Capacity of Linear Two-hop Mesh Networks with Rate Splitting, Decode-and-forward Relaying and Cooperation

O. Simeone, O. Somekh, Y. Bar-Ness, H. V. Poor and S. Shamai (Shitz)

Abstract— **A linear mesh network is considered in which a single user per cell communicates to a local base station via a dedicated relay (two-hop communication). Exploiting the possibly relevant inter-cell channel gains, rate splitting with successive cancellation in both hops is investigated as a promising solution to improve the rate of basic single-rate communications. Then, an alternative solution is proposed that attempts to improve the performance of the second hop (from the relays to base stations) by cooperative transmission among the relay stations. The cooperative scheme leverages the common information obtained by the relays as a by-product of the use of rate splitting in the first hop. Numerical results bring insight into the conditions (network topology and power constraints) under which rate splitting, with possible relay cooperation, is beneficial. Multi-cell processing (joint decoding at the base stations) is also considered for reference.**

I. INTRODUCTION

Wireless mesh networks are currently being investigated for their potential to resolve the performance limitations of both infrastructure (cellular) and multi-hop (ad hoc) networks in terms of quality-of-service and coverage [1]. Basically, mesh networks prescribe the combination of communication via direct transmission to infrastructure nodes (base stations) and via multi-hop transmission through intermediate nodes (relay stations). The latter can generally be mobile terminals, or fixed stations appropriately located by the service provider. The assessment of the performance of such networks is an open problem that has attracted interest from different communities and fields, especially information-theory [2] [3] and networking [4]. Recently, there has also been considerable interest in further enhancing the performance of infrastructure or mesh networks by endowing the system with a central processor able to pool the received signals by the base stations and perform joint processing (this scenario is usually referred to as distributed antennas systems or multi-cell processing) [5].

In this paper, we focus on a linear mesh network as sketched in Fig. 1. It is assumed that one mobile terminal (MT) is active in each cell in a given time-frequency resource
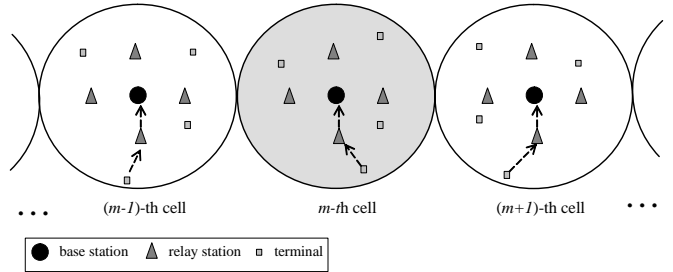
Fig. 1. A linear two-hop mesh network.

(as for intra-cell TDMA or FDMA) and that each active MT communicates with the same-cell base station (BS) via a dedicated relay station (RS) (two-hop transmission). In order to allow meaningful analysis and insight, this scenario is modelled as illustrated in Fig. 2, where symmetry is assumed in the channel gains, i.e., every cell is characterized by identical intra- and inter-cell propagation conditions. This framework follows the approach of [6] (see also [5]), which extends the model of [7] to mesh networks.

The basic premise of this work is that the model in Fig. 2 can be seen as the cascade of two interference channels, one for each hop, with many sources and corresponding receivers (border effects are neglected). Therefore, from the literature on interference channels, a promising approach is that of employing rate splitting with successive interference cancellation at the receivers [10] [11]. It is recalled that the rationale of rate splitting is that joint decoding of (at least part of) the transmitted signals at the receivers has the potential to improve the achievable rates with respect to single-user decoders that treat signals other than the intended as noise. The main contributions of this work concerning the analysis of a mesh network modelled as in Fig. 2 are:

- derivation of the performance of rate splitting applied to both hops with decode-and-forward relaying (Sec. III);
- proposal of a cooperative transmission scheme for the RSs that leverages the common information obtained by the relays as a by-product of the use of rate splitting in the first hop (Sec. IV);
- analysis of the cooperative transmission scheme above in the presence of multi-cell processing (Sec. IV); and
- performance evaluation of rate splitting, with possible relay cooperation in the second hop, via numerical simulations; comparison with the reference cases of single-rate transmission and multi-cell processing is provided as well (Sec. V).

Related work was recently reported in [6] [8] [9], where a cellular model similar to the one in Fig. 2 was addressed under the assumption of amplify-and-forward [6] [8] or decode-and-forward (DF) relaying [9] with single-rate transmission.

II. SYSTEM MODEL

We study the abstraction of the two-hop mesh network of Fig. 1 as sketched in Fig. 2. Cells are arranged in a linear fashion, one user transmitting on a given time-frequency resource in each cell. Moreover, we focus on non-faded Gaussian channels and assume homogeneous conditions for the channel power gains so that the intra-cell MS-to-RS (first hop) and RS-to-BS (second hop) power gains are β^2 and γ^2, respectively, for all cells, and, similarly, the inter-cell power gains between adjacent cells are $\alpha^2 \leq \beta^2$ and $\eta^2 \leq \gamma^2$ for first and second hop, respectively. Notice that as in [7] each cell receives signals only from adjacent cells. Moreover, here there exist no direct paths between MTs and BSs and no relevant inter-channels between RSs in adjacent cells. Because of the latter assumptions, we can deal with either full duplex or half duplex transmission at the relays with minor modifications, as explained below. Considering, for simplicity of exposition, full-duplex transmission (by means of perfect echo-cancellation), the signal received at each time by the mth RS (first hop) can be written as

$$Y'_m = \beta X_m + \alpha(X_{m-1} + X_{m+1}) + N_m, \qquad (1)$$

where β and α are the (real) channel gains, and we assume the symbols transmitted by the MTs, X_m, to be drawn from a circularly symmetric complex Gaussian distribution with power $E[|X_m|^2] = P_1$. Moreover, the additive noise N_m is complex Gaussian with $E[|N_m|^2] = 1$. Similarly, the signal received by the mth BS is

$$Y_m = \gamma Z_m + \eta(Z_{m-1} + Z_{m+1}) + M_m, \qquad (2)$$

where the symbols transmitted by the RSs satisfy $E[|Z_m|^2] = P_2$ and the additive Gaussian noise is such that $E[|M_m|^2] = 1$.

