Optimal Design of a Multi-Antenna Access Point with Decentralized Power Control Using Game Theory

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***Abstract*— Power control for uplink channels with non-cooperative and rational mobile stations (MS) can be studied in the framework of game theory. In this paper, we investigate the optimal design of a multi-antenna access point (AP) in such a scenario by modelling the interaction between the AP on one side, and the distributed set of MSs on the other, as a *Stackelberg game*. As a game leader, the AP determines the network parameters (bandwidth and the number of receiving antennas) for the power control game played between the MSs (follower), so as to maximize the network utility per system resource (bandwidth and antennas). Two game models are considered, whereby the network utility is measured either in terms of power minimization or power efficiency maximization. Extensive numerical results provide insights into the properties of the optimal design.**

I. INTRODUCTION

Power control is typically employed in uplink wireless channels in order to guarantee a sufficient strength of the user's signal while limiting its interfering effect on signals belonging to other users [1]. Optimal power control mechanisms require the access point (AP) to be able to control directly the power transmitted by mobile stations (MSs). This cannot be guaranteed in some wireless networks, such as in systems complying with the cognitive radio principle, where competitive behavior is expected to be predominant [2].

Game theory, a mathematical framework thoroughly investigated and employed in economic field [3], is a promising paradigm for modelling the performance of wireless networks that involve multiple nodes (i.e., MSs) not controlled by some central authority [4]. As these independent nodes (*players* in the game-theoretic jargon) have goals that are usually in conflict with each other, their selfish behavior might lead to extremely poor network performance. Game theory allows to predict the possible outcomes of interaction (game) between the competitive MSs, in terms of *Nash Equilibria* (NE). Therefore, it is a powerful tool for defining a set of rules to be enforced on the players that would lead to more desirable outcome.

In this paper, we consider a system with decentralized power control (see, e.g., [5]). Moreover, we use the fact that, although the MSs are not directly controlled by the AP, the game they participate in, along with its NE, is strongly dependent on the network parameters set by the AP (for example, available bandwidth and number of AP antennas). Therefore, the optimal system design requires the AP to set those parameters in a manner that provokes the most desirable power allocation (NE) from the MSs [6]. This framework where one agent (set of MSs) acts subject to the strategy that the other agent (AP) chose (with the latter aware that his action is observed), is referred to as a *Stackelberg game* [3]. Moreover, the corresponding optimal pair of system parameters and power allocation is referred to as a *Stackelberg Equilibrium* (SE). A related work is in [6], where the provider (AP) acts as a Stackelberg leader whose goal is to encourage the cooperative transmission between terminals (follower), by optimizing the service prices and possible reimbursements.

In this work, we consider two network models. The first assumes that the MSs' actions are dictated by the transmission power minimization under minimum capacity (transmission rate) constraints, while the second model is concerned with maximizing the power efficiency of the MSs. The service provider (AP) is consumer-oriented, and it aims at maximizing the users' preferences, while saving on investments such as bandwidth and network infrastructure (namely, AP antennas).

II. SYSTEM SETUP AND PROBLEM DEFINITION

A. System Setup

Consider a set $\mathcal{K}$ of K single-antenna MSs that are transmitting in the same time-frequency resource towards an AP with transmission powers P_i, $i = 1, .., K$, using asynchronous code-division access with processing gain $G \geq 1$. The set of all transmission powers is $\mathbf{P} = (P_1, P_2.., P_K)^T \in \mathcal{P}$, where $\mathcal{P}$ is the set of allowed MSs' powers, and the maximum transmission power per user is denoted as $P_{\max}$. The AP is equipped with N (receiving) antennas, and the independent identically distributed (iid) complex Gaussian channel gains between ith MS and jth AP antenna are denoted as h_{ij}. Using a vector notation, the set of channels between user i and N antennas is $\mathbf{h}_i = (h_{i1}, .., h_{iN})^T$, while the set of all channel gains is given by $N \times K$ matrix $\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_K)$. We assume matched filtering (MF) at the AP with no interference cancellation. White Gaussian noise at any of the AP antennas is independent, with single-sided power spectral density N_o. Interference coming from other users' signals is modelled as Gaussian noise. Assuming that the station are sending "Gaussian codewords" and, without loss of generality, that the used bandwidth is G Hz, the maximum achievable rate for the ith MS, C_i (in bit/sec), can be written as:

