Space-Time Coded Cooperative Multicasting with Maximal Ratio Combining and Incremental Redundancy

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Abstract— **The performance of wireless multicasting is negatively affected by multipath fading. To improve reliability, cooperation among the nodes of the network can be used to create spatial diversity. This paper analyzes a two phase, space-time coded, cooperative multicast protocol and investigates two transmission and combination strategies: Maximal Ratio Combining and Incremental Redundancy. Moreover, it addresses two different channel state information (CSI) scenarios: *i*) no transmit CSI within the network and *ii*) *broadcast* transmit CSI at the source.**

I. Introduction

Multicast networks consist of a single source node transmitting simultaneously a unique message to $N > 1$ destination nodes. Multicasting is one of the most widespread transmission modes in wireless networks, e.g., for radio and television broadcasting. However, due to its multi-destination nature, it is also one of the most defenseless schemes with respect to multipath fading. Indeed, for $N \to \infty$, the reliable transmission rate[1] of one-hop multicasting, under independently distributed fading, almost surely converges to zero [1]. One approach to solve such reliability problem is *Cooperative Diversity*, as discussed in [2], [3].

The use of cooperative relaying among wireless nodes was first considered in [4]–[6], based upon the landmark work by Cover and El Gamal [7]. In cooperative networks, users relay each other's messages creating spatial diversity through signal combination at the receiver end. Three main relaying strategies have been proposed up to now: *i*) decode-and-forward, which was presented in [7] for wireless links with one relay node, *ii*) amplify-and-forward, described in [6] and recently extended in [8] to linear relaying, and, *iii*) compress-and-forward, considered in [9]. The application of node cooperation within multicast networks was first carried out in [10], aiming at reducing the total transmit power in ad-hoc environments. Nevertheless, the fundamental capacity limits of cooperative multicasting remained unknown until recent results in [2]. It is therein shown that, for $N \to \infty$, cooperative multicasting allows to transmit with arbitrarily small error probability at the rate [2, Theorem 1]:

$$\mathcal{C} = \log_2 \left(1 + \frac{\mathrm{P}}{\sigma_o^2} \right), \quad (1)$$

[1]The reliable transmission rate of one-hop multicasting equals the channel capacity of the user with worst source-to-user channel (considering transmit channel knowledge at the source).

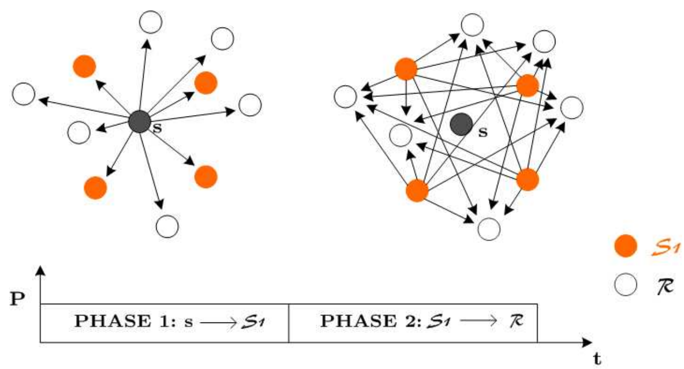

Fig. 1. Space-Time coded cooperative multicasting

where P is the total transmit power and σ_o^2 the noise power at the receiver nodes. Zero-mean, unitary-variance Rayleigh faded channels were assumed. Recall that for non-cooperative multicasting, the reliable rate $\mathcal{C} \to 0$ as the number of users tends to ∞.

In this paper, we analyze a cooperative multicast protocol composed of two consecutive, identical phases as in [3] (see Fig. 1). The first phase of the protocol is used by the source to broadcast data within the network. During phase 2, the set of nodes that has successfully decoded during phase 1 relay the multicast data using distributed space-time codes, hence creating spatial diversity in the system. Finally, the receiving nodes of phase 2 combine the received signal during the two phases and attempt to decode. Our contribution is the evaluation of the outage capacity of the protocol for two transmission and combination strategies: Maximal Ratio Combining (MRC) and Incremental Redundancy (IR). MRC considers that the space-time codeword transmitted by the relays carries a copy of the signal originally transmitted by the source. On the contrary, with IR, the relays transmit extra parity bits of the codeword transmitted by the source, and thus, increases the mutual information.

Moreover, we consider two different assumptions on channel state information (CSI): *i*) *no transmit CSI* neither at source nor at the relay nodes, but receive CSI at all multicast users, and *ii*) *broadcast CSI* at the source (i.e., source-to-users transmit channel knowledge), but no user-to-user transmit channel awareness neither at the source nor at the relays. As before, receive channel knowledge at the receiving nodes is considered. This study complements the analysis of [3], where the outage analysis of cooperative multicasting with multi-hop is carried out (i.e, nodes in phase 2 decode only with signal received in phase 2). Results obtained in [3] showed that cooperation is always worthwhile for the no transmit CSI case. However, multi-hop strategy is shown to excessively penalize the broadcast CSI scenario. This drawback is solved here by means of MRC and IR transmission.

The remainder of the paper is organized as follows: In section II we present the network and signal model for the proposed cooperative multicast network. In Section III, the outage analysis of the cooperative multicasting without CSI is carried out, while in Section IV the broadcast CSI case is addressed. Finally, Section V depicts the numerical results and Section VI summarizes conclusions.