By symmetry, we are interested in evaluating the common rate achievable by all of the MTs over the network described by Fig. 2 and equations (1)-(2). In order to simplify the treatment, we will assume that the number of cells is large enough in order to neglect border effects (see [5] for further discussion on this point in the context of the cellular model of [7]).

III. ACHIEVABLE RATE WITH RATE SPLITTING

As mentioned above, in this paper we focus for simplicity of exposition on full-duplex RSs. Accordingly, we assume a delayed block-by-block transmission strategy whereby the information is transmitted through multiple blocks, and the number of blocks is large enough so that we can neglect the loss in spectral efficiency associated with the transmission of first (MT to RS) and last (RS to BS) blocks. More specifically, in each block, the MTs communicate new information to the RSs, and, at the same time, the RSs forward (after decoding) the information received in the previous block to
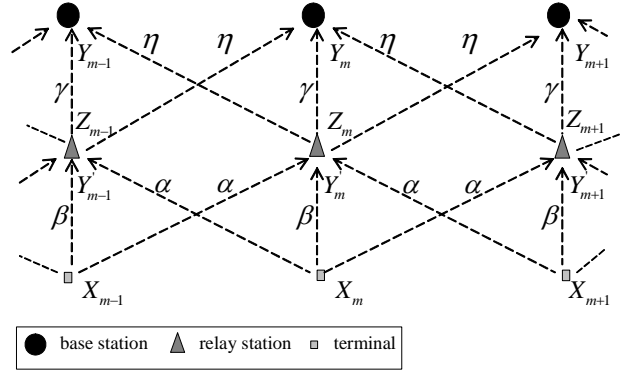

Fig. 2. A schematic model of the linear two-hop mesh network.

the BSs. The absence of a direct path between MTs and BSs allows RSs and BSs to perform block-by-block decoding without resorting to more complicated decoding strategies [11]. Moreover, for the same reason, the full-duplex coding schemes considered in this paper can be easily adapted to half-duplex RSs by simply alternating transmission from MT or RS in each block. In the case of half-duplex then, since the MTs transmit new information only once every two blocks, the corresponding achievable rates are easily seen to be just half of the corresponding rates with full duplex derived here[1].

In this section, we first review the basic reference case of single-rate transmission (Sec. III-A) and then evaluate the achievable rate with rate splitting in both hops (Sec. III-B and III-C).

A. The reference case: single-rate transmission

As a preliminary example and reference case, consider the following simple coding scheme based on DF relaying (further analyzed in a more general framework in [9]). In every block, each MT transmits to the same-cell RS a Gaussian codeword taken from a rate-R codebook. The RS decodes the message treating the signals from adjacent cells as Gaussian interference (single-user decoding), and forwards it in the next block to the same-cell BS, that finally performs single-user decoding. The maximum achievable rate per user of this scheme is easily shown to be

$$R_o = \mathrm{C}\left(\min\left(\frac{\beta^2 P_1}{1 + 2\alpha^2 P_1}, \frac{\gamma^2 P_2}{1 + 2\eta^2 P_2}\right)\right), \qquad (3)$$

where we have defined the function $\mathrm{C}(x) = \log(1 + x)$ and the two terms inside the inner parentheses correspond to the signal-to-interference-plus-noise ratios (SINRs) at the RS and BS, respectively. The performance of this scheme is poor when the inter-cell interference, i.e., the value of parameters α^2 and η^2, is large. In the next section, we attempt to alleviate this problem by leveraging on the idea of rate splitting with Multiple Access Channel (MAC) decomposition, first employed in [10] in the context of the conventional (2×2) interference channel (see also [11]).

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[1]Strictly speaking, under average power constraint, the power used with half-duplex by both sources and relays can be doubled with respect to the full-duplex case.

B. Rate splitting for transmission to the RSs

In this section, we focus on the first hop, between MTs and RSs, and propose a coding scheme based on the principle of rate splitting for the interference channel [10]. Accordingly, each MT transmits the sum of two random Gaussian codebooks,

$$X_m = X_{p,m}(W_{p,m}) + X_{c,m}(W_{c,m}): \qquad (4)$$

a *private* codebook $X_{p,m}(\cdot)$ encoding a message $W_{p,m}$ intended to be decoded only at the same-cell RS, and a *common* codebook $X_{c,m}(\cdot)$ that carries a message $W_{c,m}$ to be decoded not only at the same-cell RS but also at the two adjacent-cell RSs[2]. The rate of the private and common codebooks are denoted as R_{1p} and R_{1c}, respectively (i.e., $W_{p,m} \in \{1, 2, ..., 2^{nR_{1p}}\}$ and $W_{c,m} \in \{1, 2, ..., 2^{nR_{1c}}\}$), whereas the corresponding powers are $P_{1p} = E[|X_{p,m}|^2]$ and $P_{1c} = [|X_{c,m}|^2]$. The total power per MT P_1 is divided among the two codebooks as $P_1 = P_{1p} + P_{1c}$. Similarly, the total rate transmitted by the user to the same-cell RS is given by $R_{rs,1} = R_{1p} + R_{1c}$. Notice that each RS is informed of the private codebook used by the same-cell MT and of the common codebooks employed by the same-cell MTs and the two adjacent-cell MTs.

