Efficient Spectrum Leasing via Randomized Silencing of Secondary Users

Rocco Di Taranto, Petar Popovski, Osvaldo Simeone, and Hiroyuki Yomo

***Abstract*—In this paper, a primary (licensed) user leases part of its resources to independent secondary (unlicensed) terminals in exchange for a tariff in dollars per bit, under the constraint that secondary transmissions do not cause excessive interference at the primary receiver (PRX). The PRX selects a power allocation (PA) for the secondary user that maximizes the secondary rate (and thus its revenue) and enforces it by the following mechanism: Upon violation of a predefined interference level, PRX keeps silencing randomly selected secondary users, until the aggregate secondary interference is below the required threshold. This mechanism ensures that secondary users may not be willing to deviate from the allocated PA. Specifically, the scenario gives rise to a Stackelberg game, in which the primary determines the PA and a Nash equilibrium (NE) constraint is imposed on the PA to ensure that secondary users do not have incentives to deviate, given their knowledge of the silencing mechanism run at the PRX. In principle, the primary should find the set of all PAs that are NE and among them choose the one that maximizes the aggregate secondary utility, and thereby the revenue of the primary. For the most general setting of channel gains, we investigate the conditions for NE for a subset of PAs. When the scenario is symmetric in the sense that all secondary users have the same channel gains in the direct/interfering links, we prove that only two optimal power allocations exist. Finally, for the case of general channel gains with strong interference, we show that there is a unique NE of the game.**

***Index Terms*—Cognitive radio, dynamic spectrum sharing, game theory, power control, Nash equilibrium, Stackelberg game, silencing process.**

I. INTRODUCTION

THE main idea behind the concepts of cognitive radio [1] and dynamic spectrum access [2] is to allow coexistence on the same spectral resource among primary (licensed) and secondary (unlicensed) users. In this paper, we focus on the *property-rights model* [2]: Primary users are aware of the existence of secondary terminals and they can grant the secondary users the possibility to use the primary band, provided that such a secondary operation does not provoke excessive interference to the primary receivers (PRXs). The rationale is that primary users lease portion of their resources and charge the secondaries in price per bit, such that a primary user is motivated to maximize the aggregate secondary throughput.

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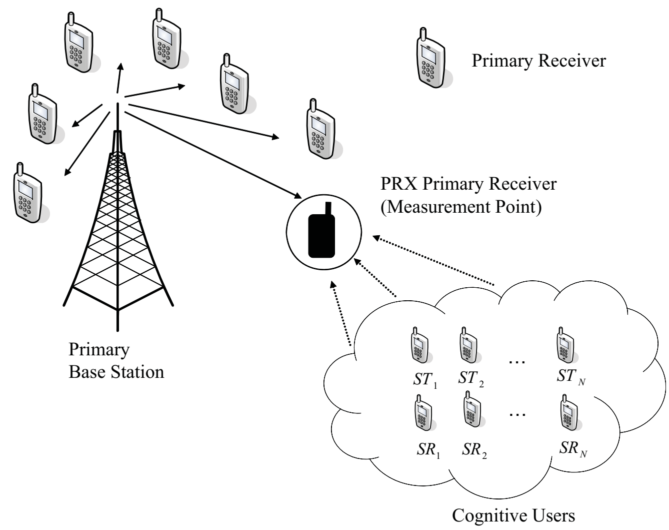

Fig. 1. System model for spectrum sharing among primary and cognitive users.

In turn, the secondary users can start their own transmissions using the primary band.

In our scenario, depicted on Fig. 1, there is one PRX which communicates with the primary Base Station (downlink) and sets the maximum amount of tolerated interference, Q, from the N secondary users toward the PRX. We assume that the PRX actively participates in dynamic spectrum management with the aim of guaranteeing protection to a larger set of primary users by serving as interference measurement point. Notice that since the PRX, by keeping the interference power below the threshold Q, guarantees non-harmful interference to a set of primary receivers, the maximum allowed interference Q should be determined accordingly by accounting for the desired primary quality-of-service requirements. This aspect is not further studied here. The PRX is assumed to know all the channel gains in the system and allocates secondary powers aiming at maximizing secondary utility, and thus its revenue, while guaranteeing that the interference constraint is met. Moreover, we assume for simplicity that all the secondary users know all the channel gains and behave selfishly, thus deviating from the prescribed power allocation (PA) if they can increase their utility. In order to discourage the secondaries from deviating and to enforce a desired PA, the primary uses a shutting-down or silencing (used as synonyms throughout the text) mechanism, such that a secondary user that is shut-down needs to cease transmission.

There are many options on how to select users to be shut down upon violation of Q, but we advocate randomized shut-down as an effective strategy: The PRX randomly selects one secondary user to silence at a time, until the interference level Q is no more violated. The rationale for random secondary silencing can be stated as follows. The PRX knows the identities of the secondary users, since it admits them in the primary spectrum, but it does not have the ability to assess the interference created by each individual secondary user. We assume that PRX can measure the total interference from all the secondary users, but not the interference contribution from each individual secondary user. Thus, upon violation of the interference constraint Q, PRX cannot pinpoint which secondaries are responsible for the violation and punish them. Therefore the silencing process meant to enforce the desired behavior to the secondary users must be random: The PRX silences randomly selected secondary users regardless of the actual interference provoked by each of them[1]. This assumption is realistic because no decoding of the interference signal is needed at the primary receiver PRX.

We assume that the secondary users have best-effort traffic and care only about the average rate, where the average is taken with respect to the random silencing mechanism enforced by the PRX. This models reasonably the secondary users' preferences while keeping the analysis simple. A completely novel model and analysis would be required to reckon with more sophisticated QoS (Quality of Service) parameters, such as guaranteed delay for example. Moreover, we assume that secondary users, selfish and willing to maximize their own utility, follow the power allocation (PA) selected by the PRX only if they have no incentives to deviate. User's average utility in our game is the result of a tradeoff between its transmission probability and its own SINR: A secondary user can always increase its SINR by unilaterally increasing its transmitting power, but at the same time its transmitting probability decreases if its action results in violation of the interference threshold Q at the PRX. This is the key point: Is it worth for a secondary user to increase its transmitting power knowing that this could result in a lower transmitting probability?

This fact is modeled via a a non cooperative power control game so that the corresponding NEs [4] [5] (and the references therein) are taken as stable operating points for the secondary users. *Stackelberg games* [9] can be used as the appropriate analytical framework to study the scenario of spectrum leasing considered in this paper. In such a hierarchical game model, one agent (the competitive secondary network) acts subject to the strategy chosen by the other agent (the PRX leasing the primary spectrum). The latter in turn seeks maximization of the sum-secondary utility, under a maximum interference constraint to the primary system, and under an equilibrium (NE) constraint on the secondary activity. PRX's strategy that yields the optimal solution and the corresponding power response of the secondary network are jointly referred to as *Stackelberg equilibrium* [10]-[18][2].

[1]The architecture and method to realize such signaling between primary and secondary are out of the scope of the paper.

[2]A natural extension of this work is a repeated game where the secondary utility can be seen as an average over multiple stages of a game in which users repeat the same action in all stages: The secondary utility is determined not only by the current channel realizations at a given instant, but also by the statistics of all the channel realizations.

A. Related Work

Previous works have investigated the application of Stackelberg games to modeling resource allocation problems in the context of cognitive radios [10]. In [11] the authors propose a game theoretical approach that allows master-slave cognitive radio pairs to update their transmission powers and frequencies simultaneously. In [12] the authors propose a cooperative cognitive radio framework, where primary users may select secondary users to be cooperative relay, and in turn lease portion of the channel access time to them for their own data transmission. Reference [13] studies the problem of pricing uplink power in wide-band cognitive radio networks. In [14] a Stackelberg game between three entities (spectrum owner, primary users and secondary users) is presented. In reference [15] the concept of Stackelberg equilibrium is introduced in order to characterize the strategic behavior of a user by considering the response of its competing users. In [16] a "waterfilling" game in fading multiple access channels is studied. In [17] the authors propose a distributed buyer/seller Stackelberg game to stimulate cooperation and improve the system performance. The work [18] is primarily interested in the optimal design of an access point in a decentralized network.