II. Network and Signal Model

We consider a wireless multicast network with a source node s (the base station) and a set of receiver nodes $\mathcal{N} = \{1, 2, \cdots, N\}$ (the multicast users). Aiming at transmitting data to users, a cooperative multicast protocol based upon two concatenated, identical phases, is established (see Fig. 1) [2], [3]. Phase 1 of the protocol is used by the source node to broadcast data. However, considering independently faded source-to-users channels, only a subset $\mathcal{S}_1 \subset \mathcal{N}$ of users is able to reliably decode data during this phase (henceforth, this subset will be referred to as decoding set of phase 1). Next, phase 2 is utilized by the set of nodes $\mathcal{S}_1$ to relay the multicast data, using a distributed space-time code (DSTC) [11] and, thus, creating spatial diversity. Relaying can consist of the transmission of either a copy of the original message or of new parity bits, according to whether MRC or IR schemes are used (see below). The receiving nodes in phase 2, i.e., $\mathcal{R} = \mathcal{N} - \mathcal{S}_1$, attempt to decode by combining the received signal during the two phases. As previously, a subset of users $\mathcal{S}_2$ will be able to decode, and the outage event (denoted by $\mathcal{O}$) occurs whenever $S_1 \cup S_2 \neq \mathcal{N}$.

As anticipated, we investigate two approaches for signal transmission and combining: MRC and IR.

1) With **MRC**, the DSTC codeword carries a copy of the signal originally transmitted by the source. At the receiver end, nodes belonging to $\mathcal{R}$ combine the signal received during the two phases using a maximal ratio combiner.

   To implement this transmission strategy we use the following coding scheme: the source encodes information messages into an independently, identically distributed (i.i.d.) Gaussian codebook $\mathcal{X} \in \mathrm{C}^{\frac{n}{2}}$ with an encoding function $x(\omega) \in \mathcal{X}$, where message $\omega \in \left[1, \lceil 2^{nR} \rceil\right]$ is the transmitted message (n is the total number of transmitted symbols during the two phases and R the multicast rate). Receiving nodes attempt to decode, but just a subset $\mathcal{S}_1$ can do so, being:

$$\mathcal{S}_1 = \{i \in \mathcal{N} : \mathcal{I}(X; Y_{i1}) \geq \mathrm{R}\}, \tag{2}$$

   where Y_{i1} is the received signal at node i during phase 1. Once decoded $x(\omega)$, nodes in $\mathcal{S}_1$ generate the linear DSTC codeword $\mathbf{x}_{\mathrm{DSTC}}(\omega)$ using a linear transformation $\mathcal{L} : \mathrm{C}^{\frac{n}{2}} \rightarrow \mathrm{C}^{|\mathcal{S}_1| \times \frac{n}{2}}$ on $x(\omega)$ [12, Chapter 7]. The codeword[2] is then jointly sent during phase 2. Finally, users in $\mathcal{R}$ attempt to decode in two steps: first they apply an inverse linear transformation to its received signal during phase 2 in order to decode the DSTC codeword; then, they combine the output with the received signal during phase 1 using MRC. Thus, node $i \in \mathcal{R}$ will be able to decode data if and only if:

$$\mathrm{R} \leq \frac{1}{2}\mathcal{I}(X; Y_{i1}, Y_{i2}), \tag{3}$$

   where factor $\frac{1}{2}$ follows from the time-division nature of the protocol, and Y_{i2} the received signal at node i during phase 2.

2) With **IR**, the DSTC codeword carries extra parity bits of the codeword transmitted by the source [14]. The received mutual information at node $i \in \mathcal{R}$, at the end of phase 2, can be proved to be equal to the sum of the received mutual information during the two phases.

   The coding scheme is as follows: the source encodes information messages into a i.i.d Gaussian codebook $\mathcal{X} \in \mathrm{C}^n$ with a function $x(\omega) \in \mathcal{X}$, where $\omega \in \left[1, \lceil 2^{nR} \rceil\right]$ is the transmitted message. The codeword $x(\omega)$ is divided into two independent blocks of length $\frac{n}{2}$, i.e., $x(\omega) = [x_1(\omega), x_2(\omega)]$. The first block is transmitted by the source during phase 1, and decoded by the set $\mathcal{S}_1$ that satisfies:

$$\mathcal{S}_1 = \{i \in \mathcal{N} : \mathcal{I}(X_1; Y_{i1}) \geq \mathrm{R}\}. \tag{4}$$

   Later, once decoded ω and knowing the encoding function $x(\omega)$, nodes in $\mathcal{S}_1$ calculate $x_2(\omega)$. Then, they linearly transform it using $\mathcal{L} : \mathrm{C}^{\frac{n}{2}} \rightarrow \mathrm{C}^{|\mathcal{S}_1| \times \frac{n}{2}}$ to obtain the DSTC codeword $\mathbf{x}_{\mathrm{DSTC}}(\omega)$, that is jointly transmitted in phase 2.

   Nodes in $\mathcal{R}$ receive in two steps: first the linearly transform their input during phase 2 in order to decode the DSTC codeword; then, they concatenate the received signals during the two phases to decode $x(\omega)$. Hence, node $i \in \mathcal{R}$ will decode data free of errors if and only if:

$$\begin{aligned} \mathrm{R} &\leq \frac{1}{2}\mathcal{I}(X_1, X_2; Y_{i1}, Y_{i2}) \\ &= \frac{1}{2}\mathcal{I}(X_1; Y_{i1}) + \frac{1}{2}\mathcal{I}(X_2; Y_{i2}), \end{aligned} \tag{5}$$

   where second equality follows from memoryless transmission within the network.

We constrain the per-phase transmitted power to P. However, during phase 2, the power constraint takes different forms depending on the amount of channel state information. As previously pointed out, our analysis considers two CSI cases: the no transmit CSI case and the broadcast CSI.