From (1) and (4), the signal received at each mth RS can be written as (dropping the arguments of the codewords):

$$
\begin{aligned}
Y'_m = & \ \beta(X_{p,m} + X_{c,m}) + \alpha(X_{c,m-1} + X_{c,m+1}) + (5) \\
& + S_m + N_m,
\end{aligned}
$$

where

$$S_m = \alpha(X_{p,m-1} + X_{p,m+1}). \qquad (6)$$

Based on (5), we assume that each mth RS jointly decodes four messages: the private message $W_{p,m}$ and the common message $W_{c,m}$ of the same-cell MT, and the common messages $W_{c,m-1}$ and $W_{c,m+1}$ of the two adjacent-cell MTs. The private messages $W_{p,m-1}$ and $W_{p,m+1}$ of the two adjacent-cell MTs are instead considered as the (Gaussian) interference terms S_m (6) with power $E[|S_m|^2] = 2\alpha^2 P_{1p}$. The channel (5) seen at any mth RS is then a four-user MAC with inputs $X_{p,m}$, $X_{c,m}$, $X_{c,m-1}$ and $X_{c,m+1}$ and equivalent Gaussian noise with power $1 + 2\alpha^2 P_{1p}$. Accordingly, for each choice of the power allocation (P_{1p}, P_{1c}), the achievable rates R_{1p} and R_{1c} are limited by the fifteen inequalities defining the capacity region $\mathcal{R}_{rs,1}(P_{1p}, P_{1c})$ of the Gaussian MAC at

hand [15], which are easily shown to boil down to:

$$R_{1p} \leq \mathrm{C}\left(\frac{\beta^2 P_{1p}}{1 + 2\alpha^2 P_{1p}}\right) \triangleq R_{1p}^{\max}(P_{1p}) \qquad (7a)$$

$$
\begin{aligned}
R_{1c} \leq & \ \min\left\{\frac{1}{2}\mathrm{C}\left(\frac{2\alpha^2 P_{1c}}{1 + 2\alpha^2 P_{1p}}\right), \right. \qquad (7b) \\
& \left. \frac{1}{3}\mathrm{C}\left(\frac{(2\alpha^2 + \beta^2)P_{1c}}{1 + 2\alpha^2 P_{1p}}\right)\right\} \\
\triangleq & \ \min\{R_{1c}^{\max,1}(P_{1p}, P_{1c}), R_{1c}^{\max,2}(P_{1p}, P_{1c})\}
\end{aligned}
$$

$$
\begin{aligned}
R_{1p} + 2R_{1c} \leq & \ \mathrm{C}\left(\frac{\beta^2 P_{1p} + 2\alpha^2 P_{1c}}{1 + 2\alpha^2 P_{1p}}\right) \qquad (7c) \\
\triangleq & \ R_1^{\mathrm{sum},1}(P_{1p}, P_{1c})
\end{aligned}
$$

$$
\begin{aligned}
R_{1p} + 3R_{1c} \leq & \ \mathrm{C}\left(\frac{\beta^2 P_{1p} + (2\alpha^2 + \beta^2)P_{1c}}{1 + 2\alpha^2 P_{1p}}\right) \qquad (7d) \\
\triangleq & \ R_1^{\mathrm{sum},2}(P_{1p}, P_{1c}).
\end{aligned}
$$

Notice that in writing the conditions (7) we have removed dominated inequalities.

In order to obtain some insight into the properties of the achievable rate region of private and common messages $\mathcal{R}_{rs,1}(P_{1p}, P_{1c})$ defined by inequalities (7), Fig. 3 shows the region $\mathcal{R}_{rs,1}(P_{1p}, P_{1c})$ for $P_{1p} = 1$, $P_{1c} = 1$, $\beta^2 = 1$ and different values of α^2. According to the value of the inter-cell parameter α^2, the achievable region $\mathcal{R}_{rs,1}(P_{1p}, P_{1c})$ is a polyhedron with different corner points. Fig. 3 shows three illustrative cases for small ($\alpha^2 = 0.4$ in the figure), intermediate ($\alpha^2 = 0.65$) and moderate inter-cell factor α^2 ($\alpha^2 = 0.8$)[3]. In all cases, *vertex A* has a simple interpretation in terms of successive interference cancellation: in fact, it can be achieved by first jointly decoding the common messages ($W_{c,m}$, $W_{c,m-1}$ and $W_{c,m+1}$), treating the private information as noise, then cancelling the decoded common messages and finally decoding the same-cell private message $W_{p,m}$. To show this, notice that, since in the first decoding stage the channel seen by the three common messages at any RS is a three-user MAC with noise power $1 + (2\alpha^2 + \beta^2)P_{1p}$ (due to the interference from the primary messages), the common rate at vertex A is given by $\min(R_{1c}^1, R_{1c}^2)$, with

$$R_{1c}^1(P_{1p}, P_{1c}) = \frac{1}{2}\mathrm{C}\left(\frac{2\alpha^2 P_{1c}}{1 + (2\alpha^2 + \beta^2)P_{1p}}\right) \qquad (8a)$$

$$R_{1c}^2(P_{1p}, P_{1c}) = \frac{1}{3}\mathrm{C}\left(\frac{(2\alpha^2 + \beta^2)P_{1c}}{1 + (2\alpha^2 + \beta^2)P_{1p}}\right). \qquad (8b)$$

Our focus on vertex A in the achievable rate region $\mathcal{R}_{rs,1}(P_{1p}, P_{1c})$ is justified by the following fact. Given the slope of the side of the polyhedron $\mathcal{R}_{rs,1}(P_{1p}, P_{1c})$ determined by conditions (7c)-(7d), it can be easily seen that *for each power allocation (P_{1p}, P_{1c}) vertex A corresponds to the point where the rate on the first hop $R_{rs,1} = R_{1p} + R_{1c}$*

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[2]Notice that the definition of private and common messages here is receiver-centric, whereas elsewhere (see, e.g., [22] [17] [18]) it refers to the message availability at the transmitters (but see also Sec. IV-A).