Our work is not concerned with any of the aspects above. The central concept introduced here is the mechanism used by the primary in order to enforce certain efficient PAs in a spectrum leasing framework. To the best of our knowledge, this scenario has not been studied hitherto.

B. Paper Organization

In Section II we define our target scenario and system model. Specifically, we discuss options for selecting users that are shut-down or silenced and we single out random selection as an effective option in our work. In Section III, in a game with arbitrary channel gains, we identify conditions for a wideband state (where all users transmit at the same time without provoking excessive interference at the primary system) to be a NE. In section IV we focus on the special case of a symmetric network and we prove that only two optimal power allocations exist: a wideband state defined above, and an orthogonal state (e.g., TDMA) in which only one user transmits at a time. In Section V, in the case of asymmetric network, but strong interference, we show that the orthogonal state is the unique NE of the game. Section VI concludes the paper.

II. Target Scenario and System Model

We consider coexistence between primary and secondary (cognitive) users, as illustrated in Fig. 1. In our model, there is one primary receiver, PRX, that has a possibility to admit secondary users in the primary spectrum, in exchange for a certain revenue (paid in price per bit), provided that the interference at the PRX is maintained below a threshold Q. The value of Q is chosen by the PRX and is not object of study in this paper. We assume that the PRX can ideally measure and quantify the real-time interference toward the primary system. Moreover, we consider an arbitrary number N of secondary users (pair of transmitter and receiver) that are willing to access the primary spectrum and are ready to pay for it. Secondary users follow the power allocation (PA) selected by the PRX only if they have no incentives to deviate. Upon violation of Q, one secondary user at a time is shut-down, until Q is no more violated. In the next subsection, we discuss three different options for selecting the users to shut down upon violation of Q.

A. Silencing Mechanisms to Enforce Policy

Since secondary users are selfish, there is no guarantee that they will adhere to the assigned PA unless it is a NE of the game modeling their preferences, discussed below. Here we list three different strategies which the PRX could possibly use and we advocate the choice of a strategy based on iterative random shutting-down.

Strategy 1: Assume that the PRX has the ability to decode the signals and thus quantify the interference from each secondary user. In this condition, the PRX can enforce any PA, as each user is individually responsible for its interference at the PRX. For example, in [19], the authors assume that the primary system has the ability to trace the interference of individual secondary users and prove the existence of a unique NE. This scenario is not of interest here, as we assume that the PRX cannot decode the secondary signals.

Strategy 2: Assume again that the PRX does not decode signals from secondary users. However, given that both PRX and all secondary users know all the channel gains, all are aware of, for given PA, which secondary users are disadvantaged and thus have incentives to deviate. If some users deviate from the allocated PA and cause violation of Q, then the PRX can decide to silence the subset of penalized secondary users, because they have incentive to deviate. Such a strategy at the PRX can easily give rise to unfair behavior: For example, user A, knowing that a given power allocation penalizes user B, will most likely deviate by increasing its power, because he is aware that in any case B will be shut down as a "usual suspect".

Strategy 3: Assume that the PRX does not have the ability to decode the interference signals[3]. In this case, upon violation of the Q, PRX starts an iterative shutting-down process: Until Q is no longer violated, it randomly selects one cognitive user to be shut down, i.e., to shut off its transmitter. The key point here is that secondary users are gambling whenever they increase their transmitting power: There is no certainty upon user deviation. For example, let us consider two cognitive users whose interference exceeds the interference limit Q. In this case, the PRX first shuts down a randomly selected secondary users. Then, if Q is no longer violated, the second cognitive transmitter can continue its transmissions. Otherwise, no secondary transmission is allowed. Generally, for an arbitrary number of users N, at the end of the shutting-down process, the protection of primary users is guaranteed with certainty.

[3]It should be noted that the fact that the PRX knows the instantaneous α_i ($i = 1, \ldots, N$), as it assumed in our model, does not imply that the PRX can also quantify the interference from each secondary user. In fact, the latter depends on the selected secondary transmission powers. Channel gains may be estimated in an initial training phase.

We assume that secondary users have the freedom to deviate from the first PA, but they are obliged to obey a shut-down command, because generally the shut-down is done by user and thus the *culprit* can always be identified. Note that when certain type of PA is imposed by the PRX shutting-down is part of the allocation, i.e., the primary knows in advance that shut-down should occur: in these cases some shut-down may include multiple users if necessary. For example, when PRX allocates the PA in correspondence of which each secondary generates the maximum tolerated interference Q at the PRX, it knows in advance that, in a N-player game, $N-1$ users have to be shut down: In this case the PRX randomly selects all the $N-1$ users to shut down at once, instead of shutting-down one of them at a time. On the other hand, if a user deviates from the PA, then there may be shutting-down that were not initially planned by the PRX. One might ask: How can the primary be sure that a given secondary will obey the shut-down command? As a slight digression on this issue, one can consider ordinary framed ALOHA: it is designed under the assumption that all users will follow the command to enlarge the backoff window upon collision. If not, then additional inference algorithms should be applied in order to assess whether the terminals are obeying the commands. The complete mechanism for verifying that the users are obedient to the shut-down command is outside the scope for this paper. For the purpose of this paper, we can assume that punishment can be in the form of a reputation mechanism. If a disobedient secondary user is found, then this user gains a bad reputation in a centralized reputation system and no primary user will grant access to that secondary user for extended (practically infinite) period of time. In short, we assume that there is a system that strongly discourages deviation upon a shut-down command, but detailed specification and analysis is outside the scope for this work.

III. General Channel Gains

Here we formalize the game to model preferences and competitive behavior of the secondaries under a shutting-down process. Later we discuss the optimal power allocation used by PRX.

Let $\mathcal{G} = [\mathcal{N}, \{\mathcal{P}_i\}, \{R_i\}]$ denote the secondary non-cooperative power control game where the N players in the game correspond to the N secondary users. Each player $i \in \mathcal{N} = \{1, 2, \ldots, N\}$ selects a transmitting power, i.e., strategy, $p_i \in \mathcal{P}_i = \{p_i \geq 0\}$ such as to maximize its utility R_i. The joint strategy space $\mathcal{P}_i = \mathcal{P}_1 \times \mathcal{P}_2 \times \ldots \times \mathcal{P}_N$ is the Cartesian product of the individual strategy sets for the N players. As shown in Fig. 2, γ_i denotes the channel gain for secondary user i, ϵ_{ij} is the intra-secondary system gain between transmitter i and receiver j, and α_i represents the channel gain between transmitter i and the PRX.