$$C_i(\mathbf{P}, \mathbf{H}, N, G) = \log_2(1 + SINR_i), \quad (1)$$

where the Signal to Noise plus Interference Ratio for the ith MS, $SINR_i$, at the output of the MF is easily shown to be:

$$SINR_i = \frac{P_i \left\|\mathbf{h}_i\right\|^2}{N_0 + \frac{1}{G}\sum_{\substack{k=1\\k\neq i}}^{K} \frac{\left|\mathbf{h}_i^H \mathbf{h}_k\right|^2}{\left\|\mathbf{h}_i\right\|^2} P_k}. \quad (2)$$

In (1) we emphasized the dependence of the achievable rate C_i on the set of transmission powers $\mathbf{P}$, channel gain matrix $\mathbf{H}$ and the parameters set by the AP, N and G.

B. Problem Definition

We distinguish between two system entities, namely the set of MSs on one side and the AP on the other. The goal of the AP is the maximization of a long-term revenue function $\rho(N, G)$ that depends on both the network parameters (number of antennas N and processing gain G), that are under the direct control of the AP, and the behavior of the MSs that cannot be directly controlled by the AP. The revenue function $\rho(N, G)$ is defined as an average over the statistics of channel gains $\mathbf{H}$ in order to account for different (fading) scenarios.

The goal of each MS is to maximize its own (instantaneous) utility function $u_i(\mathbf{P}; N, G, \mathbf{H})$, $i = 1, .., K$, defined as to reflect MS's preferences, usually in terms of achievable transmission rate and/or consumed power. The degree of freedom of each MS, say ith, is its transmission power P_i, while the parameters N and G, and the channel matrix $\mathbf{H}$, are given. To emphasize this point, we will equivalently use the notation $u_i(P_i, \mathbf{P}_{-i}; N, G, \mathbf{H})$, where $\mathbf{P}_{-i}$ stands for the vector containing all but the ith element of $\mathbf{P}$ (i.e., it denotes the set of other MSs' strategies). Furthermore, the MSs are independent and behave in a *selfish* and *rational*[1] manner, with goals typically in direct conflict. The whole set of MSs can be presented as one entity that receives as input the network parameters set by the AP (N and G), and produces an output defined by a Nash Equilibrium (NE), $\hat{\mathbf{P}}(N, G, \mathbf{H}) = \left(\hat{P}_1, \hat{P}_2 .., \hat{P}_K\right)^T$, of the non-cooperative game $\langle \mathcal{K}, \mathcal{P}, \{u_i(\cdot)\}\rangle$ played by MSs (see fig. 1).

The interaction between AP and the set of MSs described above can be studied in the framework of Stackelberg games. The AP represents the authority of the game (Stackelberg leader), playing the first move by setting the network parameters (N and G) towards the aim of increasing its revenue function $\rho(N, G)$. The MSs on the other side (Stackelberg follower) respond with the NE $\hat{\mathbf{P}}(N, G, \mathbf{H})$ of their non-cooperative game. In principle, this interchange of parameters and MS game outcomes continues until the Stackelberg Equilibrium (SE) is reached, i.e., until the AP finds the set of parameters (N and G) that, together with the corresponding NEs of the MS game, maximize its long-term (i.e., average over channel fading $\mathbf{H}$) revenue function $\rho(N, G)$.

[1]The selfish player is interested solely in maximizing its own benefit, without concern for the collective good; the rational player chooses only those strategies that are best responses to his opponents' strategies.

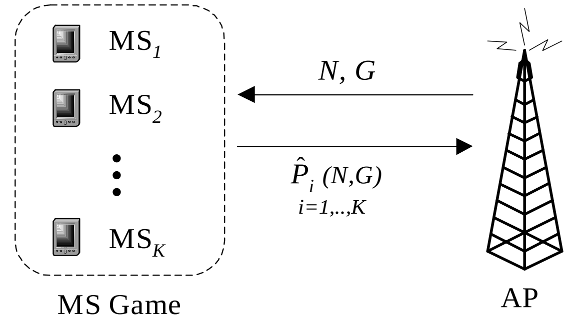

Fig. 1. Overview of Stackelberg game between the AP and the MSs.