[3]Notice that an exact determination of the threshold values of α that lead to different regions is conceptually simple but algebraically involved given the characterization (7). Moreover, we remark that we avoided the use of the term "strong interference" in this context in order to be consistent with the conventional use of the term (see, e.g., [11]).

is maximum and reads

$$R_{rs,1}^{\max}(P_{1p}, P_{1c}) = R_{1p}^{\max}(P_{1p}) + \qquad (9)$$
$$+ \min(R_{1c}^1(P_{1p}, P_{1c}), R_{1c}^2(P_{1p}, P_{1c})),$$

with definitions (7a) and (8). We remark the decoding order that leads to vertex A (first common information, then private), coupled with a specific power allocation, was recently shown in [13] to attain every point in the capacity region of the conventional interference channel to within one bit. Finally, vertex points B and B′ also have similar interpretations in terms of successive interference cancellation. This is further discussed in Appendix-A.

Remark 1 (very strong interference): Similarly to the case of a conventional interference channel [12], it can be shown that, if α^2 is sufficiently larger than the direct channel β^2 (thus contradicting our assumption that $\alpha^2 \leq \beta^2$), transmission of only common messages ($P_{1p} = 0$ and $P_{1c} = P_1$) is an optimal strategy that is able to achieve the single-user upper bound to the achievable rate, $R_{rs,1} = \log(1 + \beta^2 P_1)$. The exact condition on α^2 is derived in Appendix-B.

C. Rate splitting in the second hop

With rate splitting in the first hop, each RS, say the mth, decodes in each block the private message $W_{p,m}$ and the common message $W_{c,m}$ of the same-cell MT, along with the common messages of the adjacent cells $W_{c,m-1}$ and $W_{c,m+1}$. The mth relay can then neglect the knowledge of $W_{c,m-1}$ and $W_{c,m+1}$, and attempt to transmit to the mth BS the two messages of the same-cell user $W_{p,m}$ and $W_{c,m}$ by using rate splitting and interference cancellation exactly as explained in the previous section for the first hop. Notice that the total rate $R_{rs,1} = R_{1p} + R_{1c}$, delivered to the RSs by the MTs, can be now split into two streams, one private and one common, in a generally different share with respect to the first hop. In particular, the signal transmitted by the mth RS is given by

$$Z_m = Z_{p,m}(V_{p,m}) + Z_{c,m}(V_{c,m}), \qquad (10)$$

where $Z_{p,m}(\cdot)$ corresponds to a Gaussian codebook of rate R_{2p} for the private message $V_{p,m}$ ($V_{p,m} \in \{1, 2, ..., 2^{nR_{2p}}\}$) and $Z_{c,m}(\cdot)$ is the R_{2c}-rate code for the common message $V_{c,m}$ ($V_{c,m} \in \{1, 2, ..., 2^{nR_{2c}}\}$). The total rate achievable on the second hop thus becomes $R_{rs,2} = R_{2p} + R_{2c}$. Moreover, the power allocation is $P_2 = P_{2p} + P_{2c}$, where $P_{2p} = E[|Z_{p,m}|^2]$ and $P_{2c} = E[|Z_{c,m}|^2]$. Similarly to the first hop, each BS is informed of the private codebook used by the same-cell MT and of the common codebooks employed by the same-cell MTs and the two adjacent-cell MTs.

Following the previous section, we can define the rate region $\mathcal{R}_{rs,2}(P_{2p}, P_{2c})$ achievable in the second hop with rate splitting for a given power allocation. This is easily shown to be defined by inequalities (7), where subscript "2" should be substituted for "1" and parameters (γ^2, η^2) should be written in lieu of (β^2, α^2). Accordingly, the maximum rate
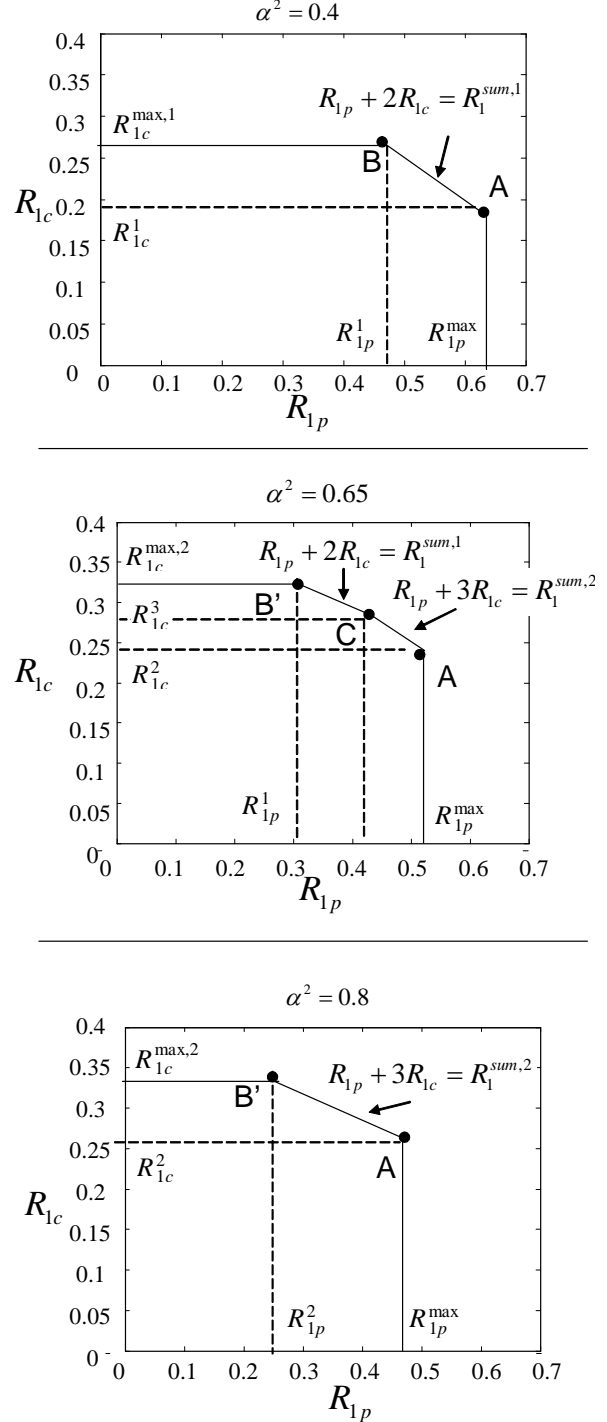