- For the latter, $\mathcal{S}_1$ is known at the source node, and then an instantaneous power constraint during phase 2 is forced by setting the power transmitted by $n \in \mathcal{S}_1$ to be $\mathrm{P}_n = \frac{\mathrm{P}}{|\mathcal{S}_1|}$, where $|\mathcal{S}_1|$ is the cardinality of set $\mathcal{S}_1$.
- On the other hand, for the no transmit CSI case, $\mathcal{S}_1$ is unknown and random at the transmitted node, as discussed in [5]. Therefore, an instantaneous power constraint can not be considered. In contrast, an average power constraint is enforced by setting $\mathrm{P}_n = \frac{\mathrm{P}}{\mathcal{E}\{|\mathcal{S}_1|\}}$[3] for $n \in \mathcal{S}_1$, and thus $\mathcal{E}\left\{\sum_{n \in \mathcal{S}_1} \mathrm{P}_n\right\} = \mathrm{P}$.

[2]DSTC design is beyond of the scope of the paper. We address the interested reader to [13].

[3]$\mathcal{E}\{\cdot\}$ denotes expectation.

The signal received at the multicast users during the two phases is written as:

$$\begin{aligned} Y_{i1} &= \mathrm{a}_{s,i} \cdot X_s + Z_i,\ i \in \mathcal{N} \\ Y_{i2} &= \sum_{n \in \mathcal{S}_1} \mathrm{a}_{n,i} \cdot X_n + Z_i,\ i \in \mathcal{R} \end{aligned} \tag{6}$$

where $\mathrm{a}_{s,i} \sim \mathcal{CN}(0,1)$ and $\mathrm{a}_{n,i} \sim \mathcal{CN}(0,1)$ are the fading coefficients between source node and receiver i, and node n to receiver i, respectively, assumed independently distributed and invariant during the entire frame duration. $Z_i \sim \mathcal{CN}(0,1)$ is AWGN at the receiver end i, with normalized power $\sigma_o^2 = 1$. Finally, $X_s \sim \mathcal{CN}(0, \mathrm{P})$ is the complex Gaussian codeword transmitted by the source and $\left[X_1, \cdots, X_{|\mathcal{S}_1|}\right] \sim \mathcal{CN}\left(0, \frac{\mathrm{P}}{\alpha} \cdot \mathbf{I}\right)$ the complex Gaussian DSTC transmitted by $\mathcal{S}_1$ during phase 2. The normalization constant reads $\alpha = |\mathcal{S}_1|$ for the broadcast CSI and $\alpha = \mathcal{E}\{|\mathcal{S}_1|\}$ for the no CSI case.

According to (6), the mutual information at the receiver nodes at the end of phase 1 and 2 respectively is computed as:

$$\mathcal{I}_{i1} = \frac{1}{2}\log_2\left(1 + |\mathrm{a}_{s,i}|^2 \mathrm{P}\right),\ i \in \mathcal{N} \tag{7}$$

$$\mathcal{I}_{i2} = \frac{1}{2}\log_2\left(1 + \mathcal{H}_{i2} \cdot \mathrm{P}\right),\ i \in \mathcal{R} \tag{8}$$

where the equivalent channel gain $\mathcal{H}_{2i}$ is defined for the MRC and IR combination modes as follows:

$$\text{MRC}:\ \mathcal{H}_{i2} = |\mathrm{a}_{s,i}|^2 + \sum_{n \in \mathcal{S}_1} \frac{|\mathrm{a}_{n,i}|^2}{\alpha} \tag{9}$$

$$\text{IR}:\ \mathcal{H}_{i2} = |\mathrm{a}_{s,i}|^2 + \sum_{n \in \mathcal{S}_1} \frac{|\mathrm{a}_{n,i}|^2}{\alpha} + |\mathrm{a}_{s,i}|^2 \sum_{n \in \mathcal{S}_1} \frac{|\mathrm{a}_{n,i}|^2}{\alpha} \cdot \mathrm{P} \tag{10}$$

Notice that (9) and (10) comes out directly when applying the mutual information in (3) and (5), respectively, to Gaussian channels in (6).

Throughout the paper, we consider the multicast outage probability and the multicast outage capacity as the performance metric of the network. Both are defined as follows:

Definition 1: The *multicast outage probability* is the probability that the transmitted message with a given rate R [bps/Hz], is not correctly decoded by at least one multicast user at the end of both phases, i.e.,

$$P_o(\mathrm{R}) = Pr\{\mathcal{O}|\mathrm{R}\}\ . \tag{11}$$

Definition 2: The *multicast outage capacity* at the outage level p is the maximum transmission rate R [bps/Hz] that guarantees a multicast outage probability lower than or equal to p, i.e,

$$\mathcal{C}_o(p) = \max\{\mathrm{R} : P_o(\mathrm{R}) \leq p\}\ . \tag{12}$$

III. Cooperative multicasting without CSI

We first consider a cooperative multicast network without transmit CSI neither at the base-station, nor at the users. In this scenario, the decoding set during phase 1, i.e., $\mathcal{S}_1$, is unknown and random. Therefore, the multicast outage probability for a given multicast transmission rate R [bps/Hz] is:

$$P_o(\mathrm{R}) = \sum_{\mathcal{S}_1} Pr\{\mathcal{S}_1\} \cdot Pr\{\mathcal{O}|\mathrm{R}, \mathcal{S}_1\}, \tag{13}$$

where $Pr\{\mathcal{S}_1\}$ is the probability of a given decoding set $\mathcal{S}_1$ during phase 1 to occur and $Pr\{\mathcal{O}|\mathrm{R}, \mathcal{S}_1\}$ the outage probability at phase 2 given such decoding set at phase 1. The former can be computed from the mutual information in (7) as:

$$Pr\{\mathcal{S}_1\} = \prod_{i \in \mathcal{S}_1} Pr\{\mathcal{I}_{i1} \geq \mathrm{R}\} \prod_{i \notin \mathcal{S}_1} Pr\{\mathcal{I}_{i1} < \mathrm{R}\}. \tag{14}$$