User i's evaluation of the spectrum is characterized by a utility function $R_i(\mathbf{p})$, with $\mathbf{p} = [p_1, \ldots, p_N]$. This depends on received SINR η_i at user i's receiver, given by

$$\eta_i(\mathbf{p}) = \frac{p_i \gamma_i}{\sum\limits_{j \neq i} p_j \epsilon_{ji} + N_0} \tag{1}$$

where p_i is user i's transmission power. N_0 is the noise power (including the interference power from the PTX) assumed to

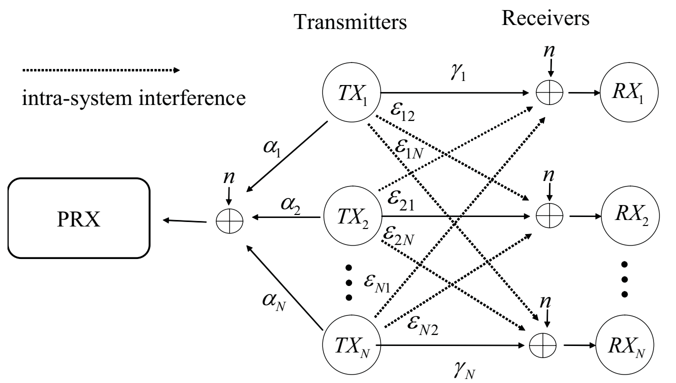

Fig. 2. System model for spectrum sharing.

be the same for all users. The total interfering power at the PRX must satisfy

$$P_I = p_1\alpha_1 + p_2\alpha_2 + ... + p_N\alpha_N \le Q, \tag{2}$$

where Q is the the maximum amount of tolerable interference at the PRX. The PRX shuts down randomly selected secondary users, one at a time, until condition (2) is satisfied. The final set $\mathcal{S} \subseteq \mathcal{N}$ of non-shut-down users represents the set of users that are allowed to transmit satisfying the interference constraint. We define the *capacity* function r_i as

$$r_i(\mathbf{p}, \mathcal{S}) = \begin{cases} \log_2(1 + \frac{p_i\gamma_i}{\sum\limits_{j\in\mathcal{S}, j\neq i} p_j\epsilon_{ji} + N_0}) & \text{if } i \in \mathcal{S} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The utility function is defined as the average rate with respect to the shut-down process as

$$R_i(\mathbf{p}) = E_{\mathcal{S}}[r_i(\mathbf{p}, \mathcal{S})]. \tag{4}$$

The distribution of the set of transmitting users $\mathcal{S}$ depends on $\mathbf{p}$, Q and the channel gains α_i ($i = 1, ..., N$), since PRX shuts down one randomly selected user at a time until (2) is satisfied. For example, assume that for fixed α_i, Q and $\mathbf{p}_{-i}$ condition (2) is satisfied. If user i increases its transmitting power such that the interference threshold Q is violated, then (at least) one shutting-down is needed at the PRX, thus determining a new set $\mathcal{S}'$ of users transmitting at the end of each shutting-down realization. Similar considerations hold for variations of α_i and Q.

The ith user utility is written as:

$$R_i(\mathbf{p}) = \sum_{\mathcal{S}\subseteq\mathcal{N}: i\in\mathcal{S}} \Phi(\mathbf{p}, \mathcal{S}) r_i(\mathbf{p}, \mathcal{S}) \qquad \text{where} \tag{5}$$

$$\Phi(\mathbf{p}, \mathcal{S}) = Pr[\textit{set S selected at the end of the shut-down process}|\mathbf{p}]. \tag{6}$$

For instance, suppose that $N = 2$, $\alpha_i = \epsilon_{ij} = \gamma_i = 1$, and $p_i \le \frac{Q}{2}$. The utility is $R_i(\mathbf{p}) = \log_2(1 + \frac{p_i}{p_j + N_0})$ ($i = 1, 2$ and $j \neq i$), because no shutting-down process at the PRX is needed, and the two players can always transmit at the same time, i.e., $\Phi(\mathbf{p}, \{1, 2\}) = 1$ and $r_i(\mathbf{p}, \{1, 2\}) = \log_2(1 + \frac{p_i}{p_j + N_0})$. On the contrary, if $\frac{Q}{2} < p_i \le Q$ ($i = 1, 2$), the interference at the PRX exceeds Q and the PRX randomly selects one user to shut down so that there is always one user transmitting at any time. The utility can be therefore written as $R_i(\mathbf{p}) = \frac{1}{2}\log_2(1 + \frac{p_i}{N_0})$ because $\Phi(\mathbf{p}, \{1, 2\}) = 0$, $\Phi(\mathbf{p}, \{i\}) = \frac{1}{2}$ and $r_i(\mathbf{p}, \{i\}) = \log_2(1 + \frac{p_i}{N_0})$.

Note that the utility (5) is not necessarily equal to the achievable rate, which would entail that the secondary operate over multiple blocks and thus attain a rate averaged over time. Equation (5) may as well be taken as a reasonable metric to be used by the secondary users to guide their choices. The PRX attempts to maximize the sum-utility of the secondaries $\sum\limits_{i=1}^{N} R_i(\mathbf{p})$, under the condition that no secondary will deviate its allocation from $\mathbf{p}$. In order to address the latter point, we utilize the concept of Nash Equilibrium (NE).

Definition 1: A power profile $\mathbf{p}^* = (p_1, p_2, ..., p_N)$ is said to be a NE of the strategic game $\mathcal{G}$ if for every $i = 1, 2, ..., N$

$$R_i(p_i, p_{-i}) \ge R_i(p_i', p_{-i}) \tag{7}$$

for any $p_i' \neq p_i \in \mathcal{P}_i$, where p_{-i} denotes the transmission powers of all the users except user i.
At a NE, given the powers of the other users, no user can increase its utility by deviating through unilateral changes in its power. We define $\mathcal{NE}$ such that $\mathbf{p} \in \mathcal{NE}$ if and only if it is NE.

Defining as $\mathcal{NE}$ the set of all NEs, the problem to be solved at the PRX is then:

$$\max_{\mathbf{p}} \sum_{i=1}^{N} R_i(\mathbf{p}) \qquad \textbf{s.t.} \quad p_1\alpha_1 + ... + p_N\alpha_N \le Q \qquad \textbf{AND} \quad \mathbf{p} \in \mathcal{NE}. \tag{8}$$

In correspondence of (8), the PRX allocates the secondary powers $\mathbf{p} = (p_1, ..., p_N)$ so as to maximize the secondary system utility under the maximum interference constraint to the primary system, and under the constraint that $\mathbf{p}$ is one of the NEs of the game. This is an optimization problem with equilibrium constraints also referred to as Stackelberg game [10]-[18].

A. Solving the General Problem

In general, solving (8) requires to explore all NEs. However, finding the set of all PAs that are NE is, in general, a tedious task. Besides $\mathbf{p}_{OR}$, nothing guarantees on the existence and the number of all the possible NEs of the game. The PRX could try to identify all the NEs numerically in a centralized approach; this is a big computational task for PRX, especially when considering games with many players and moreover, in a fading scenario, time constraints on the durations of these calculations should be taken into account. Additionally, it would be disputable whether a centralized solution can be applied in the context of independent secondary users. In this section, instead, for the general model at hand, we focus on two specific solutions: The wideband allocation $\mathbf{p}_{WB} = (\frac{Q}{N\alpha_1}, \frac{Q}{N\alpha_2}, ..., \frac{Q}{N\alpha_N})$, where each user provokes identical interference $\frac{Q}{N}$ and there is no shut-down, and the orthogonal allocation $\mathbf{p}_{OR} = (\frac{Q}{\alpha_1}, \frac{Q}{\alpha_2}, ..., \frac{Q}{\alpha_N})$ that has $N-1$ planned shutting-down, as only one user can be left to transmit.

$\mathbf{p}_{WB}$ is relevant when channel gains in the direct links are sufficiently high. For example, when the intended receiver is much closer to the transmitter compared to the other receivers that are interfered by that transmitter, likely no secondary will have incentives to deviate and no shut-down will be caused. This can be advantageous both for the primary (PRX need not to shut down users) and for secondaries that are interested in non-bursty communications (i.e., a secondary prefers to transmit for long time at low rates rather than at high rates in short time).

The next proposition identifies the conditions upon which the wideband transmission $\mathbf{p}_{WB}$ and the orthogonal transmission $\mathbf{p}_{OR}$ are NEs in our game. It should be noted that the policy $\mathbf{p}_{OR}$ provides the same utility, when (5) is interpreted as achievable rate, as one in which the PRX selects randomly one on the N users for transmission.

Proposition 1: In the game $\mathcal{G}$ defined above *i)* $\mathbf{p}_{OR}$ is always a NE, and *ii)* there exists a minimum direct link gain γ^* such that if $\gamma_i > \gamma^*$, (where γ^* is generally a function of the interference gains ϵ_{ij}) for $i = 1, ..., N$, then $\mathbf{p}_{WB}$ is NE in our game.

Proof: See Appendix A.