III. Game Models

In the following, two game models are presented. In the first game, the MSs (follower) tackle the problem of minimizing the transmission power under minimum transmission rate constraint, while in the second they aim at the (unconstrained) maximization of power efficiency (bit/sec/W). For each game, the AP optimizes the network utility (in terms of collective MSs' preferences) per invested system resource, i.e., per antenna and bandwidth. Performance of the considered distributed models is assessed by comparison with the corresponding centralized scenarios.

A. Minimizing the Power under Capacity Constraints

1) MS Game: For given network parameters N and G, the goal of the MS i is to minimize its own transmission power P_i under minimum transmission rate constraint, $C_{i,\min}$:

$$\begin{aligned} &\text{minimize } P_i,\ i = 1, .., K, \\ &\text{subject to } C_i(\mathbf{P}, \mathbf{H}, N, G) \geq C_{i,\min}, \\ &\qquad P_i \in [0, P_{\max}], \end{aligned} \quad (3)$$

This problem can be formulated as the non-cooperative power control game (NPG) $\langle \mathcal{K}, \mathcal{P}, \{u_i(P_i, \mathbf{P}_{-i})\}\rangle$, where we recall that $\mathcal{K} = \{1, 2, .., K\}$ denotes the set of K players (MSs), the players' set of strategies $\mathcal{P}$ reads

$$\mathcal{P} = \{\mathbf{P} | P_i \in [0, P_{\max}],\ C_i(\mathbf{P}, \mathbf{H}, N, G) \geq C_{i,\min}, \forall i \in \mathcal{K}\}, \quad (4)$$

and the ith player's utility function is defined as

$$u_i(P_i, \mathbf{P}_{-i}) = -P_i,\ i \in \mathcal{K}. \quad (5)$$

Notice that the strategy sets for different users are coupled according to (4). Furthermore, the parameters set by the AP, i.e., N and G, and the channel gains $\mathbf{H}$, influence the game through its constraints and not through its utility $u_i(P_i, \mathbf{P}_{-i})$. To conclude on the game setup, we note that in a game theory framework a strictly concave utility function is preferred, so we equivalently replace (5) with

$$u_i\left(P_i, \mathbf{P}_{-i}\right) = -\log_2 P_i,\ i \in \mathcal{K}, \tag{6}$$

where the base 2 of the log function is chosen purely for the sake of consistency with the definition of capacity (1).

The NPG $\langle \mathcal{K}, \mathcal{P}, \{u_i(\cdot)\}\rangle$, is easily shown to be an (exact) potential game[2], $\langle \mathcal{K}, \mathcal{P}, U\rangle$, with the following potential function:

$$U(\mathbf{P}) = -\sum_{i=1}^{K} \log_2 P_i. \tag{7}$$

Assuming the optimization problem (3) is feasible, the set of strategies $\mathcal{P}$ is compact. Furthermore, $U(\mathbf{P})$ is a continuous and strictly concave function on the interior of $\mathcal{P}$. It follows that a strategy $\mathbf{P}_{\text{opt}}$ that maximizes the potential $U(\mathbf{P})$, $\mathbf{P}_{\text{opt}} = \arg\max_{\mathbf{P}} U(\mathbf{P})$, is also a NE of the NPG $\langle \mathcal{K}, \mathcal{P}, \{u_i(\cdot)\}\rangle$ [9]. Furthermore, since the set $\mathcal{P}$ is also convex (in fact, it is a cone), following [9] the optimal $\mathbf{P}_{\text{opt}}$, and therefore the NE, $\hat{\mathbf{P}}(N, G, \mathbf{H}) = \mathbf{P}_{\text{opt}}$, is unique.