Furthermore, noting that $|\mathrm{a}_{s,i}|^2$ is a unitary-mean exponential random variable with $Pr\{\mathcal{I}_{i1} \geq \mathrm{R}\} = e^{-\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}}$. Then, we rewrite (14) as:

$$Pr\{\mathcal{S}_1\} = \left(e^{-\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}}\right)^{|\mathcal{S}_1|} \left(1 - e^{-\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}}\right)^{N-|\mathcal{S}_1|}. \tag{15}$$

The computation of $Pr\{\mathcal{O}|\mathrm{R}, \mathcal{S}_1\}$ requires an independent analysis for the IR and MRC combination modes, due to the different $\mathcal{H}_{i2}$ in the resultant mutual information at the receiving nodes of the second phase in (8):

$$\begin{aligned} Pr\{\mathcal{O}|\mathrm{R}, \mathcal{S}_1\} &= 1 - \prod_{i \notin \mathcal{S}_1} \left(1 - Pr\{\mathcal{I}_{i2} < \mathrm{R}\}\right) \\ &= 1 - \prod_{i \notin \mathcal{S}_1} \left(1 - Pr\left\{\mathcal{H}_{i2} < \frac{2^{2\mathrm{R}}-1}{\mathrm{P}}\right\}\right). \end{aligned} \tag{16}$$

A. MRC scheme

With MRC, the equivalent channel gain $\mathcal{H}_{i2}$ is defined in (9) with $\alpha = \mathcal{E}\{|\mathcal{S}_1|\}$. Randomness of $\mathcal{H}_{i2}$ is then characterized in terms of the distributions of $|\mathrm{a}_{s,i}|^2$ and $\sum_{n \in \mathcal{S}_1} |\mathrm{a}_{n,i}|^2$. First, we notice that $|\mathrm{a}_{s,i}|^2 \sim \mathcal{X}_2^2$ is a unitary-mean exponential random variable. However, we are considering the source-to-node i fading channel, given that i does not belong to the decoding set of phase 1. Therefore, we need the probability density function (pdf) of $|\mathrm{a}_{s,i}|^2$ conditioned on $i \notin \mathcal{S}_1$, which is given by:

$$f_{|\mathrm{a}_{s,i}|^2}(\mathrm{a}) = \begin{cases} 0 & \text{for } \mathrm{a} \geq \frac{2^{2\mathrm{R}}-1}{\mathrm{P}} \\ \frac{e^{-\mathrm{a}}}{1 - e^{-\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}}} & \text{for } 0 \leq \mathrm{a} < \frac{2^{2\mathrm{R}}-1}{\mathrm{P}} \end{cases} \tag{17}$$

Furthermore, $\sum_{n \in \mathcal{S}_1} |\mathrm{a}_{n,i}|^2 \sim X^2_{2|\mathcal{S}_1|}$ is a chi-square distributed random variable with $2 \cdot |\mathcal{S}_1|$ degrees of freedom. Hence, its pdf is given by $f_{\mathcal{X}^2_{2|\mathcal{S}_1|}}(\mathrm{x}) = \frac{1}{|\mathcal{S}_1|-1} \mathrm{x}^{|\mathcal{S}_1|-1} e^{\mathrm{x}}$, and its cumulative density function (cdf) by the regularized incomplete Gamma function[4], $F_{\mathcal{X}^2_{2|\mathcal{S}_1|}}(b) = \gamma(|\mathcal{S}_1|, b)$. Thus, we have:

$$\begin{aligned} &Pr\left\{\mathcal{H}_{i2} < \frac{2^{2\mathrm{R}}-1}{\mathrm{P}}\right\} = \\ &\qquad Pr\left\{|\mathrm{a}_{s,i}|^2 + \sum_{n \in \mathcal{S}_1} \frac{|\mathrm{a}_{n,i}|^2}{\mathcal{E}\{|\mathcal{S}_1|\}} < \frac{2^{2\mathrm{R}}-1}{\mathrm{P}}\right\} \\ &= \int_0^{\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}} f_{|\mathrm{a}_{s,i}|^2}(\mathrm{a}) \int_0^{\left(\frac{2^{2\mathrm{R}}-1}{\mathrm{P}} - \mathrm{a}\right)\mathcal{E}\{|\mathcal{S}_1|\}} f_{\mathcal{X}^2_{2|\mathcal{S}_1|}}(\mathrm{x})\, d\mathrm{a} d\mathrm{x} \end{aligned} \tag{18}$$

Computation in (18) has been obtained considering the sum of independent random variables. Finally, making use of the