The main conclusion here is that, $\mathbf{p}_{OR}$ is always a NE and thereby considered as an allocation strategy by the PRX. When the channel gain in the direct link of each secondary user is sufficiently large with respect to the interference (see previous Proposition), $\mathbf{p}_{WB}$ becomes a NE and thus appears as an alternative to the primary. When $\mathbf{p}_{WB}$ is NE, no user i ($i = 1, ..., N$) can increase its utility by unilaterally increasing its transmitting power to $\frac{Q}{\alpha_i}$. On the other hand with this deviation, the utility of user i equals the utility it would gain if $\mathbf{p}_{OR}$ were allocated by the PRX: User i generates an interference Q at the PRX therefore it transmits with probability $\frac{1}{N}$ without intrasystem interference exactly as in correspondence of $\mathbf{p}_{OR}$.

But it can be seen from (5) that the utility of user i, with this deviation, will be equal to the one obtained with $\mathbf{p}_{OR}$. Therefore the aggregate secondary utility in correspondence of $\mathbf{p}_{WB}$, when is NE, is higher than in correspondence of $\mathbf{p}_{OR}$. Since the primary is paid in price per bit, it will always allocate $\mathbf{p}_{WB}$ when it is NE. To summarize, whenever $\mathbf{p}_{WB}$ is a NE, the PRX will tend to select this PA over $\mathbf{p}_{OR}$. Otherwise, it can always select $\mathbf{p}_{OR}$, this being a NE of the considered game, albeit not guaranteed to maximize the aggregate secondary utility.

To illustrate Proposition 1, on Fig. 3 we plot γ^*, which is the minimal value of γ for which $\mathbf{p}_{WB}$ is a NE, assuming symmetry in channel conditions $\gamma_i = \gamma$, $\alpha_i = \alpha = 0.1$, $\epsilon_{ij} = \epsilon = 0, 5, 10, 20$ (for $i, j = 1, ..., N$ and $i \neq j$) and $Q = 1$. The x-axis represents the number of secondaries. It can be seen that in the absence of intra-system interference, $\epsilon = 0$, $\mathbf{p}_{WB}$ is always a NE in a game with $N = 2$ and 3 secondaries, and no minimal value of γ^* is needed to support this power allocation. In fact, due to (5) the user's utilities are the result of a tradeoff between transmission probability and SINR. In general, the transmission probabilities decrease upon violation of Q and SINR is directly/inversely proportional to transmitting power/intra-secondary system interference. However, when $\epsilon = 0$, SINR equals SNR and therefore the advantage for the deviating user when transmitting alone or with only one secondary user, relies only on the higher transmitting power

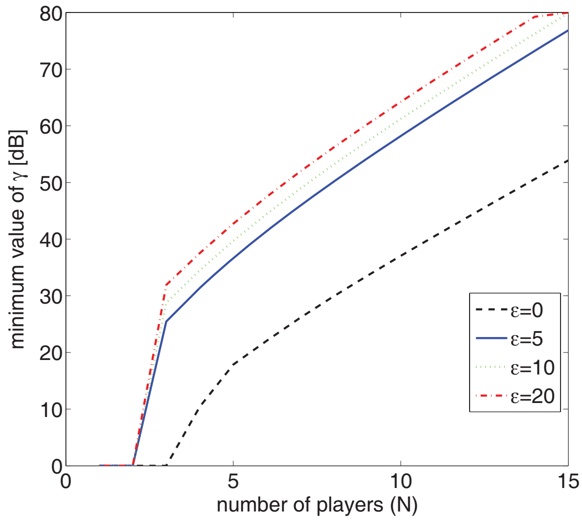

Fig. 3. Minimum γ for $\mathbf{p}_{WB}$, power allocation without violation of Q, to be NE (Q=1, $\alpha = 0.1$).

(and not on the lower intra-secondary system interference). As it is shown in Fig. 3, when $N = 2$ or $N = 3$, the probability to be shut down for the deviating secondary is considerable and always dominates the advantages from transmitting with higher power (and lower probability).

When $N > 3$, the situation is different and a minimum value of $\gamma^* > 0$ is needed to sustain $\mathbf{p}_{WB}$ (for any value of ϵ). Note that γ^* increases with N. On the one hand, the probability to be shut down decreases as the number of secondaries increases, thus giving (higher) motivation for deviation (even with $\epsilon = 0$). On the other hand, when γ is large users prefer to transmit for the longest time possible without violation of Q. These two conflicting needs explain why γ^* increases with the number of secondaries N.

Figure 3 shows also the influence of the intra-system interference, corresponding to different values of ϵ. For fixed N, the value of γ^* increases with ϵ. This is because upon increasing ϵ, the secondaries harm each other more and more and therefore a given user may sensibly increase its utility upon deviation (not only by increasing its transmitting power, but also be receiving reduced intra-secondary system interference). Similarly as before, it is always possible to find a minimum value γ^* where all the secondaries prefer not to violate the Q. This happens because the tradeoff between the higher transmitting power, lower intra-system interference and the shutting-down probability upon deviation is not advantageous for the deviating secondary when γ is very high.

B. Simulation results for Rayleigh fading channels

Here we evaluate the secondary aggregate utility in correspondence of the PA selected by the PRX. As a reference, we evaluate the PA that maximizes the aggregate secondary utility, while respecting the interference constraint (2), but is not necessarily a NE. That PA is obtained as a solution to the optimization problem modeling the case where secondary

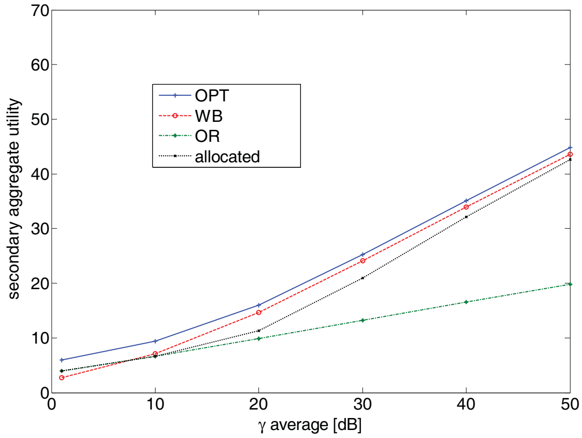

Fig. 4. Secondary aggregate utility versus average direct link γ average (N=3).

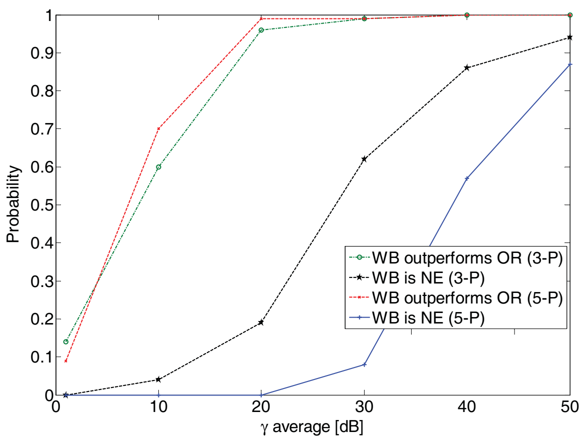

Fig. 5. Probability of allocation of wideband state $\mathbf{p}_{WB}$ (N=3,5).

users are not selfish and can be written as:

$$\max_{\mathbf{p}} \sum_{i=1}^{N} R_i(\mathbf{p}) \quad \textbf{s.t.} \quad p_1\alpha_1 + ... + p_N\alpha_N \leq Q. \qquad (9)$$

We consider a Rayleigh fading scenario, where the channels γ_i, α_i and ϵ_{ij} fade independently. We assume that all the α_i $(i = 1, ..., N)$ fades independently with average $\bar{\alpha}$, all the γ_i $(i = 1, ..., N)$ fade independently with average $\bar{\gamma}$, and all the ϵ_{ij} $(i = 1, ..., N, j = 1, ..., N, i \neq j)$ fade independently with average $\bar{\epsilon}$. Recall (Proposition 1) that $\mathbf{p}_{OR}$ is always NE while it exists a condition on the direct link of each secondary user for $\mathbf{p}_{WB}$ to be NE.