Both Gauss-Seidel and Jacobi algorithms, implementing best response, better response or the gradient projection rule, are guaranteed to reach the NE of the potential game at hand [9], [7]-[8]. Here we detail the Gauss-Seidel algorithm with the best response rule. The MSs play sequentially, and at the $(t+1)$th iteration the ith MS updates its transmission power following:

$$P_i^{t+1} = \min\left(P_i^*, P_{\max}\right), \tag{8}$$

where P_i^* is the minimum power satisfying the constraint $C_i = C_{i,\min}$ (recall (1) and (2)):

$$P_i^* = \frac{\left(2^{C_{i,\min}} - 1\right)}{\left|\left|\mathbf{h}_i\right|\right|^2} \times \left(N_0 + \right. \tag{9}$$

$$\left. + \frac{1}{G} \sum\nolimits_{k=1}^{i-1} \frac{\left|\mathbf{h}_i^H \mathbf{h}_k\right|^2}{\left|\left|\mathbf{h}_i\right|\right|^2} P_k^{t+1} + \frac{1}{G} \sum\nolimits_{k=i+1}^{K} \frac{\left|\mathbf{h}_i^H \mathbf{h}_k\right|^2}{\left|\left|\mathbf{h}_i\right|\right|^2} P_k^{t} \right).$$

The converging point of the algorithm is the NE strategy set $\hat{\mathbf{P}}(N, G, \mathbf{H})$, where $\hat{\mathbf{P}}(N, G, \mathbf{H}) = \mathbf{P}_{\text{opt}}$.

2) AP Revenue Function: The revenue function accounts for the preferences of the service provider, e.g., profit (if it is charging the users for the service while investing in equipment) or quality of service (measured in SINR ratios, achievable rates, the probability of error, etc.). Here we assume a service provider that, following the users' interest, strives to minimize the total power expenditure. However, it is also interested in reducing the cost of the two primary resources: number of antennas and bandwidth. We propose the following revenue function that measures the overall average network utility per system resource:

$$\rho(N, G) = \frac{-\sum_{i=1}^{K} E_{\mathbf{H}}\left[\log_2\left(\hat{P}_i\left(N, G, \mathbf{H}\right)\right)\right]}{NG}. \tag{10}$$

The expectation $E_{\mathbf{H}}[\cdot]$ is taken with respect to fading, since decentralized power control by the MSs is operated according to the instantaneous channel realization, while the system optimization is based on (long-term) channel statistics. Note that the revenue function in (10) depends on the NE of the MS game $\hat{\mathbf{P}}_i$, which in turn is a function of parameters N and G, set by the AP.

3) Centralized Scenario: For reference, we also consider the case where the AP is able to control optimally not only the network parameters N and G, but also the MS transmission power, $\mathbf{P}(N, G, \mathbf{H})$, toward the goal of maximizing (10) (where the NE $\hat{\mathbf{P}}(N, G, \mathbf{H})$ is substituted with the variable $\mathbf{P}(N, G, \mathbf{H})$**)**. From the discussion above, the decentralized solution of the power control (NE) for given N and G is the one that maximizes the potential (7). Comparing (7) with (10), it is easy to see that decentralized and centralized solution coincide in this case (see also [9]). Section III-B.3 will discuss a scenario where this does not hold true.

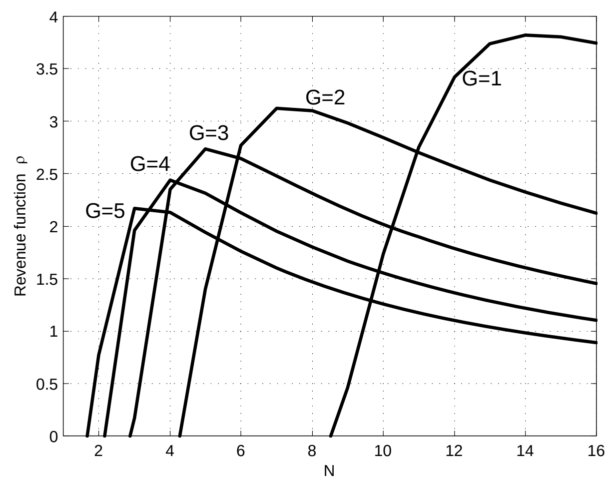

Fig. 2. Revenue function of the AP, ρ, versus number of AP antennas N for different values of processing gain (bandwidth) G.