[4]The regularized incomplete Gamma function is defined as $\gamma(\mathrm{n}, b) = \frac{1}{(\mathrm{n}-1)!}\int_0^b \mathrm{x}^{\mathrm{n}-1} e^{\mathrm{x}}$

$$P_o(\mathrm{R}) = \sum_{\kappa=0}^{N} \binom{N}{\kappa} \left(e^{-\frac{2^{2\mathrm{R}-1}}{\mathrm{P}}}\right)^{\kappa} \left(1-e^{-\frac{2^{2\mathrm{R}-1}}{\mathrm{P}}}\right)^{N-\kappa} \left(1-\left(1-\int_0^{\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}} \frac{e^{-\mathrm{a}}}{1-e^{-\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}}} \gamma\left(\kappa, \left(\tfrac{2^{2\mathrm{R}}-1}{\mathrm{P}}-\mathrm{a}\right)\mathcal{E}\{|\mathcal{S}_1|\}\right) d\mathrm{a}\right)\right)^{N-\kappa} \quad (20)$$

$$P_o(\mathrm{R}) = \sum_{\kappa=0}^{N} \binom{N}{\kappa} \left(e^{-\frac{2^{2\mathrm{R}-1}}{\mathrm{P}}}\right)^{\kappa} \left(1-e^{-\frac{2^{2\mathrm{R}-1}}{\mathrm{P}}}\right)^{N-\kappa} \left(1-\left(1-\int_0^{\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}} \frac{e^{-\mathrm{a}}}{1-e^{-\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}}} \gamma\left(\kappa, \frac{\frac{2^{2\mathrm{R}}-(1+\mathrm{a}\cdot\mathrm{P})}{\mathrm{P}}}{\mathcal{E}\{|\mathcal{S}_1|\}\cdot(1+\mathrm{a}\cdot\mathrm{P})}\right) d\mathrm{a}\right)\right)^{N-\kappa} \quad (22)$$

pdf's defined previously, we obtain:

$$Pr\left\{\mathcal{H}_{i2} < \frac{2^{2\mathrm{R}}-1}{\mathrm{P}}\right\} = = \int_0^{\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}} \frac{e^{-\mathrm{a}}}{1-e^{-\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}}} \gamma\left(|\mathcal{S}_1|, \left(\tfrac{2^{2\mathrm{R}}-1}{\mathrm{P}}-\mathrm{a}\right)\mathcal{E}\{|\mathcal{S}_1|\}\right) d\mathrm{a}. \quad (19)$$

Plugging the integral (19) in (16), we can derive the outage probability at phase 2, given a decoding set $\mathcal{S}_1$ during phase 1. Then, putting together (15) with (16) into (13), we compute the outage probability of the cooperative multicasting with MRC as (20). Notice that, to derive the combinatorial in (20), we use the fact that for two different decoding sets during phase 1, $\mathcal{S}_1 = \mathcal{A}$ and $\mathcal{S}_1 = \mathcal{B}$, the probabilities $Pr\{\mathcal{A}\} \cdot Pr\{\mathcal{O}|\mathrm{R},\mathcal{A}\} = Pr\{\mathcal{B}\} \cdot Pr\{\mathcal{O}|\mathrm{R},\mathcal{B}\}$ if $|\mathcal{A}| = |\mathcal{B}|$. Finally, the outage capacity of the proposed cooperative multicasting with MRC transmission is obtained by inverting equation (20) following (12).

B. IR scheme

With IR, the equivalent channel gain $\mathcal{H}_{i2}$ in (16) is obtained from (10). Therefore, the outage probability of user $i \in \mathcal{R}$ is computed as:

$$Pr\left\{\mathcal{H}_{i2} < \frac{2^{2\mathrm{R}}-1}{\mathrm{P}}\right\} = \int_0^{\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}} f_{|\mathrm{a}_{s,i}|^2}(\mathrm{a}) \int_0^{\frac{\frac{2^{2\mathrm{R}}-(1+\mathrm{a}\cdot\mathrm{P})}{\mathrm{P}}}{\mathcal{E}\{|\mathcal{S}_1|\}\cdot(1+\mathrm{a}\cdot\mathrm{P})}} f_{\mathcal{X}^2_{2|\mathcal{S}_1|}}(\mathrm{x})\, d\mathrm{a} d\mathrm{x}.$$
$$= \int_0^{\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}} \frac{e^{-\mathrm{a}}}{1-e^{-\frac{2^{2\mathrm{R}}-1}{\mathrm{P}}}} \gamma\left(|\mathcal{S}_1|, \frac{\frac{2^{2\mathrm{R}}-(1+\mathrm{a}\cdot\mathrm{P})}{\mathrm{P}}}{\mathcal{E}\{|\mathcal{S}_1|\}\cdot(1+\mathrm{a}\cdot\mathrm{P})}\right) d\mathrm{a}, \quad (21)$$

The outage probability, conditioned to a decoding set $\mathcal{S}_1$, is achieved inserting (21) into (16). Finally, the multicast outage probability is derived putting together (15) with (16) into (13). The resulting outage probability is expressed in (22), and the outage capacity obtained by inverting such equation using numerical methods.

IV. Cooperative Multicasting with Broadcast CSI

We analyze now a multicast network where the source node has source-to-users transmit channel knowledge. Moroever, we assume that the source, and the decoding set $\mathcal{S}_1$, are unaware of the user-to-user transmit channel coefficients.

Transmit channel knowledge is commonly used in wireless communications to optimally allocate resources. In our approach, we have forced the transmit power to be P and the slot interval to be a half of the frame duration. Thus, those parameters may not be taken into account in optimization. However, the source still can exploit one more degree of freedom in order to optimize performance: adaptive modulation. In non-cooperative multicasting, adaptive modulation is able to overcome fading, an thus eliminate outage, by adapting the multicast transmission rate R to be

$$\mathrm{R} \le \min_{1\le i\le N} \log_2\left(1+|\mathrm{a}_{s,i}|^2\mathrm{P}\right), \quad (23)$$

which is referred to as zero-outage capacity. On the contrary, in cooperative multicasting, outage can not be eliminated through adaptive modulation since the user-to-user channels are still unknown within the network; nevertheless, it can be used to optimally select the decoding set during phase 1 and thus to minimize the outage probability during the second. The optimal way for the source to implement such a selection is shown in [3, Proposition 1], which is revisited here. Due to the above mentioned fact, the outage capacity (and probability) of cooperative multicasting is a function of the channel knowledge at the source, i.e., the source-to-users channels.