Figure 4 refers to scenario with three secondary users (N=3) where $\bar{\alpha} = 1$ dB, $\bar{\epsilon} = 1$ dB and $\bar{\gamma}$ ranges between 1 and 50 dB. We average the performances of four different power allocations over 10000 channel realizations. We plot the average secondary aggregate utility with the four following power allocation: *a)* optimal power allocation (approximated for each fading realization by numerical search), *b)* $\mathbf{p}_{WB}$, regardless if it is a NE, *c)* $\mathbf{p}_{OR}$, *d)* $\mathbf{p}_{WB}$ if it is NE, and $\mathbf{p}_{OR}$ otherwise. It can be noted that, for small values of $\bar{\gamma}$, the secondary aggregate utility is larger with $\mathbf{p}_{OR}$ than with $\mathbf{p}_{WB}$. This happens because when the channel gain in the direct link is, on average, equal to channel gain of the interfering links between the secondary users, the intra-secondary interference strongly affects the SINR at the receiver. Therefore, each user benefits transmitting without intra-secondary system interference, as with $\mathbf{p}_{OR}$. The situation is reversed when $\bar{\gamma}$ increases: in this case the direct link is on average sensibly larger than the interfering links and favors the non-risky $\mathbf{p}_{WB}$, since the interference is not very significant. Note that for $\bar{\gamma} < 10$, $\mathbf{p}_{WB}$ is not a NE and thus its aggregate secondary utility may be smaller than for $\mathbf{p}_{OR}$.

Figure 5 shows the probability *a)* that the secondary aggregate utility in correspondence of $\mathbf{p}_{WB}$ is higher than in correspondence of $\mathbf{p}_{OR}$, and *b)* that $\mathbf{p}_{WB}$ is NE and therefore allocated by the PRX. It is important to note that in order for $\mathbf{p}_{WB}$ to be NE, the condition in *a)* is not sufficient; on the contrary we know with certainty that if $\mathbf{p}_{WB}$ is NE, then the aggregate secondary utility in its correspondence is higher than with $\mathbf{p}_{OR}$. When $\bar{\gamma}$ increases, the probability that $\mathbf{p}_{WB}$ outperform $\mathbf{p}_{OR}$ increases considerably towards 1. The same tendency can be observed about the probability for $\mathbf{p}_{WB}$ to be NE, albeit this probability attains much lower values than 1. For fixed value of $\bar{\gamma}$, the probability for $\mathbf{p}_{WB}$ to be a NE is higher for $N = 3$ compared to $N = 5$. This happens because in our game setting, users are randomly selected for shutting-down upon violation of Q, and therefore the probability to be shut down decreases as the number of secondaries increases: a higher value of $\bar{\gamma}$ is needed with $N = 5$ for $\mathbf{p}_{WB}$ to be a NE.

In this section, we have compared only two specific PAs, i.e., $\mathbf{p}_{WB}$ and $\mathbf{p}_{OR}$: In a game with arbitrary channel gain it is very difficult to identify all the possible NEs. Nevertheless, when considering specific cases, e.g., symmetric networks (Section IV) and strongly interfered scenarios (Section V), we are able to approach the problem of optimal PA in a more comprehensive and exhaustive way.

IV. Symmetric Channel Gains

In this section, we restrict our attention to a symmetric scenario where *a)* all the secondary users have the same channel gain toward the intended receiver $\gamma_i = \gamma$ for $i = 1, ..., N$, *b)* all the users have the same channel gain toward the PRX $\alpha_i = \alpha$, for $i = 1, ..., N$ and *c)* the intra-secondary interference between transmitter i and received j is the same for all the users $\epsilon_{ij} = \epsilon$ for all $i, j = 1, ..., N$ with $i \neq j$. In Section III-A, we have compared two specific PAs in terms of overall secondary utility, while here we attack problem (8) more thoroughly. We identify the optimal power vector $\mathbf{p}$ allocated by the PRX to the secondary users in two different cases. We first find $\mathbf{p}$ that maximizes the aggregate secondary utility without necessarily being a NE. Next, we study the maximization problem as defined in (8).

A. Without NE constraint

Here we study problem (9) restricted to a symmetric network.

Proposition 2: There exists a $\epsilon^* \geq 0$ such that the solution to (9) is *a) wideband transmission* $\mathbf{p}_{WB} = (\frac{Q}{N\alpha}, \frac{Q}{N\alpha}, ..., \frac{Q}{N\alpha})$ when $\epsilon < \epsilon^*$; *b)* $\underline{\mathbf{p}}$, in which all non-zero elements are equal to $\frac{Q}{\alpha}$.

Proof: See Appendix B.

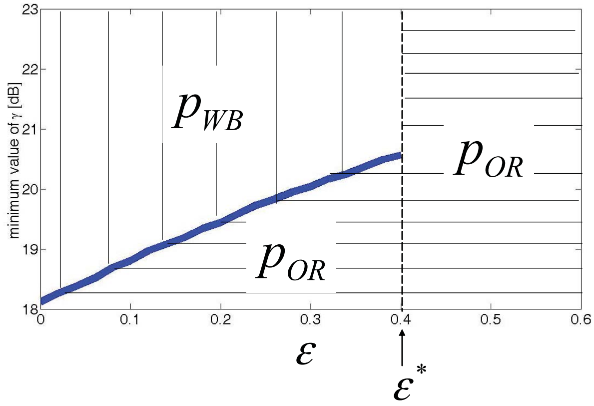

Fig. 6. Optimality region for the two power allocations $\mathbf{p}_{WB}$ and $\mathbf{p}_{OR}$ for problem (9).

The threshold ϵ^* generally changes with the number N of secondaries and is function of Q and noise level at the receiver. It is worth noticing, that in a scenario with arbitrary channel gain, the solution to (9) can in principle be any power allocation satisfying the interference constraint (2) at the PRX. Note that $\mathbf{p}_{OR}$ is a special case of $\underline{\mathbf{p}}$, in which no component is zero.

B. With NE constraint

Here we deal with problem (8). Given the result of the previous section, $\mathbf{p}_{WB}$ and the orthogonal transmissions $\underline{\mathbf{p}}$ are the only possible solutions. However, in (8) they have to satisfy an additional constraint: they must be a NE of the game $\mathcal{G}$. Note that only $\mathbf{p}_{OR}$ among all possible vectors $\underline{\mathbf{p}}$ is a NE: This happens because, for any orthogonal transmission $\underline{\mathbf{p}}$ (defined above) with at least one zero-element, there is at least one user (gaining utility zero) which simply increasing its transmitting power (originally equal to zero) to a value less or equal than $\frac{Q}{\alpha}$ can gain a positive utility.

The next proposition identifies the conditions under which *a)* they are NEs and *b)* they maximize the aggregate system throughout in (8).

Proposition 3: If $\gamma > \gamma^*$, with γ^* defined in Proposition 1, and if $\epsilon < \epsilon^*$, with ϵ^* defined in Proposition 1, $\mathbf{p}_{WB}$ solves problem (8); otherwise the solution is $\mathbf{p}_{OR}$.

