1}k_i$). In particular, the rate achievable by the destination at transmission $n$ reads:

$$C_D(n,k_1,..,k_{n-1})=\log_2\left[1+\sum_{l=1}^{n}\left(\left|h_{SD}^{(l)}\right|^2+\right.\right. \tag{10}$$
$$\left.\left.+\sum_{j=1}^{\tilde{k}_n}\left|h_{R_jD}^{(l)}\right|^2 1(j\leq\tilde{k}_{l+1})\right)\frac{P}{N_0}\right],$$

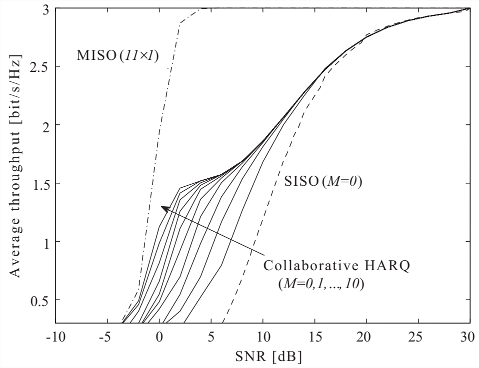

Fig. 2. Average throughput versus SNR for collaborative HARQ-TI ($C_0 = 3$ $bit/s/Hz$).

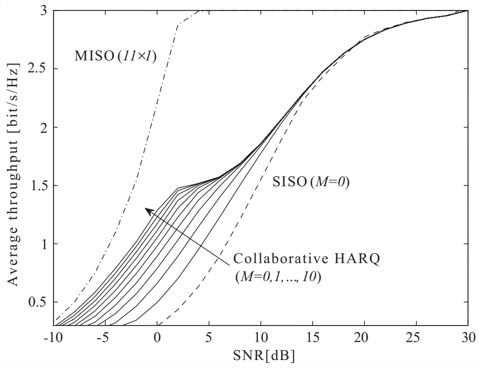

Fig. 3. Average throughput versus SNR for collaborative HARQ-CC ($C_0 =$ $3$ $bit/s/Hz$).

where the indicator function $1(\cdot)$ equals 1 or 0 according to whether its argument is satisfied or not, and guarantees that at each time instant $l = 1, ..., n$ only packets received by active relays are combined. Notice that at time-slot $n$, the fading term in (10) is a chi-square random variable with $n+\sum_{j=1}^{n-1} k_j(n-j)$ degrees of freedom. Thus, the probability of unsuccessful decoding at the $n$th transmission trial conditioned on the event that the previous $(n-1)$ transmissions were unsuccessful is

$$P\left\{C_D(n,k_1,..,k_{n-1}) < C_0|C_D(n-1,k_1,..,k_{n-2}) < C_0\right\} = \\ = \frac{F\left[\mu, 2\left(n+\sum_{j=1}^{n-1} k_j(n-j)\right)\right]}{F\left[\mu, 2\left(n-1+\sum_{j=1}^{n-2} k_j(n-j)\right)\right]}. \tag{11}$$

Accordingly, the probability of unsuccessful decoding at the $n$-th transmission (given that the previous $(n-1)$ transmissions were unsuccessful) can be written as (recall (2))

$$p_e(n) = \sum_{\mathcal{K}} p_R(k_1, ..., k_{n-1}) \times \tag{12}$$
$$\times P\left\{C_D(n,k_1,..,k_{n-1}) < C_0|C_D(n-1,k_1,..,k_{n-2}) < C_0\right\},$$

where $p_R(k_1, ..., k_{n-1})$ is derived in the following. To this end, the achievable rate at the relay $R_i$ in the $n$-th transmission slot is, similarly to (10),

$$C_{R_i}(n;k_1,..,k_{n-1}) = \log_2\left[1+\sum_{l=1}^{n}\left(\left|h_{SR_i}^{(l)}\right|^2 + \right.\right. \tag{13}$$
$$\left.\left.\sum_{j=1}^{\tilde{k}_n}\left|h_{R_jR_i}^{(l)}\right|^2 1(j \le \tilde{k}_{l+1})\right)\frac{\alpha P}{N_0}\right].$$

Therefore, the probability that in the $n$-th trial the relay $R_i$ still does not successfully decode is

$$\bar{p}_{R_i}(n,k_1,..,k_{n-1}) = \\ = P\left\{C_{R_i}(n,k_1,..,k_{n-1}) < C_0|C_{R_i}(n-1,k_1,..,k_{n-2}) < C_0\right\} \\ = \frac{F\left[\mu/\alpha, 2\left(n+\sum_{j=1}^{n-1} k_j(n-j)\right)\right]}{F\left[\mu/\alpha, 2\left(n-1+\sum_{j=1}^{n-2} k_j(n-j)\right)\right]}. \tag{14}$$

It follows that the probability that $k_n$ relays successfully decode in the current slot is

$$p_R(k_n|k_1, ...k_{n-1}) = \\ = P_{bin}(\bar{p}_{R_i}(n,k_1,..,k_{n-1}), M-\tilde{k}_n, k_n), \tag{15}$$

and, finally, the probability $p_R\left(k_1, ..., k_{n-1}\right)$ in (12) reads

$$p_R(k_1,k_2,...,k_{n-1}) = \prod_{i=1}^{n} p_R(k_i|k_1,...,k_{i-1}). \tag{16}$$

## IV. NUMERICAL RESULTS

Fig. 2 and Fig. 3 show the average throughput $C_0/E[T]$ versus $SNR = P/N_0$ for collaborative HARQ systems with $M = 1,..,10$ relays, $C_0 = 3bit/s/Hz$, and HARQ-TI and HARQ-CC protocols respectively (throughput is computed through (4) and (12) in combination with (1)-(2)). Performance bounds set by Single-Input-Single-Output (SISO) systems, i.e., no collaboration, and $11 \times 1$ MISO systems (perfect collaboration among source and 10 relays) are shown. Increasing the protocol complexity (i.e., using HARQ-CC instead of HARQ-TI) or adding relay stations yields relevant performance benefits in the relatively low SNR region, where the number of retransmissions is relevant. However, if the average number of retransmission is less than one (i.e., for average throughput larger than $C_0/2 = 1.5bit/s/Hz$), cooperation does not actually occur and the performance reduces to the lower bound given by the SISO scenario.

## V. CONCLUSION

In this letter, analysis of multi-relay collaborative HARQ systems with both memoryless and packet combining reception has been presented. Numerical results have been provided as well to corroborate the study and provide insight into the performance benefits achievable by the technology.

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