Let us consider first, without loss of generality, that the base station orders the multicast users according to:

$$|\mathrm{a}_{s,1}| \ge ... \ge |\mathrm{a}_{s,\kappa}| \ge ... \ge |\mathrm{a}_{s,N}|, \quad (24)$$

and define $\mathcal{S}_1^{\kappa}$ as the set of nodes $[1,\cdots,\kappa]$.

***Proposition** 1: The outage capacity of a two-phase space-time coded cooperative multicasting with broadcast CSI is given by* [3]

$$\mathcal{C}_o(p) = \max_{1\le\kappa\le N-1} \min\left\{\mathcal{C}_1^{\kappa}, \mathcal{C}_2^{\kappa}(p)\right\}, \quad (25)$$

where

$$\mathcal{C}_1^{\kappa} := \frac{1}{2}\log_2\left(1+|\mathrm{a}_{s,\kappa}|^2\mathrm{P}\right) \quad (26)$$
$$\mathcal{C}_2^{\kappa}(p) := \max\left\{\mathrm{R} : Pr\{\mathcal{O}|\mathrm{R},\mathcal{S}_1^{\kappa}\} \le p\right\}. \quad (27)$$

Proof: see [3] for a proof. §

When interpreting (25), it should be noted that $\min\{\mathcal{C}_1^{\kappa}, \mathcal{C}_2^{\kappa}(p)\}$ is the outage capacity when the source selects the set $\mathcal{S}_1^{\kappa}$ to decode during phase 1. $\mathcal{C}_1^{\kappa}$ is the maximum reliable rate at which the base station communicates with arbitrary small error probability with $\mathcal{S}_1^{\kappa}$. Moreover, $\mathcal{C}_2^{\kappa}(p)$ is the maximum transmission rate during phase 2 that, given the decoding set $\mathcal{S}_1^{\kappa}$, guarantees an outage level lower than or equal to p. Hence, the minimum of both determines the outage capacity for $\mathcal{S}_1^{\kappa}$, and the minimization over κ, the optimum decoding set.

We apply separately Proposition 1 to the MRC and IR strategies.

A. MRC scheme

As mentioned above, the term $\mathcal{C}_2^{\kappa}(p)$ is a function of the conditioned outage probability $Pr\{\mathcal{O}|\mathrm{R},\mathcal{S}_1^{\kappa}\}$. Such probability is computed from equation (16), which in turn depends upon variable $\mathcal{H}_{i2}$ in (9) with $i \in \mathcal{R} = \mathcal{N} - \mathcal{S}_1^\kappa$. In contrast to the no CSI case, the broadcast CSI case entails that $|\mathrm{a}_{s,i}|^2$ is random but known at the source. Therefore, the only uncertainty within the network is the space-time coded gain[5] during phase 2, i.e, $\frac{\sum_{n\in\mathcal{S}_1^\kappa}|\mathrm{a}_{n,i}|^2}{\kappa}$. Hence, the probability:

$$Pr\left\{\mathcal{H}_{i2} < \frac{2^{2\mathrm{R}}-1}{\mathrm{P}}\right\} = Pr\left\{\sum_{n\in\mathcal{S}_1^\kappa}|\mathrm{a}_{n,i}|^2 < \left(\tfrac{2^{2\mathrm{R}}-1}{\mathrm{P}} - |\mathrm{a}_{s,i}|^2\right)\kappa\right\} \quad (28)$$
$$= \gamma\left(\kappa, \left(\frac{2^{2\mathrm{R}}-1}{\mathrm{P}} - |\mathrm{a}_{s,i}|^2\right)\kappa\right).$$

Introducing the equation above in (16), and applying definition (27), we obtain:

$$\mathcal{C}_2^\kappa(p) = \max\left\{\mathrm{R} : 1 - \left(1 - \gamma\left(\kappa, \left(\tfrac{2^{2\mathrm{R}}-1}{\mathrm{P}} - |\mathrm{a}_{s,i}|^2\right)\kappa\right)\right)^{N-\kappa} \le p\right\}. \quad (29)$$

The outage capacity is derived applying (25).

B. IR scheme

With IR, the equivalent channel gain $\mathcal{H}_{i2}$ is defined as in (10), and therefore:

$$Pr\left\{\mathcal{H}_{i2} < \frac{2^{2\mathrm{R}}-1}{\mathrm{P}}\right\} = Pr\left\{\sum_{n\in\mathcal{S}_1^\kappa}|\mathrm{a}_{n,i}|^2 < \frac{2^{2\mathrm{R}} - \left(1+|\mathrm{a}_{s,i}|^2\mathrm{P}\right)}{\frac{\mathrm{P}}{\kappa}\left(1+|\mathrm{a}_{s,i}|^2\mathrm{P}\right)}\right\}$$
$$= \gamma\left(\kappa, \frac{2^{2\mathrm{R}} - \left(1+|\mathrm{a}_{s,i}|^2\mathrm{P}\right)}{\frac{\mathrm{P}}{\kappa}\left(1+|\mathrm{a}_{s,i}|^2\mathrm{P}\right)}\right).$$

Finally, considering definition (27) we derive:

$$\mathcal{C}_2^\kappa(p) = \max\left\{\mathrm{R} : 1 - \left(1 - \gamma\left(\kappa, \tfrac{2^{2\mathrm{R}} - \left(1+|\mathrm{a}_{s,i}|^2\mathrm{P}\right)}{\frac{\mathrm{P}}{\kappa}\left(1+|\mathrm{a}_{s,i}|^2\mathrm{P}\right)}\right)\right)^{N-\kappa} \le p\right\}, \quad (30)$$

while the outage capacity is obtained through (25), and the outage probability inverting the function according to (12).