Fig. 3. Three illustrative cases for the capacity region (in terms of rates of private message, R_{1p}, and common message, R_{1c}) of rate splitting on the first hop, corresponding to different values of the inter-cell power gain α^2 ($P_{1p} = 1$, $P_{1c} = 1$, $\beta^2 = 1$). The rate-maximizing vertex A is achievable by successive interference cancellation where common messages are decoded first followed by the same-cell private message.

in the second hop reads (recall (9))

$$R_{rs,2}^{\max}(P_{2p}, P_{2c}) = R_{2p}^{\max}(P_{2p}) + \qquad (11)$$
$$+ \min(R_{2c}^1(P_{2c}, P_{2c}), R_{2c}^2(P_{2p}, P_{2c})),$$

where $R_{2p}^{\max}(P_{2p})$, $R_{2c}^1(P_{2c}, P_{2c})$ and $R_{2c}^2(P_{2p}, P_{2c})$ are obtained from (7a) and (8), respectively, following the rules mentioned above.

Since with rate splitting in both hops the two hops are operated independently, the optimal strategy is to transmit in both hops at the maximum sum-rates $R_{rs,i}^{\max}(P_{ip}, P_{ic})$ in (9) and (11) for given power allocations (P_{ip}, P_{ic}), $i = 1, 2$. It follows that, optimizing over the power allocation on both hops, the rate achievable with rate splitting in both hops is

$$R_{rs} = \min_{i=1,2} R_{rs,i}^{\max}, \qquad (12)$$

with $(i = 1, 2)$

$$R_{rs,i}^{\max} = \max_{P_{ip}, P_{ic}} R_{rs,i}^{\max}(P_{ip}, P_{ic})$$
$$\text{s.t. } P_{ip} + P_{ic} = P_i. \qquad (13)$$

IV. IMPROVING THE ACHIEVABLE RATE IN THE SECOND HOP

In this section, we investigate the performance of an alternative transmission scheme for the second hop that leverages the common information gathered at the RSs as a by-product of the use of rate splitting in the first hop. This contrasts with the naive scheme discussed in Sec. III-C whereby the common messages from adjacent cells were neglected when transmitting in the second hop. Moreover, for reference, we evaluate the rate achievable with rate splitting and multi-cell processing at the BSs (as in the case where BSs are connected via a high capacity backbone) in Sec. IV-B.

A. Cooperative transmission at the relays

The rate splitting-based scheme discussed in Sec. III-C for transmission from RSs to BSs fails to exploit the knowledge of the common messages of adjacent cells $W_{c,m-1}$ and $W_{c,m+1}$ at any mth RS. Based on this side information, any mth cell could cooperate with the adjacent cells $m - 1$ (and $m + 1$) in order to deliver the common message $W_{c,m-1}$ (and $W_{c,m+1}$) to the intended BS in cell $m - 1$ (and $m + 1$). The presence of shared information among the transmitters has been previously considered in the context of conventional (2×2) interference channels in different scenarios. In particular, a model in which the two transmitters have common information to deliver to *both* receivers has been considered in [17] [18], whereas an asymmetric case where one transmitter has knowledge of the message of the other transmitter was studied in [19] [20] [21]. Also relevant is the case of a MAC channel with common information studied in [22].

Similarly to the above mentioned works, here we adopt a superposition scheme whereby transmitters cooperate for transmission of common information towards the goal of achieving coherent power combining at the BSs. In particular, the signal transmitted by the mth RS according to this scheme is given by

$$Z_m = Z_{p,m}(W_{p,m}) + \sum_{i=-1}^{1} Z_{c,m+i}(W_{c,m+i}), \qquad (14)$$

where $Z_{p,m}(\cdot)$ is defined as above and $Z_{c,m}(\cdot)$ accounts for a common Gaussian codebook employed by the $m - 1$, m and $(m + 1)$th RSs for cooperative relaying of the common messages $W_{c,m}$. Notice that variables $Z_{p,m}(\cdot)$ and $Z_{c,m}(\cdot)$ are uncorrelated. The private $(W_{p,m})$ and common $(W_{c,m})$ messages are the ones sent in the first hop by the MTs and therefore have rates R_{1p} and R_{1c}, respectively. We focus on a simple power allocation among the transmitted codewords in (14), whereby the total power P_2 is divided as $P_2 = P_{2p} + P_{2c}$ with $P_{2p} = E[|Z_{p,m}|^2]$ for the private part and the power P_{2c} equally shared among the three cooperative common signals as $P_{2c} = 3E[|Z_{c,m}|^2]$. Moreover, as in the previous section, each BS is assumed to know the private codebook used by the same-cell MT and of the common codebooks employed by the same-cell MTs and the two adjacent-cell MTs. It should be remarked that a more general transmission scheme than the one considered here could be employed where joint encoding of private $W_{p,m}$ and common $W_{c,m}$ messages takes place at each mth RS (instead of the independent encoding by which we interpret (14)), similarly to [22]. Here, for simplicity, we do not further pursue the analysis of this scenario.