4) System Performance: The results in this section are obtained for the following parameters: $E\left[\left|h_{ij}\right|^2\right] = 1$, $C_{i,\min} = 1\ \text{bit/sec}$, $P_{\max} = 2$ W and the average Signal to Noise Ratio (defined as $SINR$ for $N = 1$, $K = 1$ and $P = P_{\max}$) is $SNR = 13$ dB. Figure 2 shows the revenue function $\rho(G, N)$ in (10) versus the number of antennas N for $K = 10$ MSs and different values of processing gain G. It can be observed that, for fixed G, the revenue increases with N up to a certain (optimal) point, after which the collective MS utility (7) (i.e., the numerator in (10)) becomes less then linearly proportional to N. In other words, ρ has a unique maximum, that is a SE,

[2]Analysis of the game, namely the assessment, existence and uniqueness of NEs, is significantly simplified for the class of potential games [4], [9]. For a strategic game, say $\left\langle \mathcal{K}', \mathcal{P}', \left\{u_i'(\cdot)\right\}\right\rangle$, to be a potential game, there needs to exist a function $U' : \mathcal{P}' \to \mathbb{R}$ such that for all $i \in \mathcal{K}'$ and $(P_i', \mathbf{P}'_{-i})$, $(P_i'', \mathbf{P}'_{-i}) \in \mathcal{P}'$, it satisfies either $u_i'(P_i', \mathbf{P}'_{-i}) - u_i'(P_i'', \mathbf{P}'_{-i}) = U'(P_i', \mathbf{P}'_{-i}) - U'(P_i'', \mathbf{P}'_{-i})$, in which case it is called an *exact* potential game; or $u_i'(P_i', \mathbf{P}'_{-i}) - u_i'(P_i'', \mathbf{P}'_{-i}) > 0 \Leftrightarrow U'(P_i', \mathbf{P}'_{-i}) - U'(P_i'', \mathbf{P}'_{-i}) > 0$, in which case it is an *ordinal* potential game. The function $U'(\cdot)$ is called a *potential function*.

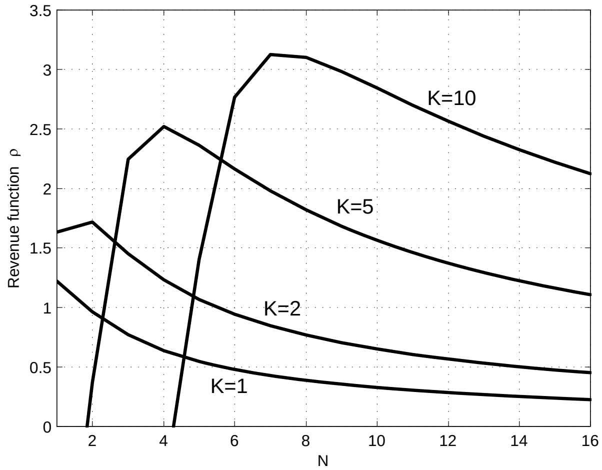

Fig. 3. Revenue function of the AP, ρ, versus number of AP antennas N for different number of users K.

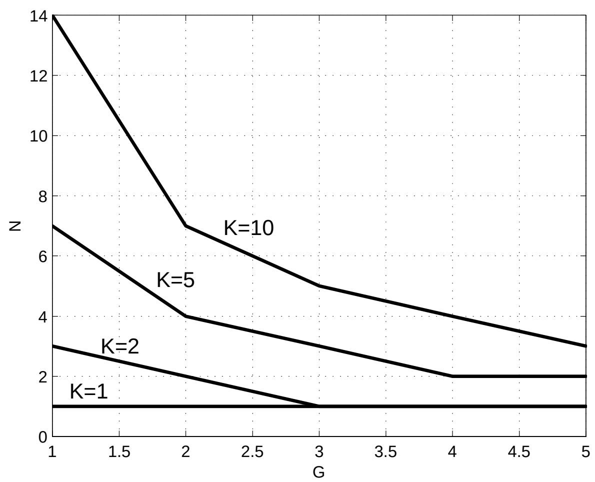

Fig. 4. Stackelberg Equilibrium: dependence between number of antennas N and processing gain (bandwidth) G, with one parameter fixed and another optimally chosen by the AP, for different number of users K.

over N for fixed G. Moreover, increasing the processing gain G decreases the optimal value of N. While the reverse also holds, i.e., there is a unique maximum of revenue over G for fixed N, it is interesting to note that investing in antennas N has better effect on revenue function than buying more bandwidth (increasing G). The reason behind this can be explained by pointing out that the number of antennas has a two-fold effect on the SINR (2), i.e., power gain (in the numerator) and interference mitigation (in the denominator); on the other side, G results in interference mitigation only.