V. Numerical results

In this section, we evaluate the outage capacity of the proposed cooperative multicasting under both CSI scenarios of no transmit CSI and broadcast CSI. Moreover, the non-cooperative case is also considered for comparison. For convenience of representation, we normalize the outage capacity in all plots with respect to the bound in (1).

Fig. 2 and Fig. 3 depict the outage capacity of the proposed protocol versus the total number of multicast nodes. A transmit signal-to-noise ratio (SNR) equal to 10 dB, i.e., $\frac{\mathrm{P}}{\sigma_o^2}$, and an outage level $p = 10^{-1}$ are considered. On the first one, results for the no transmit CSI case are shown. Firstly, we notice that the protocol clearly outperforms non-cooperative systems. However, for number of nodes ≤ 100, it only reaches a 45% of the capacity bound proposed in [2]. As expected, the outage capacity grows with N, but not fast, and the use of IR only increases performance by a 5% with respect to that of MRC mode. On Fig 3, the broadcast CSI case is considered. We plot the expected value of the outage capacity of cooperative multicasting, averaged over the source-to-users channel distribution. Moreover, the expected zero-outage capacity (23) of non cooperative multicasting with broadcast is shown for comparison. Here, we notice that cooperation is

[5]For the broadcast CSI case, $\alpha = |\mathcal{S}_1^\kappa| = \kappa$.

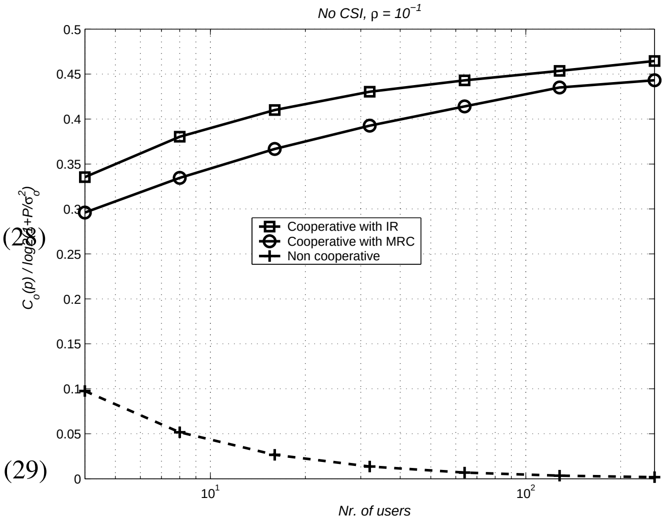

Fig. 2. Outage capacity of cooperative and non-cooperative multicasting with no transmit CSI versus the total number of network users. Transmit SNR = 10 dB is assumed and a multicast outage level $p = 10^{-1}$.

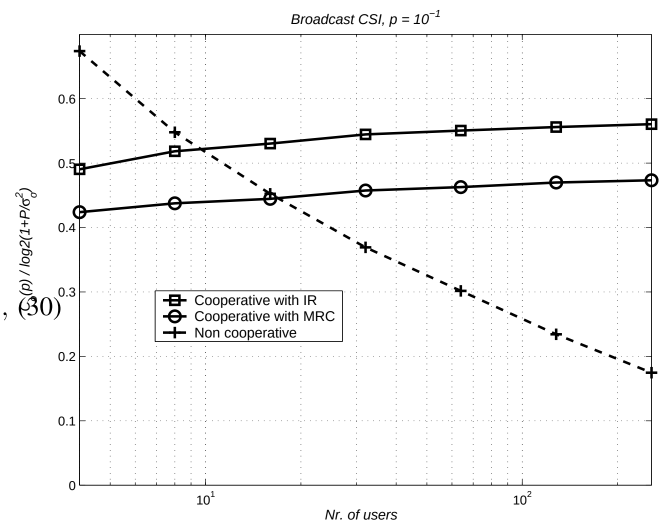

Fig. 3. Outage capacity of cooperative multicasting with broadcast CSI versus the total number of network users. The zero-outage capacity of non-cooperative multicasting is plotted for comparison. Transmit SNR = 10 dB is assumed and a multicast outage level $p = 10^{-1}$.

only worthwhile for $N \ge 10$. Moreover, almost 55% of the bound is reached with cooperation and the gain between IR and MRC modes has increased up to 8%.

Fig. 4 and Fig. 5 compare the outage capacity of cooperative and non-cooperative multicasting versus the total transmit power. Again, $p = 10^{-1}$ is considered, and we define a network with 50 users. For the no CSI case (Fig. 4), we realize that the percentage of bound (1) reached by the protocol increases with the transmitted power. Moreover, we clearly notice that the difference between IR and MRC also increases. On the contrary, results for the broadcast CSI scenario (Fig. 5) shows that cooperation is only advantageous in the low SNR regime. This can be explained noting that decode-and-forward relaying is considered in the protocol, which is more spectral efficient for low power budget [15]. Finally, we notice that with channel knowledge the variation with the SNR is almost