In order to derive the achievable rates of this scheme, let us substitute (14) in the received signal (2) at the BSs (dropping the arguments of the codewords):

$$Y_m = \gamma Z_{p,m} + (\gamma + 2\eta)Z_{c,m} + (\gamma + \eta)Z_{c,m-1} + \qquad (15)$$
$$+ (\gamma + \eta)Z_{c,m+1} + S'_m + M_m,$$

where S'_m represent the nuisance term due to the private messages of adjacent cells and the common messages of cells $m - 2$ and $m + 2$:

$$S'_m = \eta Z_{p,m-1} + \eta Z_{p,m+1} + \eta Z_{c,m-2} + \eta Z_{c,m+2}. \qquad (16)$$

We remark that the common messages of cells $m - 2$ and $m + 2$ ($Z_{c,m-2}$ and $Z_{c,m+2}$) are considered as interference by the mth BS since they are received without the benefit of cooperation from other RSs. Therefore, adding the constraint of correct decoding of these messages at the mth BS would reduce unnecessarily the rate R_{1c} of the common codebooks $W_{c,i}$. From (15), it can be seen that any mth BS observes a four-user MAC channel with equivalent noise power $1 + E[|S'_m|^2] = 1 + 2\eta^2(P_{2p} + P_{2c}/3)$. Therefore, similarly to Sec. III-B, the achievable rates (R_{1p}, P_{1c}) of the private and common information belong to the rate region

$\mathcal{R}_{coop,2}(P_{2p}, P_{2c})$ characterized by:

$$R_{1p} \leq \mathrm{C}\left(\frac{\gamma^2 P_{2p}}{1 + 2\eta^2(P_{2p} + P_{2c}/3)}\right)$$

$$R_{1c} \leq \min\left\{\frac{1}{2}\mathrm{C}\left(\frac{2(\gamma+\eta)^2 P_{2c}}{1 + 2\eta^2(P_{2p} + P_{2c}/3)}\right),\right.$$
$$\left.\frac{1}{3}\mathrm{C}\left(\frac{(2(\gamma+\eta)^2 + (\gamma+2\eta)^2)P_{2c}}{1 + 2\eta^2(P_{2p} + P_{2c}/3)}\right)\right\}$$

$$R_{1p} + 2R_{1c} \leq \mathrm{C}\left(\frac{\gamma^2 P_{2p} + 2(\gamma+\eta)^2 P_{2c}}{1 + 2\eta^2(P_{2p} + P_{2c}/3)}\right)$$

$$R_{1p} + 3R_{1c} \leq \mathrm{C}\left(\frac{\gamma^2 P_{2p} + (2(\gamma+\eta)^2 + (\gamma+2\eta)^2)P_{1c}}{1 + 2\eta^2(P_{2p} + P_{2c}/3)}\right).$$

The maximum achievable rate with rate splitting in the first hop and cooperative transmission in the second hop, according to the coding scheme described above, can be found by solving the following optimization problem:

$$R_{coop} = \max_{R_{1p},R_{1c},P_{1p},P_{1c},P_{2p},P_{2c}} R_{1p} + R_{1c} \qquad (17)$$
$$\text{s.t.} \begin{cases} P_{ip} + P_{ic} = P_i, \ i = 1,2 \\ (R_{1p}, R_{1c}) \in \begin{matrix} \mathcal{R}_{rs,1}(P_{1p},P_{1c})\cap \\ \mathcal{R}_{coop,2}(P_{2p},P_{2c}). \end{matrix} \end{cases}$$

Notice that for each choice of the power allocation $(P_{1p}, P_{1c}, P_{2p}, P_{2c})$, the optimization problem (17) can be solved by linear programming.

B. Multi-cell processing

In this section we consider the possibility of performing joint decoding of the received signals at the BSs [5]. As mentioned above, this requires the presence of a high capacity backbone connecting all the BSs to a central processor. We assume the use of rate splitting in the first hop, whereas in the second hop the cooperative transmission scheme of Sec. IV-A, which aims at coherent power combining at the BSs for the common messages, is employed.

Similarly to [7], we can interpret the received signal (15)-(16) as an equivalent inter-symbol interference (ISI) channel over the BSs:

$$Y_m = h_{p,m} * Z_{p,m} + h_{c,m} * Z_{c,m} + M_m, \qquad (18)$$

where " $*$ " denotes convolution and the finite-impulse response filters $h_{nc,m}$ and $h_{c,m}$ are given by

$$h_{p,m} = \eta\delta_{m+1} + \gamma\delta_m + \eta\delta_{m-1} \qquad (19a)$$
$$h_{c,m} = \eta\delta_{m+2} + (\gamma+\eta)\delta_{m+1} + (\gamma+2\eta)\delta_m \qquad (19b)$$
$$+ (\gamma+\eta)\delta_{m-1} + \eta\delta_{m-2},$$

with δ_m denoting the Kronecker delta function ($\delta_m = 1$ for $m = 0$ and $\delta_m = 0$ elsewhere). The channel (18)-(19) is a Gaussian MAC with ISI [23] so that, allocating the transmission powers as in Sec. IV-A, the region $\mathcal{R}_{mcp,2}(P_{2p}, P_{2c})$ of achievable rates (R_{1p}, R_{1c}) in the second hop with multicell processing and relay cooperation is easily shown to satisfy

the following conditions:

$$R_{1p} \leq \int_0^1 \mathrm{C}\left(P_{2p}(\gamma + 2\eta\cos(2\pi f))^2\right) df$$

$$R_{1c} \leq \int_0^1 \mathrm{C}\left(\frac{P_{2c}}{3}(\gamma + 2\eta + 2(\gamma+\eta)\cos(2\pi f) + \right.$$
$$\left. +2\eta\cos(4\pi f))^2\right) df$$

$$R_{1p} + R_{1c} \leq \int_0^1 \mathrm{C}\left(P_{2p}(\gamma + 2\eta\cos(2\pi f))^2\right.$$
$$+\frac{P_{2c}}{3}(\gamma + 2\eta + +2(\gamma+\eta)\cos(2\pi f) +$$
$$\left. +2\eta\cos(4\pi f))^2\right) df.$$

Finally, accounting for both first and second hops, the rate achievable with rate splitting, relay cooperation and multicell processing can be obtained by solving the following optimization problem:

$$R_{mcp} = \max_{R_{1p},R_{1c},P_{1p},P_{1c},P_{2p},P_{2c}} R_{1p} + R_{1c} \qquad (20)$$
$$\text{s.t.} \begin{cases} P_{ip} + P_{ic} = P_i, \ i = 1,2 \\ (R_{1p}, R_{1c}) \in \begin{matrix} \mathcal{R}_{rs,1}(P_{1p},P_{1c})\cap \\ \mathcal{R}_{mcp,2}(P_{2p},P_{2c}). \end{matrix} \end{cases}$$

Notice again that, for fixed power allocation $(P_{1p}, P_{1c}, P_{2p}, P_{2c})$, problem (20) can be solved by linear programming. As a final remark, we recall that, as stated in Sec. IV-A, an alternative transmission scheme to (14) could employ joint encoding of common and private messages following [22]. The performance advantages of this solution are not further investigated here.