The revenue function versus the number of antennas N for $G = 2$ and different number of users K is presented in fig. 3. It is interesting to see that a larger number of users, though increasing the required network resources (i.e., antennas and, not shown here, bandwidth) at the optimal points, also increases the network revenue and is therefore desirable from the collective point of view. However, for large values of K, allowing additional users into the system has a negligible effect on the relative increase of the revenue function $\rho(G, N)$.

The optimal network parameter N (or G) set by the AP in the SE for fixed G (or N), is presented in fig. 4, for different number of users, K. The well known trade-off between bandwidth and spectral dimension is confirmed. Moreover, it is confirmed that increasing the number of user requires more resources.

B. Maximizing the Power Efficiency

1) MS Game: Instead of minimizing the power under the minimum transmission rate constraint as in Sec. III-A, here we consider as MSs' preference the maximization of power efficiency:

$$\begin{array}{l}\text{maximize } \frac{C_i(\mathbf{P},\mathbf{H},N,G)}{P_i},\ i = 1,..,K,\\ \text{subject to } P_i \in [0, P_{\max}].\end{array} \quad (11)$$

Under the assumption of selfish and rational MSs, problem (11) can be cast as a non-cooperative power control game (NPG) $\langle \mathcal{K}, \mathcal{P}, \{u_i(\cdot)\}\rangle$, where $\mathcal{K} = \{1, 2, .., K\}$ denotes the set of K players (MSs), the players' set of strategies $\mathcal{P}$ reads

$$\mathcal{P} = \{\mathbf{P} \mid P_i \in [0, P_{\max}],\ \forall i \in \mathcal{K}\},$$

and the ith player's utility function is defined as

$$u_i(P_i, \mathbf{P}_{-i}; N, G, \mathbf{H}) = \frac{C_i(\mathbf{P}, \mathbf{H}, N, G)}{P_i},\ i \in \mathcal{K}. \quad (12)$$

While this utility function strongly reflects the pragmatic preferences of the MSs, it needs a slight modification in order to avoid singularity at $P_i = 0$, while preserving quasi-concavity on $\mathcal{P}$:

$$u_i(P_i, \mathbf{P}_{-i}; N, G, \mathbf{H}) = \frac{C_i(\mathbf{P}, \mathbf{H}, N, G)}{P_i + P_c},\ i \in \mathcal{K}, \quad (13)$$

where P_c could be any conveniently chosen constant (for instance, it could account for the power consumed by electronic circuitry of MS [11]). Notice that the utility defined in (13) depends on AP parameters N and G, as well as the channel gains $\mathbf{H}$. A NPG with utility function as the one defined in (13) was investigated in [10].

In order to reach the NE, we can use the Jacobi algorithm, where all the users update their strategy in a parallel fashion using the Newton's method:

$$\mathbf{P}^{t+1} = \mathbf{P}^t + \alpha\left(d_1^t \frac{\partial u_1}{\partial P_1^t}, .., d_2^t \frac{\partial u_K}{\partial P_K^t}\right)^T, \quad (14)$$

where α is some conveniently chosen small number and d_i^t is chosen as $d_i^t = \left(\frac{\partial^2 u_i}{\left(\partial P_i^t\right)^2}\right)^{-1}$ [7]. The convergence point of the algorithm is the NE of the game, $\hat{\mathbf{P}}(N, G, \mathbf{H})$.

2) AP Revenue Function: As in Sec. III-A, we assume that the AP has preferences compatible with the MSs. Therefore, it aims at maximizing the (overall) power efficiency, averaged over fading, while accounting for the resource expenditure:

$$\rho(N,G) = \frac{1}{GN}\sum_{i=1}^{K} E_{\mathbf{H}}\left[\frac{C_i\left(\hat{\mathbf{P}}_i\left(N,G,\mathbf{H}\right),N,G,\mathbf{H}\right)}{\hat{P}_i + P_c}\right]. \tag{15}$$

3) Centralized Scenario: For the centrally optimal solution, the problem boils down to maximizing the revenue function (15), by assuming that the AP can also control the set of the MSs' powers $\mathbf{P}(N,G,\mathbf{H})$. Therefore, the maximization is carried out with respect to G, N and $\mathbf{P}(N,G,\mathbf{H})$. This task can be performed numerically. As shown below, in this case the decentralized solution has degraded performance as compared to the centralized scenario.