Proof: Proposition 1 states that, in a game with arbitrary channel gains, *a)* $\mathbf{p}_{OR}$ is always NE, and *b)* $\mathbf{p}_{WB}$ is NE if $\gamma_i > \gamma$, $i = 1, ..., N$. Clearly, this results holds also for a game with symmetric channel gains. Proposition 2 guarantees that, in a game with symmetric channel gains, depending on ϵ, there exist only two possible PAs which maximize the secondary aggregate utility: $\mathbf{p}_{WB}$ or $\mathbf{p}_{OR}$. Proposition 3 combines the conclusions from Proposition 1 and 2 to identify the conditions upon which $\mathbf{p}_{WB}$ maximizes the secondary aggregate utility and is NE (as it is requested in (8)). When both conditions are satisfied, $\mathbf{p}_{WB}$ solves (8); otherwise the solution is $\mathbf{p}_{OR}$. □

Figure 6 illustrates Proposition 3 through a numerical example for a game with $N = 5$. Depending on the values of γ and ϵ two regions can be identified in Fig. 6: The region with vertical lines identifies situations where $\mathbf{p}_{WB}$ is the solution of the Stackelberg game. The region with horizontal lines identifies the conditions under which $\mathbf{p}_{OR}$ is the optimal solution. For $\epsilon < \epsilon^*$, the minimum value of γ to support $\mathbf{p}_{WB}$ increases with ϵ. This happens because with the larger intra-system secondary system interference ϵ, higher γ is needed to support $\mathbf{p}_{WB}$. The vertical dashed line in correspondence of $\epsilon = \epsilon^*$ specifies other two important regions in correspondence of which either $\mathbf{p}_{WB}$ ($\epsilon < \epsilon^*$) or $\mathbf{p}_{OR}$ ($\epsilon > \epsilon^*$) solve problem (9), without NEs constraint as per Proposition 3.

V. Optimal Power Allocation with Strong Interference

In the previous section we have made a restrictive assumption of perfect symmetry in the secondary system. Here, we still assume that γ and α are the same for all the secondaries, but we relax the conditions for the interference, such that ϵ_{ij} are not necessarily equal. In our game, the PRX shuts down randomly selected secondary users until (2) is satisfied. We can therefore find the distribution of the set of transmitting users $\mathcal{S}$, and the probability $\Phi(\mathbf{p}, \mathcal{S})$ that set $\mathcal{S}$ is selected at the end of a given realization of the shutting-down process (see (5)-(6)). Therefore, our system is akin, interpreting (5) as achievable rates, to a TDMA or FDMA system, where a given set $\mathcal{S}$ of users transmits for a fraction of the whole time/frequency slot. The fraction of time/frequency slot allocated to set $\mathcal{S}$ equals $\Phi(\mathbf{p}, \mathcal{S})$ in (6).

In our scenario, under the given power constraints, TDMA and FDMA schemes are equivalent, and this allows us to reuse some interesting results presented in [21] where the authors consider the scenario of multiple multicarrier communication systems contending in a common frequency band in flat channels and prove some conditions for the optimality of flat frequency sharing and flat FDMA (frequency division multiple access). A channel is said to be *flat* in a given frequency band $W = f_1 - f_2$ if the channel gains are constant for any frequency within that frequency band. For detailed description of results in [21] see Appendix C.

Normalizing ϵ_{ij}, α and N_0 by the direct channel gain γ (see Appendix C for details), we prove the following proposition in a strong intra-secondary system interference scenario. This case is interesting because it encompasses scenarios where secondary users are located in a small geographical area, and/or with low propagation attenuation and path loss (e.g., low frequencies).

Proposition 4: In a N-player game, if $\epsilon_{ij} \geq \frac{1}{2}$, $\forall i \neq j$, $i, j = 1, 2, ..., N$, $\mathbf{p}_{OR}$ is the unique NE and therefore solves problem (8), which is an optimization problem with NE constraints.

Proof: See Appendix D

VI. Conclusions

In this paper, we have designed robust spectrum leasing solutions among N secondary users with a constraint on the total interference at a particular primary receiver (where robustness is with respect to the selfish behavior of the secondary users). We have used a secondary network power control game to model the secondary behavior. First, we have introduced a shutting-down mechanism at the primary system in order to enforce a given power allocation on the secondary users that guarantees that the primary link target quality of service is not affected. Then, with the random shutting-down mechanism in place, we have identified some special power allocations that are NEs. In a scenario with arbitrary channel gains, conditions are identified upon which a wideband state, where all users transmit at the same time and provoke the same interference at the primary, is NE. We have also studied a symmetric network and a specific asymmetric network with strong intra-secondary interference.

Our study opens a large number of interesting items for future investigations. For example, in [22] the PRX actively participates in the game by varying the maximum interference threshold Q depending on the secondary users' strategies: Such an approach can be combined with the silencing mechanism introduced in this paper to discourage a secondary from misbehaving.

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APPENDIX

A. Proof of Proposition 1 and Related Results

Lemma 1: When deviating from $\mathbf{p}_{WB} = \left[\frac{Q}{N\alpha_1}, \frac{Q}{N\alpha_2}, ..., \frac{Q}{N\alpha_N}\right]$ player i must select one of the actions $p_i^{(k)} = \frac{(k+1)Q}{N\alpha_i}$, where $k = 1, 2, ..., N-1$, in order to possibly increase its utility.

Proof: Let us assume that user 1 deviates from $\mathbf{p}_{WB}$ (while all the other secondaries stick to it) increasing p_1 to improve its utility (same considerations for the other secondaries). As in (5), user 1's average utility is a tradeoff between its transmission probability (6) and its own SINR, $r_I(\mathbf{p}, \mathcal{S})$. On the one hand, $r_I(\mathbf{p}, \mathcal{S})$ is an increasing function of p_1. On the other hand, when user 1 increases p_1, its transmitting probability certainly decreases because this results in a violation of the Q at the PRX. Since all users, except for user 1, provoke the same interference $\frac{Q}{N}$ at the PRX, the number of users which can coexist with user 1 (without violation of Q) does not depend on their identity; moreover, there are intervals of p_1 in correspondence of which user 1 can transmit with the same number of other secondaries. Specifically, for any value of p_1 in $I_k = \{\frac{kQ}{N\alpha_1} < p_1 \leq \frac{(k+1)Q}{N\alpha_1}\}$, $k = 1, , N-1$, (where k is interpreted as the number of secondaries to be shut down by the PRX) user 1 transmits with the same probability: In fact when not backed off, it can coexist with the same number of secondary users, regardless of their identities. Therefore rational user 1 will select the largest transmitting power within each interval I_k and this proves the Lemma. □

Lemma 2: In the game $\mathcal{G}$ there are only N *symmetric* PAs that are candidates for NE: $\mathbf{p}_i = \left[\frac{Q}{(N-i+1)\alpha_1}, \frac{Q}{(N-i+1)\alpha_2}, ..., \frac{Q}{(N-i+1)\alpha_N}\right]$, where $i = 1, \ldots N$. Note that $\mathbf{p}_1 = \mathbf{p}_{WB}$ and $\mathbf{p}_N = \mathbf{p}_{OR}$.

Proof: Fix a symmetric allocation $\mathbf{p} = [p_1, ..., p_N]$ such that all the secondaries generate the same interference $I = \alpha_i p_i$ at the PRX. If m^* is the largest integer for which $m^* I \leq Q$, then

$$\Phi(\mathbf{p}, \mathcal{S}) = \begin{cases} 0 & \text{if} \quad |S| \neq m^* \\ \frac{1}{\binom{N}{m^*}} & \text{if} \quad |S| = m^* \end{cases} \tag{10}$$

All the symmetric $\mathbf{p}$ in which $m^* I < Q$ cannot be NE, because each user can increase its transmitting power keeping $\Phi(\mathbf{p}, \mathcal{S})$ constant and thus increasing its utility. Conversely, if $\mathbf{p}$ is such that $m^* I = Q$ no user can increase its power without decreasing his transmission probability $\Phi(\mathbf{p}, \mathcal{S})$. The N PAs listed in Lemma 2 are the only ones that satisfy $m^* I = Q$. □

Lemma 3: In a N-player game $\mathcal{G}$ the shutting-down probabilities in correspondence of the $N-1$ *smart* actions $p_i^{(k)} = \frac{(k+1)Q}{N\alpha_i}$ (Lemma 1) are $P_b(i) = \frac{k}{N}$, for any $k = 1, 2, ..., N-1$.