V. NUMERICAL RESULTS

Here we present some numerical results in order to corroborate the analysis and gain some insight into the performance of the proposed coding schemes. Throughout this section, we set $\beta^2 = \gamma^2 = 1$ and $\alpha^2 = \eta^2$. We are interested at first in investigating the conditions under which rate splitting is advantageous over single-rate transmission. Toward this goal, we consider a symmetric scenario with $P_1 = P_2 = P$ and evaluate the optimal fraction of power f to be devoted to the private message assuming rate splitting in both hops as per (13). By symmetry, it is clear that the optimal fraction $\hat{f}$ is the same in both hops, i.e., $\hat{f} = \hat{P}_{1p}/P = \hat{P}_{2p}/P$, where the hat notation identifies optimal quantities. Fig. 4 shows the optimal fraction $\hat{f}$ versus the inter-cell gains $\alpha^2 = \eta^2$. It can be seen that for small inter-cell gains $\alpha^2 = \eta^2$, it is optimal to use single-rate transmission ($\hat{f} = 1$) until a given threshold gain, after which it is in general increasingly better to devote more power to common messages. This result is in line with the known results on the interference channel [12] [10] and confirms our initial motivation (see Sec. III). Moreover, for increasing power P the threshold gain at which common messages should carry more power decreases significantly.

We now turn to the performance assessment of rate splitting (with possible cooperation or multi-cell processing in the second hop) in terms of achievable rates. In order to obtain meaningful results, we focus on a scenario where the second hop is the bottleneck by setting $P_2 = P_1/2$
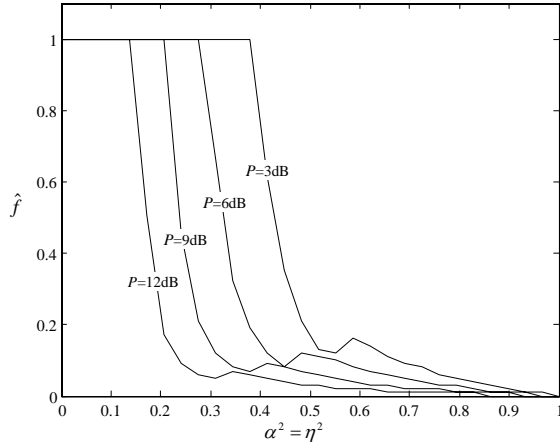
Fig. 4. Optimal fraction $\hat{f}$ of power devoted to the transmission of private messages when rate splitting is used in both hops versus inter-cell gains $\alpha^2 = \eta^2$ ($\beta^2 = \gamma^2 = 1$).

Fig. 5. Achievable rates with single-rate transmission R_o (3), rate splitting in both hops R_{rs} (12), relay cooperation in the second hop R_{coop} (17) and multi-cell processing in the second hop R_{mcp} (20) versus inter-cell gains $\alpha^2 = \eta^2$ ($P_2 = 0.5 \cdot P_1$, $P_1 = 3dB$).

Fig. 6. Achievable rates with single-rate transmission R_o (3), rate splitting in both hops R_{rs} (12), relay cooperation in the second hop R_{coop} (17) and multi-cell processing in the second hop R_{mcp} (20) versus inter-cell gains $\alpha^2 = \eta^2$ ($P_2 = 0.5 \cdot P_1$, $P_1 = 10dB$).

(to be interpreted in linear scale). While this might not be the case in typical applications where RSs are fixed and endowed with a power supply, it is an interesting case study to assess the possible benefits of more elaborate processing in the second hop. Figs. 5 and 6 show the achievable rates with single-rate transmission R_o (3), rate splitting in both hops R_{rs} (12), cooperation at the relays in the second hop R_{coop} (17) and multi-cell processing in the second hop R_{mcp} (20) for $P_1 = 3dB$ and $P_1 = 10dB$, respectively. Also shown is the maximum rate achievable on the first hop with rate splitting and optimal power allocation $R_{rs,1}^{\max}$ (13). This provides an upper bound on the overall achievable rate in the considered scenario where the second hop creates the performance bottleneck. It can be seen that: (*i*) as expected from the discussion on Fig. 4, rate splitting is advantageous with respect to single-rate transmission if the inter-cell gains $\alpha^2 = \eta^2$ are large enough; (*ii*) for sufficiently small signal-to-noise ratio (i.e., power P_1) cooperation at the relays provides relevant performance gains over rate splitting in both hops and allows to achieve the upper bound $R_{rs,1}^{\max}$ for $\alpha^2 = \eta^2$ large enough (Fig. 5); (*iii*) for signal-to-noise-ratio sufficiently large, the additional interference created by the common messages relayed with cooperative transmission in the second hop (recall the discussion in Sec. IV-A) has a deleterious effect on the rate if gains $\alpha^2 = \eta^2$ are relevant and, accordingly, the benefits of cooperation are less pronounced (Fig. 6); (*iv*) multi-cell processing in the second hop allows the system to achieve the upper bound $R_{rs,1}^{\max}$ for $\alpha^2 = \eta^2$ large enough.