4) System Performance: Figure 5 shows the revenue function $\rho(G,N)$ versus the number of antennas N for different values of processing gain G, and parameters $E\left[|h_{ij}|^2\right] = 1$, $P_c = 0.1$ W, $P_{\max} = 2$ W and $SNR = 13$ dB. The conclusions are very similar to those for the power minimization problem. Furthermore, the dependence among N, G and K for the optimal (SE) solution is shown in fig. 6, revealing the similar system behavior to that of fig. 4.

Figure 7 shows the optimal revenue function ρ versus number of antennas N, for different number of users K and for both the distributed and centralized scenarios. As expected, centralized control allows to harness a larger revenue. However, as the number of antennas increase, the difference in performance between centralized and decentralized scheme reduces. This shows that with enough interference mitigation options, decentralized power control is not as harmful for the system performance. Moreover, it is clear from fig. 7 that, by increasing the number of users, the efficiency of the distributed scheme falls behind that of the optimal (centralized) scenario, thus confirming that large distributed systems pose the major challenge. Furthermore, it is very interesting to observe that, while the increased number of users is again desirable for the network (at least in centralized scenario), the relevant lack of efficiency for large K can diminish this gain in decentralized scenario.

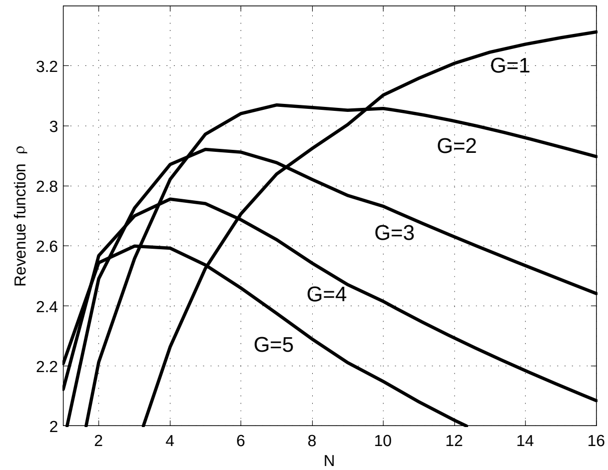

Fig. 5. Revenue function ρ of the AP versus number of AP antennas N for different values of processing gain (bandwidth) G.

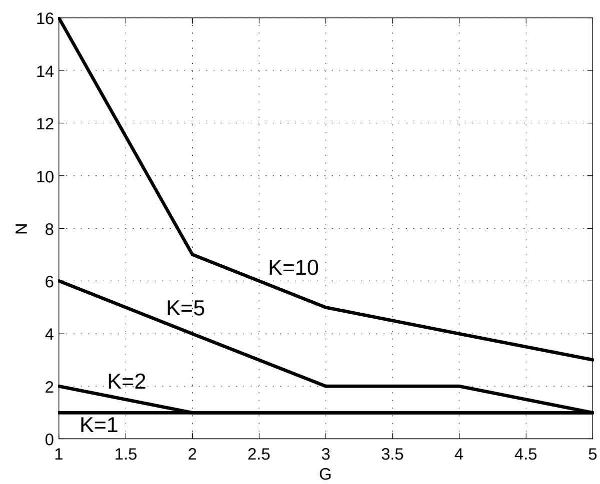

Fig. 6. Stackelberg Equilibrium: dependence between number of antennas N and processing gain (bandwidth) G, with one parameter fixed and another optimally chosen by the AP, for different number of users K.

IV. Conclusions

In this paper, we analyzed the design of a multi-antenna access point with decentralized power control in the uplink channel. The optimal solution, in terms of number of antennas and bandwidth, has been studied by modelling the problem as a Stackelberg game between the access point and competitive mobile stations. In this framework, it has been shown that a larger number of users motivates the provider (i.e., access point) to invest, as the overall performance enhancement well balances the costs. It was discussed, however, that in certain decentralized scenarios the system cannot efficiently cope with large amount of user. Furthermore, the well-known tradeoff between system resources, bandwidth and antennas, was confirmed.