Proof: With $p_i^{(k)} = \frac{(k+1)Q}{N\alpha_i}$, user i can transmit at the same time with $N-k-1$ other secondaries, i.e., k secondary users have to be shut-down. In a N-player game, if k secondaries have been randomly chosen to be shut-down, then the number of ways in which the selection can be done is given by the combination $C(N, k)$. Among this selection, if some user i has been selected, then there are $C(N-1, k-1)$ ways of selecting the other $k-1$ users from the rest of the $N-1$ group. Thus the probability of selecting a given user among k selections becomes $P_b = \frac{C(N-1,k-1)}{C(N,k)}$. Substituting the factorial terms for the combination, and canceling out the common factors, we get $P_b = k/N$.

Proof of Proposition 1: i) With $\mathbf{p}_{OR} = \left[\frac{Q}{\alpha_1}, ..., \frac{Q}{\alpha_N}\right]$ each user generates an interference at the PRX equal to Q. On the one hand, no user can increase its utility by decreasing its power. On the other hand, if any user increases its power (thus violating the Q at PRX), then it is shut down with certainty and its utility is always zero. This shows that $\mathbf{p}_{OR}$ is always a NE. *ii)* Let us first focus on user 1: Its utility in correspondence of $\mathbf{p}_{WB}$ can be written as $R_1(\mathbf{p}_{WB}) = \log_2\left(1 + \frac{x_1}{S}\right)$, where $x_1 = p_1\gamma_1$, $S = N_0 + Y_2 + ... + Y_N$ and $Y_2 = p_2\epsilon_{21} ..., Y_N = p_N\epsilon_{N1}$. Following Lemma 1, when deviating from $\mathbf{p}_{WB}$, player 1 must select one of the $N-1$ actions $p_1^{(k)} = \frac{(k+1)Q}{N\alpha_1}$, $k = 1, 2, ..., N-1$, in order to possibly increase its utility. Notice that in correspondence of $p_1^{(k)}$ ($k = 1, 2..., N-1$), user 1 can transmit at the same time with other $N-k-1$ secondaries (thus provoking interference Q at the PRX), regardless of their identity. Moreover if we assume, without loss of generality, that $Y_2 \geq Y_3 \geq ... \geq Y_N$, we can write $R_1(p_1^{(k)}, \mathbf{p}_{WB_{-1}}) \leq \frac{N-k}{N} \log_2\left(1 + \frac{(k+1)x_1}{S - \sum_{j=2}^{k+1} Y_j}\right) = V_1(k)$, where in the right side it is assumed that *a)* user 1 always transmits with the less interfering secondaries, i.e., with the lowest Y_i, $i = k+2, ..., N$, and *b)* $\frac{N-k}{N}$ is the transmitting probability of user 1 in correspondence of $p_1^{(k)}$ (Lemma 3). Then, by not considering the intra-system interference Y_i, $R_1(p_1^{(k)}, \mathbf{p}_{WB_{-1}}) \leq V_1(k) < \frac{N-k}{N} \log_2(1+(k+1)x_1)$. Define the continuous function of z, $U(z) = (N-z+1)\log_2(1+zx_1)$ by substituting the integer k with $z = k+1$. We want to show that $U(z)$ is a decreasing function of z, $z \geq 1$ (integral values of z correspond to k which is a positive integer): In fact, sufficient condition for $\mathbf{p}_{WB}$ to be a NE is that $\frac{dU}{dz} < 0, z \geq 1$, because then the user is not motivated to deviate to a higher power level, with a risk to be silenced. $\frac{dU}{dz} < 0$ implies that $a = \log_2(1 + x_1 z) > x_1 \frac{N-z+1}{1+x_1 z} = b$. Since $\lim_{x_1 \to \infty} a = \infty$ and $\lim_{x_1 \to \infty} b = \frac{N-z+1}{z}$, it follows that it always exist a value of x_1 (and thus γ_1^*) such that $\frac{dU}{dx} < 0$ for any $x \geq 1$. The same procedure can be repeated for all the N players, and finally $\gamma^* = \max(\gamma_1^*, \gamma_2^*, ..., \gamma_N^*)$.

B. Proof of Proposition 2

We start proving that the optimum value for (9) is smaller than the optimum value for the maximization problem solved in [20].

$$\max_{\mathbf{p}} \sum_{i=1}^{N} R_i(\mathbf{p}) = \max_{\mathbf{p}} \sum_{S \subseteq N} \Phi(\mathbf{p}, S) \sum_{i=1}^{N} r_i(\mathbf{p}, S) \tag{11}$$

$$\leq \max_{\mathbf{p}'} \sum_{S \subseteq N} \Phi(\mathbf{p}', S) \max_{p: \sum_s p_i \leq \frac{Q}{\alpha}} \sum_{i=1}^{N} r_i(\mathbf{p}, S) \tag{12}$$

$$\leq \max_{\mathbf{p}'} \sum_{S \subseteq N} \Phi(\mathbf{p}', S) \max_{p: \sum_N p_i \leq \frac{Q}{\alpha}} \sum_{i=1}^{N} r_i(\mathbf{p}, N) \tag{13}$$

$$\leq \max_{p: \sum_N p_i \leq \frac{Q}{\alpha}} \sum_{i=1}^{N} r_i(\mathbf{p}, N). \tag{14}$$

Problem (14) has been solved by [20]. The solution to (14) is either *a)* $[\frac{Q}{N\alpha}, \frac{Q}{N\alpha}, ..., \frac{Q}{N\alpha}] = \mathbf{p}_{WB}$ $(\epsilon < \epsilon^*)$, or *b)* $p_i = \frac{Q}{\alpha}$ and $p_j = 0$ for $j \neq i$ $(\epsilon > \epsilon^*)$. But (9) evaluated for $\mathbf{p}_{WB}$ or $p_i = \frac{Q}{\alpha}$ and $p_j = 0$ for $j \neq i$ equals the optimum value of (14). Therefore they are solutions of (9) as well.

C. Results by Zhao and Pottie from [21]

Here we define parameters ϵ_{ij}, α and N_0 which are normalized by the direct channel gain γ: $\epsilon_{ij_n} = \frac{\epsilon_{ij}}{\gamma}$, $\alpha_n = \frac{\alpha}{\gamma}$, $N_{0n} = \frac{N_0}{\gamma}$. [21] considers multiple multicarrier communication systems contending in a common frequency band in flat channels. There are two basic co-existence strategies for common flat channels: *Flat frequency sharing* and *Flat FDMA*. Given a flat channel in the band (f_1, f_2), $N_{0n}(f) = N_{0n}; \alpha_{21}(f) = \alpha_{21}; \alpha_{12}(f) = \alpha_{12}; \forall f \in (f_1, f_2)$, a flat frequency sharing of two users is defined as any power allocation in the form of $P_1(f) = p_1, P_2(f) = p_2, \forall f \in (f_1, f_2)$. On the other hand, a flat FDMA of two users is defined as any power allocation in the form of: $\{P_1(f)P_2(f) = 0, \quad P_1(f) + P_2(f) = p', \quad \forall f \in (f_1, f_2)$.

The authors introduce a basic transformation from flat frequency sharing to flat FDMA: *flat FDMA re-allocation*. By noting the bandwidth , $W = f_2 - f_1$, a flat FDMA re-allocation is defined to be the following scheme that transforms a flat frequency sharing to a flat FDMA: (1) User 1 re-allocates all of its power within a sub-band $W_1' = \frac{p_1}{p_1+p_2} W$ with a flat power spectral density (PSD) $p_1' = p_1 + p_2$; (2) User 2 re-allocates all of its power within another disjoint sub-band $W_2' = \frac{p_2}{p_1+p_2} W$ with the same flat PSD $p_2' = p_1 + p_2$. Similarly, the authors define flat frequency sharing schemes, flat FDMA schemes, and flat FDMA re-allocation in *n-user* flat channel access. The following Lemma is proved in [21]:

Lemma 4: Consider an N-user flat interference channel: $N_{0n}(f) = N_{0n}$, $\alpha_{ij}(f) = \alpha_{ij}$. Let the N users use frequency sharing: $P_i(f) = p_i, \forall f \in (f_1, f_2), i = 1, 2, 3, ..., N$. If $\alpha_{ij} \geq \frac{1}{2}, \forall i \neq j$, then with a flat FDMA re-allocation scheme, the rate of each user at least remains equal.