VI. CONCLUSIONS

In a mesh network with a regular (cellular) structure, there exists a rich structure in the underlying wireless connections that can be exploited in order to design more effective coding strategies. In this paper, we have explored one such opportunity for a two-hop mesh network with one active user (and relay) per cell. In particular, we have exploited the presence of me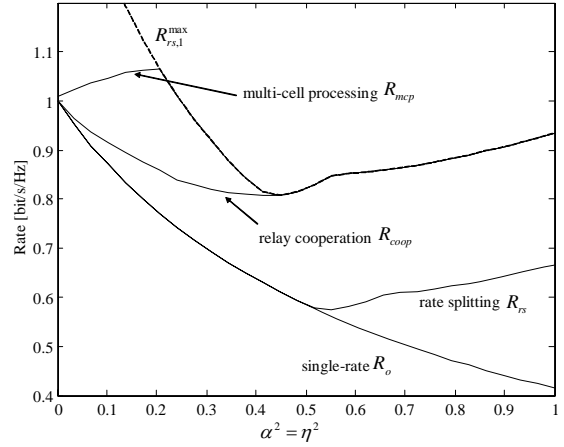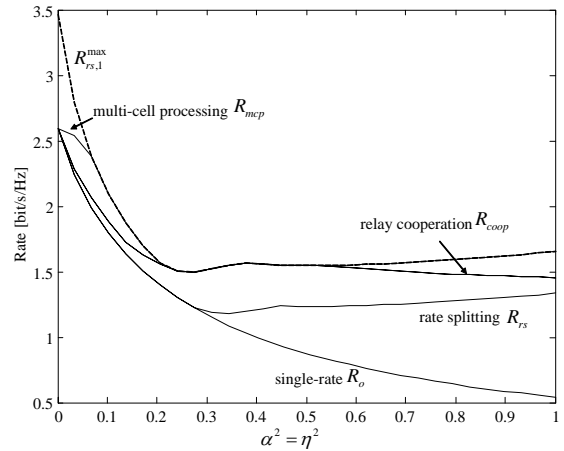aningful inter-cell propagation paths (from terminals to relays and/or from relays to base stations) by considering the use of a rate splitting coding approach, which is know to be close to optimal (or even optimal, in certain cases) for conventional interference channels. Based on this basic scheme, we have further proposed an alternative cooperative transmission scheme in the second hop, that takes advantage of the side information available at the relays as a by-product of the use of rate splitting in the first hop. Numerical results confirm that rate splitting is able to provide significant gains as long as the inter-cell power gains are large enough.

VII. APPENDIX

A. Further discussion on the capacity regions in Fig. 3

In Sec. III-B, the successive interference strategy achieving the rate-maximizing vertex A in the rate region $\mathcal{R}_{rs,1}(P_{1p}, P_{1c})$ was discussed in detail (recall Fig. 3). Here

we would like to further interpret the corner points B and B$'$ in terms of successive interference cancellation. Vertex B, arising in scenarios with weak interference, is obtained by detecting first the common message from same-cell MT, then the private message from same-cell MT and finally common messages from adjacent-cell. This leads to $R_{1c} = R_{1c}^{\max,1}$ and $R_{1p} = R_{1p}^1 = \mathrm{C}\left(\frac{\beta^2 P_{1p}}{1+2\alpha^2 P_{1p}+2\alpha^2 P_{1c}}\right)$. Similarly, vertex B$'$, arising with intermediate interference, can be achieved by first detecting the private message and then jointly recovering the common messages, leading to $R_{1c} = R_{1c}^{\max,2}$ and $R_{1p} = R_{1p}^2 = \mathrm{C}\left(\frac{\beta^2 P_{1p}}{1+2\alpha^2 P_{1p}+(2\alpha^2+\beta^2) P_{1c}}\right)$. Finally, vertex C is characterized by the common rate $R_{1c}^3 = \mathrm{C}\left(\frac{\beta^2 P_{1c}}{1+2\alpha^2 P_{1p}+\beta^2 P_{1p}+2\alpha^2 P_{1c}}\right)$.

B. Derivation of the condition of very strong interference

Following Remark 1, here we look for conditions on the inter-cell power gain α^2 that allow that system to achieve the single-user upper bound $R_{rs,1} = \mathrm{C}(\beta^2 P_1)$ on the achievable rate of the first hop, through transmission of only common messages. Setting $P_{1c} = P_1$ (and $P_{1p} = 0$), we need to impose the condition that all the rate inequalities defining the capacity region of the three-user MAC channel seen by the common messages at each RS support rates larger than $\mathrm{C}(\beta^2 P_1)$. Notice that, since here we allow $\alpha^2 > \beta^2$, we should now consider all the seven inequalities of the MAC capacity region (as opposed to (7) where some bounds were dominated under the assumption that $\alpha^2 \le \beta^2$). This leads to: (*i*) from single-user bounds, it immediately follows that we need $\alpha^2 \ge \beta^2$; (*ii*) from two-user bounds, we have

$$\frac{1}{2}\,\mathrm{C}(2\alpha^2 P_1) \ \ge \ \mathrm{C}(\beta^2 P_1) \qquad (21a)$$

$$\frac{1}{2}\,\mathrm{C}((\alpha^2 + \beta^2) P_1) \ \ge \ \mathrm{C}(\beta^2 P_1), \qquad (21b)$$

from which we obtain

$$\alpha^2 \ge \beta^2 \cdot \max\left(\frac{P_1}{2}+1, \beta^2 P_1 + 1\right); \qquad (22)$$

(*iii*) from three-user bounds, it follows that

$$\frac{1}{3}\,\mathrm{C}((2\alpha^2 + \beta^2) P_1) \ge \mathrm{C}(\beta^2 P_1), \qquad (23)$$

which implies

$$\alpha^2 \ge \beta^2 \cdot (2 + 3P_1 + \beta^4 P_1). \qquad (24)$$

Noticing that condition (24) dominates (22) for any β^2, we finally obtain the result that, in order for rate-splitting to achieve the single-user bound, we need an inter-cell power gain that satisfies the very strong interference conditions (24).

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