We believe that this work can be expanded in any of the following directions: firstly, one can investigate the performance degradation due to incomplete knowledge of the environment by the mobile stations (e.g., the knowledge limited to other players' *average* channel gains); secondly, system fairness can be assessed and optimized for systems with heterogeneous average channel conditions. Finally, if receiving antennas are thought of as different access points, this work could be extended and applied to systems employing macrodiversity.

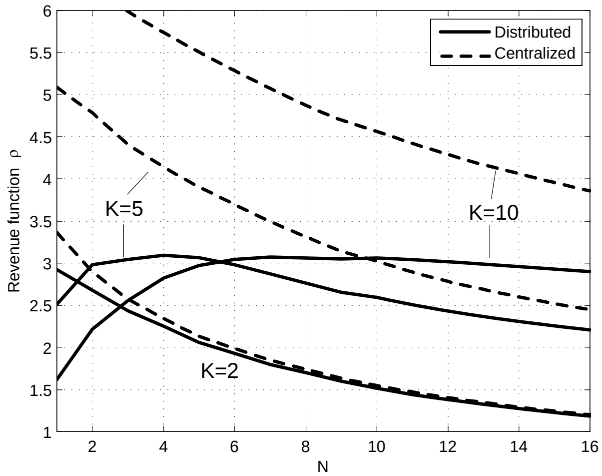

Fig. 7. Revenue function ρ of the AP versus number of antennas N for different number of users K: comparison between centralized and distributed scenarios.

V. Acknowledgment

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References

[1] R. D. Yates, "A Framework for Uplink Power Control in Cellular Radio Systems", *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1341–1347, Sep. 1995.

[2] S. Haykin, "Cognitive Radio: Brain-Empowered Wireless Communications", *IEEE J. Select. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[3] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: MIT Press, 2002.

[4] A. B. MacKenzie and L. A. DaSilva, *Game Theory for Wireless Engineers*. Morgan & Claypool, 2006.

[5] C. U. Saraydar, N. B. Mandayam and D. J. Goodman, "Efficient Power Control Via Pricing in Wireless Data Networks," *IEEE Trans. Commun.*, vol. 50, pp. 291–303, Feb. 2002.

[6] O. Ileri, S. C. Mau and N. B. Mandayam, "Pricing for Enabling Forwarding in Self-Configuring Ad Hoc Networks", *IEEE J. Select. Areas Commun.*, vol. 23, No. 1, pp. 151–162, Jan. 2005.

[7] D. P. Bertsekas and J. N. Tsitsiklis, *Nonlinear Programming*. Athena Scientific, 1997.

[8] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computations: Numerical Methods*. Athena Scientific, 1999.

[9] G. Scutari, S. Barabarossa and D. Palomar, "Potential Games: A Framework for Vector Power Control Problems with Coupled Constraints", in *Proc. ICASSP 2006.*

[10] A. Augustín, O. Muñoz and J. Vidal, "A Game Theoretic Approach for Cooperative MIMO Schemes with Cellular Reuse of the Relay Slot", in *Proc. ICASSP 2004.*

[11] S. Cui, A. J. Goldsmith and A. Bahai, "Energy-Efficiency of MIMO and Cooperative Techniques in Sensor Networks", *IEEE Journ. Selected Areas Commun.*, vol. 22, no. 6, pp. 1089-1098, Aug. 2004.

[12] S. Verdu, *Multisuer Detection*. Cambridge University Press, 1998.

[13] M. Hayajneh and C. T. Abdallah, "Distributed Joint Rate and Power Control Game-Theoretic Algorithms for Wireless Data", *IEEE Commun. Lett.*, vol. 8, no. 8, pp. 511-513, Aug. 2004.

[14] J. Juang, R. A. Berry and M. L. Honig, "Distributed Interference Compensation for Wireless Networks", *IEEE J. Select. Areas Commun.*, vol. 24, No. 5, pp. 151–162, May 2006.

[15] C. A. St Jean and B. Jabbari, "Game-Theoretic Power Control in DS-CDMA Wireless Networks with Successive Interference Cancellation", *IEEE Electr. Lett.*, vol. 42, No. 3, Feb. 2006.