D. Proof Proposition 4

Lemma 5: In a N-player game, if $\epsilon_{ij} \geq \frac{1}{2}$, $\forall i \neq j$, then $U_i(\mathbf{p}_{OR}) = U_i(\mathbf{p}_N) \geq U_i(\mathbf{p}_j)$, $\quad i = 1, 2, ..., N \quad j = 1, 2, ..., N-1$.

Proof: *a)* We first prove that, if $\epsilon_{ij} \geq \frac{1}{2}$, $\forall i \neq j$, then $U_i(\mathbf{p}_{OR}) \geq U_i(\mathbf{p}_{WB})$, where $i = 1, 2, ..., N$ *(i)*. When $\mathbf{p}_{WB}$ is allocated by the PRX each user i transmits with power $p_i = \frac{Q}{N\alpha}$ and coexists with all the other $N-1$ users. Let us describe the FDMA-reallocation of $\mathbf{p}_{WB}$ for a generic user i: $W_i' = \frac{\frac{Q}{N\alpha}}{\frac{Q}{N\alpha} + ... + \frac{Q}{N\alpha}} = \frac{1}{N}$; $\quad p_i' = \frac{Q}{N\alpha} ... + \frac{Q}{N\alpha} = \frac{Q}{\alpha}$. Note that $\mathbf{p}_{OR}$ corresponds to a FDMA reallocation of $\mathbf{p}_{WB}$: Each user transmits with power $p_i' = \frac{Q}{\alpha}$ and bandwidth $W_i' = \frac{1}{N}$. Then, we use Lemma 4 and this proves *(i)*. *b)* We prove Lemma 5. We consider the utility gained by each secondary in each of the N symmetric PA candidates for NE, see Lemma 2. For those PAs, say $\mathbf{p}_R$, $R = 1, ..., N$, any user M transmits at the same time with other $N-R$ with probability $P_b = \frac{N-R+1}{N}$. From Lemma 2 with $\mathbf{p}_R$ each player generates interference $I = \frac{Q}{N-R+1}$ at the PRX, therefore there can be only $N-R+1$ secondary users transmitting at the same time (same considerations for the other players). This also means that user M transmits with probability $P_{tx} = \frac{N-R+1}{N}$. Let us focus on user 1 (same considerations for the other users). In correspondence of any symmetric joint strategy profile $\mathbf{p}_R$, by putting $P_{tx} = \frac{N-R+1}{N}$ the expected utility of user 1 is $U_1(\mathbf{p}_R) = P_{tx} U_1(\mathbf{p}_{WB}(N-R+1)) + (1 - P_{tx}) \times 0$, where $U_1(\mathbf{p}_{WB}(N-R+1))$ is user 1's utility in a game with $N-R+1$ players, when $\mathbf{p}_{WB}$ is allocated. This happens because from user 1's point of view there are two sub-games in correspondence of $\mathbf{p}_R$: A sub-game A with $N-R+1$ players where user 1 always transmits with $N-R$ players, and a sub-game B where user 1 is always shut down. Now, from *(i)* (valid also for a game with $N-R+1$ players) $U_1(\mathbf{p}_R) = P_{tx} U_1(\mathbf{p}_{WB}(N-R+1)) \leq P_{tx} U_1(\mathbf{p}_{OR}(N-R+1))$. But, $P_{tx} U_i(\mathbf{p}_{OR}(N-R+1))$ is the utility of user 1 with $\mathbf{p}_{OR}(N)$; in fact, $P_{tx} \frac{1}{N-R+1} = \frac{N-R+1}{N} \frac{1}{N-R+1} \log_2(1 + \frac{Q}{\alpha}\gamma) = \frac{1}{N} \log_2(1 + \frac{Q}{\alpha}\gamma)$, which means that user 1 is allocated $\frac{1}{N}$ of the total bandwidth (as with $\mathbf{p}_{OR}(N)$) where it can transmit with power $\frac{Q}{\alpha}$. $\square$

Proof Proposition 4: In a N-player game there are N symmetric PAs candidates for NE. Lemma 5 implies that the only NE among them is $\mathbf{p}_{OR}$. We prove that there does not exist any asymmetric PA that can be a NE under a strong interference. Transmitting powers in an asymmetric PA can be ordered. Assume user 1 uses the lowest power. We prove that user 1 deviates from any asymmetric PA: We can always identify one symmetric PA (Lemma 2) by which user 1 gains at least equal utility. Consequently, user 1 deviates from any asymmetric joint power allocation, and this proves the Proposition. We recall (Lemma 2) that in correspondence to each symmetric PA, user 1 transmits with power $p_1 = \frac{Q}{N-i}$, and probability $P_{tx} = \frac{N-i}{N}$. We prove that *a)* for $0 < \bar{p_1} < \frac{Q}{N\alpha}$, $U_1(\bar{p_1}, \mathbf{p}_{-1}) < U_1\left(\frac{Q}{N\alpha}, ..., \frac{Q}{N\alpha}\right)$, and *b)* for $\frac{Q}{(N-i+1)\alpha} < \bar{p_1} < \frac{Q}{(N-i)\alpha}$, $U_1(\bar{p_1}, \mathbf{p}_{-1}) < U_1\left(\frac{Q}{(N-i)\alpha}, ..., \frac{Q}{(N-i)\alpha}\right)$, $i = 1, ..., N-1$. *a)* In correspondence of $\mathbf{p}_{WB} = \left(\frac{Q}{N\alpha}, ..., \frac{Q}{N\alpha}\right)$, $\Phi(\mathbf{p}_{WB}, (1, ..., N)) = 1$, (see (6)), therefore user 1 always transmits with power $p_1 = \frac{Q}{N\alpha}$. If $0 < \bar{p_1} < \frac{Q}{N\alpha}$ utility of user 1 is certainly lower than in correspondence of $\mathbf{p}_{WB}$: transmitting probability cannot be increased, and transmitting power is lower by assumption. *b)* Whenever $\frac{Q}{(N-i+1)} < \bar{p_1} < \frac{Q}{(N-i)}$, at least i users are shut down. This happens because, by assumption, $p_1 > \frac{Q}{(N-i+1)}$ and $p_1 < p_2 \leq p_3 \leq ... \leq p_N$: if all the secondaries had transmitted with power $\frac{Q}{(N-i+1)}$, $i-1$ shutting-down would have been needed, after which $N-i$ users can remain to transmit without violation of Q at the PRX, and at the end of the shutting-down process there is no room for additional interference. This means that with $(\bar{p_1}, \mathbf{p}_{-1})$ user 1 can transmit in the best case with probability $P_{tx} = \frac{N-i}{N}$. But in correspondence of the joint strategy profile $\left(\frac{Q}{(N-i)}, ..., \frac{Q}{(N-i)}\right)$ user 1 transmits with probability $P_{tx} = \frac{N-i}{N}$ and higher power. Since we have proven that the user deviates from the symmetric vector $\left(\frac{Q}{(N-i)}, ..., \frac{Q}{(N-i)}\right)$, then it follows that it will deviate from the vector $(\bar{p_1}, \mathbf{p}_{-1})$. $\square$

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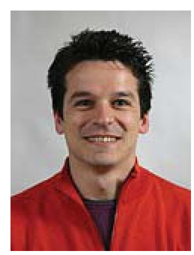

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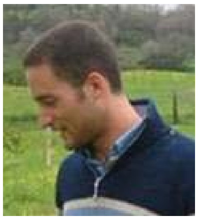

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