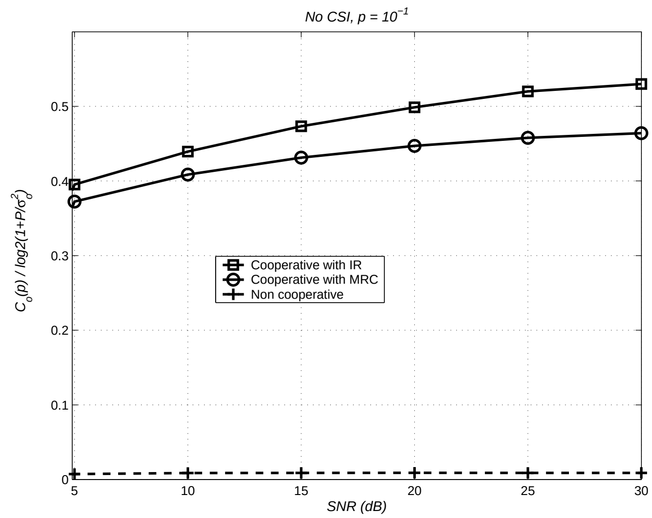

Fig. 4. Outage capacity of cooperative and non-cooperative multicasting with no transmit CSI versus transmit SNR. 50 multicast users are assumed and a multicast outage level $p = 10^{-1}$.

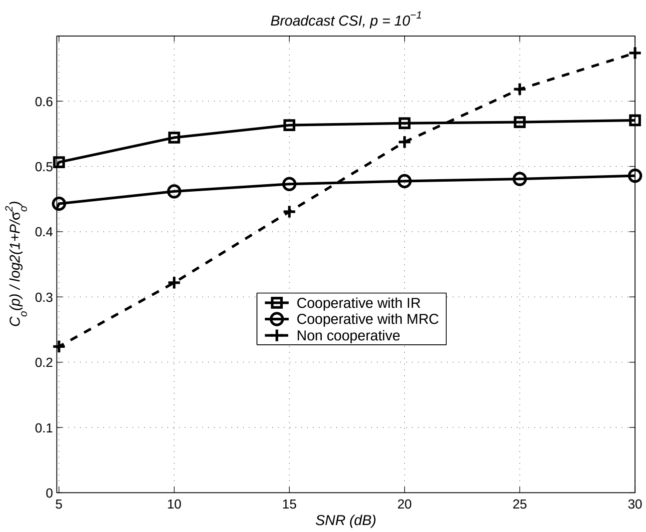

Fig. 5. Outage capacity of cooperative multicasting with broadcast CSI versus transmit SNR. The zero-outage capacity of non-cooperative multicasting is plotted for comparison. 50 multicast users are assumed and a multicast outage level $p = 10^{-1}$.

flat.

VI. Conclusions

This paper studied the outage performance of a two-phase, space-time coded, cooperative multicasting. We considered two transmission and combination modes: Maximal Ratio Combining (MRC) and Incremental Redundancy (IR). Furthermore, we analyzed the protocol under two different CSI assumptions: *i*) no transmit CSI within the network and *ii*) broadcast transmit CSI at the source. Results showed that: 1) cooperation clearly outperfoms non-cooperative multicasting for the no CSI case, yielding outage capacity gains up to 40%, 2) on the contrary, for the broadcast CSI case, cooperation is only worthwhile for low SNR and high number of users, 3) IR performs 5-10 % better than MRC.

References

[1] N. Jindal and Z.Q. Luo, "Capacity limits of multiple antenna multicast," in *Proc. International Symposium on Information Theory*, Cambridge, USA, Jul. 2006, pp. 1841–1845.

[2] A. Khisti, U. Erez, and G.W. Wornell, "Fundamental limits and scaling behavior of cooperative multicasting in wireless networks," *to appear IEEE Trans. on Information Theory*, 2006.

[3] A. del Coso, O. Simeone, Y. Bar-ness, and C. Ibars, "Outage capacity of two-phase space-time coded cooperative multicasting," in *Proc.* 40^{th} *Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, USA, Nov. 2006.

[4] A. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *Proc. International Symposium on Information Theory*, Cambridge, USA, Jul. 1998, p. 156.

[5] J.N. Laneman and G. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. on Information Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[6] R. Nabar, H. Bölcskei, and F.W. Kneuböhler, "Fading relay channels: performance limits and space-time signal design," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.

[7] T. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. on Information Theory*, vol. 525, no. 5, pp. 572–584, Sep. 1979.

[8] A. El Gamal, M. Mohseni, and S. Zahedi, "Bounds on capacity and minimum energy-per-bit for AWGN relay channels," *IEEE Trans. on Information Theory*, vol. 52, no. 4, pp. 1545–1561, Apr. 2006.

[9] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. on Information Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.

[10] I. Maric and R.D. Yates, "Cooperative multicast for maximum network lifetime," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 1, pp. 127–135, Jan 2005.

[11] L. Lampe, R. Schober, and S. Yiu, "Distributed space-time coding for multihop transmission in power line communication networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 7, pp. 1389–1400, Jul. 2006.

[12] E.G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*, 1^{st} Edition, Cambridge University Press, 2003.

[13] B. Sirkeci-Mergen and A. Scaglione, "Randomized distributed space-time coding for cooperative communication in self organized networks," in *Proc. IEEE* 6^{th} *Workshop on Signal Processing Advances in Wireless Communications*, Philadelphia, USA, Mar. 2005, pp. 500–504.

[14] S. Sesia, G. Caire, and G. Vivier, "Incremental redundancy hybrid ARQ schemes based on low-density parity-check codes," *IEEE Trans. on Communications*, vol. 52, no. 8, pp. 1311–1321, Aug. 2004.

[15] D. Tse, "Cooperative communication: diversity, freedom and energy," in *MSRI Workshop on Mathematics of Relaying and Cooperation in Communication Networks*, Berkeley, CA, USA, Apr. 2006.